SPECIAL PROJECT FINAL REPORT

All the following mandatory information needs to be provided.

Direct numerical simulation of wind wave fields in a rapidly changing environment
SPGBSHRI
2016 – 2018
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The following should cover the entire project duration.

Summary of project objectives

(10 lines max)

This project aims at developing new models and algorithms for wind wave modelling, applicable for situations with fast changes of the environment, which are beyond the limits of applicability of the classical kinetic equation. Rapid changes of wind can lead to the increased probability of extreme wave events. The generalized kinetic equation (GKE) and direct numerical simulations (DNS) of wind wave fields, based on the Zakharov equation, allow tracing the evolution of spectra and higher-order moments of the field for many thousands of wave periods (at least an order of magnitude longer than with other DNS approaches). In this project, a direct comparison of the DNS and the GKE is performed, with the aim to understand the role of coherent processes, resolved by the DNS but completely filtered out in the statistical approach. These processes result in the difference in the timescales of the evolution of spectra, and affect higher statistical moments.

Summary of problems encountered

(If you encountered any problems of a more technical nature, please describe them here.)

No particular problems encountered

Experience with the Special Project framework

(Please let us know about your experience with administrative aspects like the application procedure, progress reporting etc.)

More than 10 years of continuous experience of all aspects special projects application procedures and reporting

Summary of results

(This section should comprise up to 10 pages, reflecting the complexity and duration of the project, and can be replaced by a short summary plus an existing scientific report on the project.)

In this project, we study numerically the long-term evolution of water wave spectra using three different models, aiming at understanding the role of different sets of assumptions. The first model is the classical Hasselmann kinetic equation (KE). Numerical simulations of long time evolution of random water wave fields are almost overwhelmingly based upon this model. The KE is based on two key assumptions: quasi-gaussianity, employed in the statistical closure, and quasi-stationarity, implied by the large-time limit. The specific role of each of these assumptions in the discrepancies between the kinetic theory and the reality is not clear. Since the spectral evolution occurs only when a wave field manifests a departure from gaussianity, the assumption of quasi-gaussianity implies that the evolution is slow. However, it is reasonable to assume that during a rapid transformation of a wave field (e.g. due to a rapid change of environment) both these assumptions, especially that of quasi-stationarity, can be violated.

Two other models are new. The second model is the generalised kinetic equation (gKE), derived without the assumption of quasi-stationarity. Thus, unlike the KE, the gKE is valid when a wave spectrum is changing rapidly (e.g.\ at the initial stage of evolution of a narrow spectrum). However, the gKE employs the same statistical closure as the KE.

The third model is based on the Zakharov integrodifferential equation for water waves and does not depend on any statistical assumptions. Since the Zakharov equation plays the role of the primitive equation of the theory of wave turbulence, we refer to this model as direct numerical simulation of spectral evolution (DNS-ZE). The DNS-ZE method allows to study long-term spectral evolution (up to $O(10^4)$ periods), which was previously possible only with the KE. Thus, we are able to perform a direct comparison of spectral evolution with and without the statistical closure.

The project was developed along a few different but closely related lines.

I. Spectral evolution of weakly nonlinear random waves

Our target is to trace the evolution of initially narrow (both in frequency and angle) spectra subjected only to dissipation at high frequencies. Such spectra are far from equilibrium, and at the initial stage of evolution they undergo relatively fast broadening, which is likely to put them at, or beyond, the limit of applicability of the quasi-gaussianity and quasi-stationarity assumptions, employed in the derivation of the classical kinetic equations. Thus, a meaningful comparison of spectral evolutions obtained with different sets of assumptions becomes possible.



Figure 1: Short-term evolution of the initial spectra (a) A and (b) B, with a direct comparison of five different numerical approaches. Initial (black solid curve) and final (after 150 wave periods) omnidirectional energy spectra S(k) are plotted. Evolution is simulated the high-order spectral method (HOS); Dysthe equation (BMNLS, both from Xiao et al 2013, cf their figure 7); DNS based on the Zakharov equation (DNS-ZE); standard kinetic equation, WRT algorithm, KE(WRT); and generalized KE (gKE)

As initial conditions, we choose two narrow-banded spectra with identical frequency distributions (a JONSWAP spectrum with high peakedness $\gamma = 6$) and different directional distributions. Spectrum A is very narrow in angle (corresponding to N = 840 in the \cos^N directional model), while spectrum B is initially wider in angle (corresponds to N = 24). The reason for this particular choice of initial conditions is the vast amount of observational and numerical data relevant to these spectra. The same spectra were used as the initial conditions in the experimental study by Onorato et al (2009) and numerical studies by Toffoli et al (2010) and Xiao et al (2013). In particular, Xiao et al (2013) performed numerical simulations of the evolution (only about 150 periods) of the same initial spectra using higher-order spectral method (HOS) and broadband modified nonlinear Schrodinger equation (Dysthe equation, BMNLS). Thus, we can consider the short-term evolution of these spectra (without wind forcing) with five different approaches, based on different sets of assumptions, and use the results for comparison and validation of our algorithms.



Figure 2: Evolution of directional spread θ_m (second moment of directional distribution) of two initial spectra A and B. As in figure 1, a direct comparison of five different numerical approaches is shown

In figure 1a, b evolution of spectra A and B over the first 150 wave periods is shown, with a direct comparison of five numerical approaches: two kinetic equations (KE and gKE), two direct numerical simulations (DNS-ZE and higher-order spectral method (HOS) by Xiao et al (2013) and Dysthe equation (BMNLS, also from Xiao et al 2013). These numerical approaches fall into two categories, KE and gKE being statistical methods based on the same statistical closure, while DNS-ZE, HOS and BMNLS are dynamical methods free from statistical assumptions. To facilitate the comparison, we plot omnidirectional energy-wavenumber spectra S(k), following Xiao et al (2013).

Spectral evolutions predicted with two kinetic equations (KE and gKE) are practically identical for the case B with wider directional distribution, but for the initial spectrum A the KE slightly overestimates the amplitude of the spectral peak. The simulations with three dynamical methods DNS-ZE, HOS and BMNLS are consistent with each other, but different from both kinetic equations. In general, the kinetic equations predict more narrow spectra, with a pronounced overshoot, while the DNS algorithms give wider spectra with lower amplitude of the peak.

In figure 2 we show the evolution of mean directional spread $\theta_m = \overline{\theta_2(k)}$ for both initial spectra, defined, following Xiao et al (2013), as the average of the second moment of the directional distribution

$$\theta_2(k) = \left(\int_0^{\pi/2} \theta^2 D(k,\theta) \,\mathrm{d}\theta\right)^{1/2} \left(\int_0^{\pi/2} D(k,\theta) \,\mathrm{d}\theta\right)^{-1/2},\tag{1}$$

where $D(k, \theta)$ is the angular distribution function (Hwang et al 2010). Figure 2 shows a dramatic difference in the rate of angular broadening, which is consistent between DNS-ZE, HOS and BMNLS and much higher for gKE and KE.

In figure 3a, b we present the long-term evolution of both initial spectra obtained with the three models. Omnidirectional energy-wavenumber spectra are plotted every 150 peak periods of the initial spectrum. Thus, the curves plotted in figure 1 are shown as the first step after the initial condition. As can be seen from the figure, simulation of evolution of the same initial conditions by different models, based on different assumptions, leads to a number of interesting observations.



Figure 3: Long-term evolution of initial spectra A and B, with comparison of three different numerical approaches. (a) Evolution of spectrum A at time t = 0 (black solid curve), 50, 150, 300, 500, 800, 1200 periods (DNS-ZE, gKE and KE), t = 2000 and 3200 periods (DNS-ZE and KE only). (b) Evolution of spectrum B at time t = 0 (black solid curve), 50, 150, 300, 500, 800, 1200, 2000 periods (DNS-ZE, gKE and KE), t = 3200 and 4750 periods (DNS-ZE and KE only)

First, let us compare the two kinetic equations. In the case of spectrum B, which has moderate angular width, both kinetic models lead to nearly identical evolution, at least as far as the wavenumber (or frequency) spectrum is concerned. This is to be expected, since the KE and the gKE spectra for wind-generated waves were previously shown to be very close (Annenkov & Shrira 2016). However, for narrow directional spectra, it is reasonable to expect that the simulation results will be different, since the KE predicts no evolution in the limit of zero angular width, while the gKE is capable of modelling such situations. Indeed, in the case of spectrum A the two kinetic equations noticeably diverge, the spectral peak being appreciably higher in the case of the KE. The DNS spectral evolution in both cases predicts considerably wider spectra with lower peaks.

Although, as noted above, figure 3 demonstrates considerable differences between the predictions of the three employed models, it appears that the evolution of integral characteristics of the spectra (in particular, the wavenumber of the spectral peak, wave steepness, and therefore, the total wave energy) is close for all models. At large time, the evolution tends to be self-similar. From the theory of the KE (e.g. Badulin et al 2005) it is well known that the spectral evolution at large time approaches a self-similar state characterised by the downshift of the spectral peak $|\mathbf{k}_p| \sim t^{-2/11}$ and growth of the peak $n_p \sim t^{4/11}$. The DNS spectra have a peak with considerably lower amplitude than in the case of the KE and gKE spectra, and its time dependence does not tend to the KE asymptotic $n_p \sim t^{4/11}$, but the evolution of other integral characteristics is nearly identical.

The rates of change of the spectra obtained with the DNS-ZE scale as ε^4 , where ε is the wave steepness. This agrees with our earlier findings for waves subjected to wind forcing and corresponds to the dynamical timescale of evolution, not to the kinetic one. This difference of growth rate scaling manifests itself, in particular, in the difference between self-similar spectral shapes and rates of angular broadening.

II. Evolution of higher statistical moments and comparison with experiment

In order to predict not just the average characteristics of wave fields, but also extremes, it is necessary to know the probability density function (p.d.f.) of surface elevations. The p.d.f. can be approximately reconstructed from the wave spectrum and at least two first higher-order moments of a random wave field (skewness and kurtosis, see Janssen 2014). Most theoretical studies of higher-order moments were confined to narrowband wave fields. However, real oceanic wave fields are not narrowband and always have a certain directional distribution. Recently, Annenkov & Shrira (2014) estimated higher-order moments of JONSWAP spectra in the large-time limit, using the theory developed by Janssen (2009). The important question is on the behaviour of higher moments, especially the kurtosis, during short-term evolution in transient fast-changing situations. As it is well-known, the kurtosis evolution for a wave field with one-dimensional spectrum crucially depends on the value of the BFI (Benjamin-Feir index, representing the ratio between nonlinearity and spectral bandwidth). If BFI > 1, then the kurtosis attains large positive values during short-term evolution. Measurements of the short-term kurtosis evolution for wave fields with spectra initially corresponding to JONSWAP form with various (although in all cases relatively narrow) directional distributions were performed by Onorato et al (2009). Numerical simulations by Toffoli et al (2010) and Xiao et al (2013), using high-order spectral method and Dysthe equation, showed good agreement with the experimental results. Annenkov & Shrira (2018), using these numerical and experimental results for the validation of the DNS-ZE algorithm, showed good agreement with the measured and previously simulated kurtosis evolution.



Figure 4: Evolution of the total kurtosis for spectra (a) A and (b) B vs fetch with different numerical approaches, and comparison with observations. Black dashed curve - high-order spectral method (HOS, Xiao et al 2013), red solid curve - DNS based on the Zakharov equation (DNS-ZE), blue dashed curve - gKE. Dotted blue curve shows bound harmonics kurtosis, calculated from the DNS spectra. Black dots - observations of the total kurtosis for the same initial conditions (Toffoli et al 2010)

Results demonstrate the dependence of the kurtosis on directional distribution. In figure 4, we show the comparison of the kurtosis evolution for initial spectra A and B over the first 50 wavelengths (corresponding to 100 wave periods in the duration-limited simulations). At the start of the evolution, the wave field is created as an ensemble of uncorrelated harmonics, and the dynamical kurtosis is zero, so that the total kurtosis is equal to the bound harmonics one. Observations by Toffoli et al (2010) and numerical experiments by Xiao et al (2013) show that the total kurtosis in both cases undergoes a rapid (over the first several dozen periods) evolution, attaining a positive or negative maximum and then decreasing in absolute value. DNS-ZE simulations reproduce well the initial maxima in both cases. After the initial evolution, the total kurtosis is found to have a small negative value in both cases, gradually decreasing in absolute value with time.

In what follows we concentrate on the dynamical kurtosis $C_4^{(d)}$, since the bound harmonics component of the kurtosis $C_4^{(b)}$, as well as the skewness C_3 , are nearly constant during shortterm evolution (see Annenkov & Shrira 2013 for definition of higher moments of a wave field). In figure 5, we study the effect of wind on the evolution of the dynamical kurtosis. These simulations show that the well-known phenomenon of large positive dynamical kurtosis during the evolution of initially one-dimensional narrowband spectra can be structurally unstable with respect to wind. Similar structural instability has been found in the case of interaction with shorter waves. On the other hand, simulations with wind show that wave fields of intermediate angular width can be characterized by moderate or even large negative dynamical kurtosis. This phenomenon is known from wave tank observations, but does not have an explanation within the framework



Figure 5: Evolution of dynamical kurtosis vs time by DNS-ZE. Initial conditions correspond to initial spectrum A (red curve) and B (blue curve). Dashed curves - wind U/c = 5

of weakly nonlinear theory.

In order to get a broader picture of the kurtosis evolution for various initial spectra, we performed a number of numerical experiments with the DNS-ZE algorithm. The questions addressed are (i) role of the initial directional distribution, (2) sensitivity of results obtained without wind forcing to the presence of moderate and strong wind (iii) dependence of the kurtosis evolution on the initial amplitude (iv) evolution in the presence of the second wave system with higher peak frequency and possibly different directional distribution. Details of the numerical procedure can be found in Annenkov & Shrira (2018).

Directional distribution is specified by

$$D(\theta) = \begin{cases} \frac{2}{\Omega} \cos^2\left(\frac{\pi\theta}{\Omega}\right) & \text{for } |\theta| \le \Omega/2\\ 0 & \text{for } |\theta| > \Omega/2 \end{cases}$$
(2)

where θ is the mean propagation direction and Ω is the directional spreading width. Figure 6 shows evolution of the dynamical kurtosis over 100 wave periods for JONSWAP spectra with $\gamma = 6$ and $\gamma = 3$, initial wave steepness 0.11, initial wave period $T_p = 1$ s and different directional distributions, ranging from almost unidirectional waves to a directionally wide spectrum typical of oceanic conditions. When the angular distribution is very narrow, the kurtosis initially evolves to high positive values, as predicted by the theory for one-dimensional wave fields. However, even



Figure 6: Evolution of the dynamical kurtosis for initial JONSWAP spectra with the significant wave height $H_s = 0.08$ m, peak period $T_p = 1$ s, and peakedness parameter (a) $\gamma = 6$ (b) $\gamma = 3$. Directional distribution is specified by (2), with different values of Ω (in degrees) corresponding to different curves. Values of N in the equivalent \cos^N directional model are also shown

for a slightly wider directional width this behaviour changes qualitatively, and for wave fields of intermediate angular width the kurtosis becomes negative during the initial evolution. For directionally wide wave fields the kurtosis still evolves towards negative values, but remains close to zero.

Overall, numerical simulations of the kurtosis evolution largely confirm the established picture of the kurtosis evolution, which is based on two key points (i) for a one-dimensional, or nearly one-dimensional spectrum, the dynamical kurtosis can be large provided that the spectrum is sufficiently narrow (ii) for directionally wide wave fields, typical of real oceanic conditions, the dynamical kurtosis is small and can be neglected. However, this picture is substantially enriched. First, for initially one-dimensional spectra the established behaviour of the kurtosis can be considerably affected by any factor that can enhance the rate of angular broadening (large amplitude, wind forcing, presence of another wave system in higher frequencies, etc). Second, it was found that wave fields of intermediate directional width (say, close to that of oceanic swell) show a qualitatively different behaviour, with negative dynamical kurtosis that can be moderately large in absolute value (say, -0.3–0.4 for wave steepness O(0.1)).

III. Evolution of spectra under constant and changing wind and comparison with experiments

Simulations with DNS and kinetic equations demonstrate substantial differences between dynamical and statistical models with respect to spectral shapes, growth rates and timescales of evolution. For simulations without wind forcing, these differences were discussed in part 1 of this report. The important question is whether the DNS results are supported by observations of real wind waves. To this end, we have performed comparisons between the numerical simulations of the evolution of wind-generated waves and observations of fetch-limited waves generated by



Figure 7: Spectral evolution by Hasselmann kinetic equation and the DNS vs measurements during the Tehuantepec experiment

strong offshore winds in the Gulf of Tehuantepec by Romero & Melville (2010). Both DNS-ZE and KE demonstrate close evolution of integral characteristics of spectra, in good agreement with the experiment. However, the DNS-ZE results show different spectral shapes, considerably wider and with a less pronounced peak than in the simulations obtained with the kinetic equation (figure 7). There is a striking discrepancy between the spectral peak amplitudes, which are, for large times, considerably larger in the case of the kinetic equations than the DNS-ZE prediction. The observed spectra for large times are shown to have the spectral width and the shape of the peak very close to those of the DNS-ZE spectra.

At the same time, we note again that integral characteristics of spectra demonstrate very close evolution for all spectra, including those simulated with the kinetic equations, this evolution being consistent with the data. Discrepancies between the DNS and the data, on the one hand, and the simulations with the kinetic equations, on the other, involve spectral shapes and spectral peak amplitudes.

An important implication of these results is the role played by spectral shapes in the estimation of extreme waves probability, which is based on the values of kurtosis. As it was shown in part 2 of this report (see also Annenkov & Shrira 2013, 2014), the dynamic component of the kurtosis is, generally speaking, small for broadband wind wave fields. The bound harmonics kurtosis and skewness are integrals of spectra, and the difference of spectral shapes manifests itself in their values. Previously it was shown for JONSWAP spectra that an increase of γ (at constant wave steepness) leads to a decrease of the kurtosis (Annenkov & Shrira 2014). Figure 8, which shows the evolution of the bound harmonics kurtosis calculated from the DNS, the kinetic equations, and the observed spectra, demonstrates that the difference between the kurtosis



Figure 8: Evolution of bound harmonics kurtosis evaluated from the DNS and kinetic equation spectra, and from the spectra measured during the Tehuantepec experiment

values due to differences in spectral shapes is very significant.

References

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Annenkov, S.Y. & Shrira, V.I. 2014 Evaluation of skewness and kurtosis of wind waves parameterized by JONSWAP spectra. J. Phys. Oceanogr. 44, 1582–1594.

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Romero, L. & Melville, W.K. 2010 Airborne observations of fetch-limited waves in the Gulf of Tehuantepec. J. Phys. Oceanogr. **40**, 441–465.

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Xiao, W., Liu, Y., Wu, G. & Yue, D. K. 2013 Rogue wave occurrence and dynamics by direct simulations of nonlinear wave-field evolution. *J. Fluid Mech.* **720**, 357–392.

List of publications/reports from the project with complete references

1. Annenkov S.Y., Shrira V.I. 2016 Spectral evolution of weakly nonlinear random waves: kinetic description vs direct numerical simulations. Geophysical Research Abstracts Vol. 18, EGU2016-10523.

2. Annenkov S.Y., Shrira V.I. 2016 New approaches to numerical simulation of random wave field spectral evolution. Wave Interaction (WIN-2016) 25 - 28 April 2016, Linz, Austria

http://www.dynamics-approx.jku.at/lena/Workshop2016/A_Annenkov.pdf

3. Annenkov S.Y., Shrira V.I. 2016 Spectral evolution of wind waves: classical vs generalised kinetic equations vs DNS. WISE Meeting CNR-ISMAR Venice, May 22-26, 2016

4. Annenkov, S. Y., Shrira, V. I. 2016 Modelling transient sea states with the generalised kinetic

equation. In: Rogue and Shock Waves in Nonlinear Dispersive Media, Springer, pp. 159--178

5. Annenkov, S. Y., Shrira, V. I. 2017 Modelling of kurtosis evolution for wind waves. The Onset of Rogue Waves, 7-8 April 2017, Northumbria University, Newcastle

6. Annenkov, S. Y., Shrira, V. I. 2017 DNS modelling of evolution of kurtosis for wind waves. 2017 WISE meeting, 14-18 May 2017, Victoria University, Victoria, B.C., Canada.

7. Annenkov, S.Y. & Shrira, V.I. 2018 Spectral evolution of weakly nonlinear random waves: kinetic description vs direct numerical simulations. *J. Fluid Mech.* **844**, 766-795.

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9. Annenkov, S.Y., Shrira, V.I. & Caulliez, G. 2018 Evolution of water wave spectra under a sharp increase of wind. Geophysical Research Abstracts Vol. 20, EGU2018-11054.

10. Annenkov, S.Y. & Shrira, V.I. 2018 Long term spectral evolution of wind waves: direct numerical simulations vs kinetic equations modelling and observations. WISE 2018, Tel Aviv University, 22--26 April 2018.

11. Annenkov S.Y., Shrira V.I. 2018 Modelling evolution of directional spectra of water waves. Workshop on nonlinear water waves, RIMS Kôkyûroku No.2109, Kyoto University.

12. Annenkov S.Y., Shrira V.I. 2019 When is the dynamic non-Gaussianity essential for water wave fields? WISE 2019, Jozankei, Hokkaido, Japan, 12--16 May 2019

Future plans

(Please let us know of any imminent plans regarding a continuation of this research activity, in particular if they are linked to another/new Special Project.)

In the new Special Project (started 2019) we plan to further explore the discrepancy between the shape of the DNS and the Hasselmann equation predictions, including the implications for probability of freak waves, mixing via the vortex force and other processes sensitive to the shape of spectra. The new set of data (obtained via collaboration with LATMOS, France) includes measurements in the Mediterranean Sea, collected in various wave regimes, different from the Tehuantepec observations. Objectives include getting new insights into the input and dissipation functions, and exploring the causes of the discrepancies in spectral shapes.