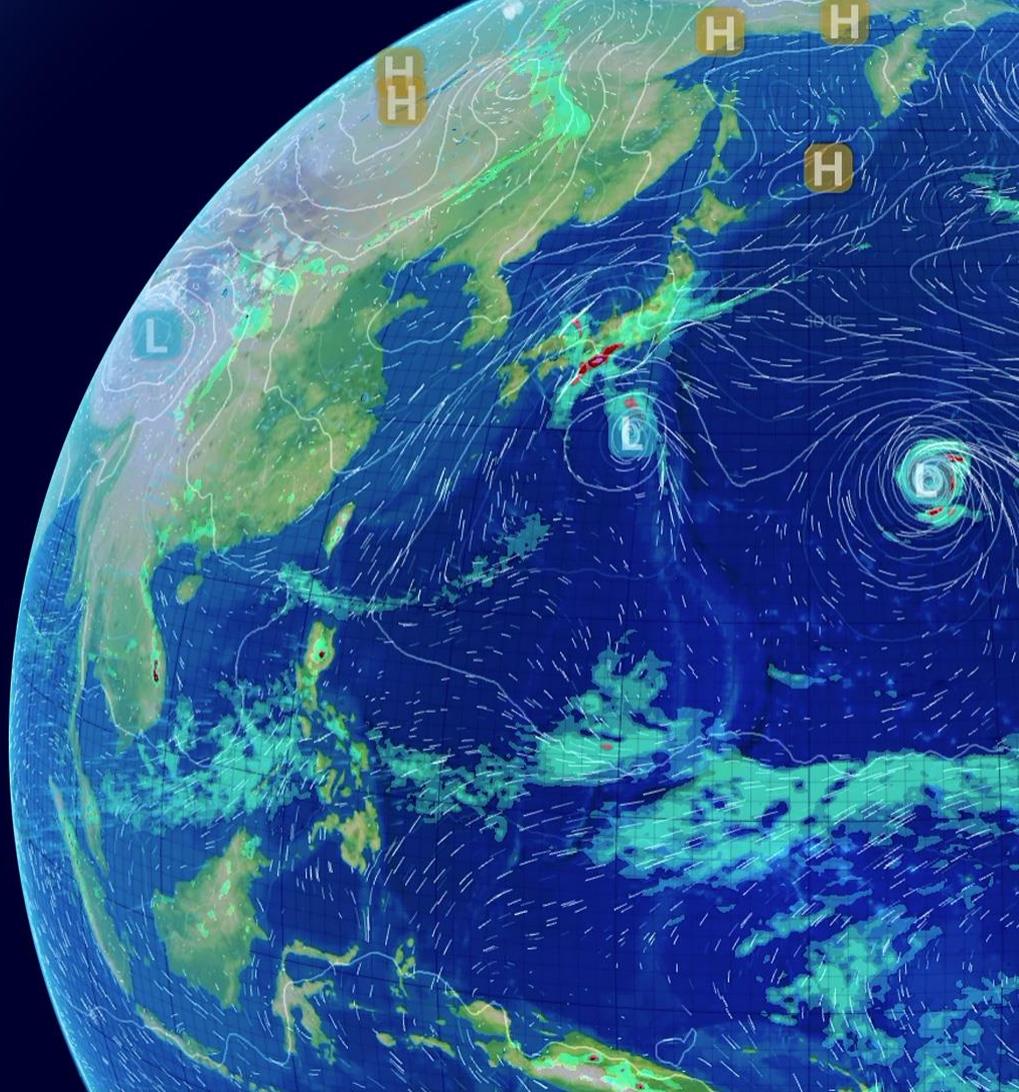


# Model error and parameter estimation

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ECMWF Annual Seminar, 11 September 2018



# Summary

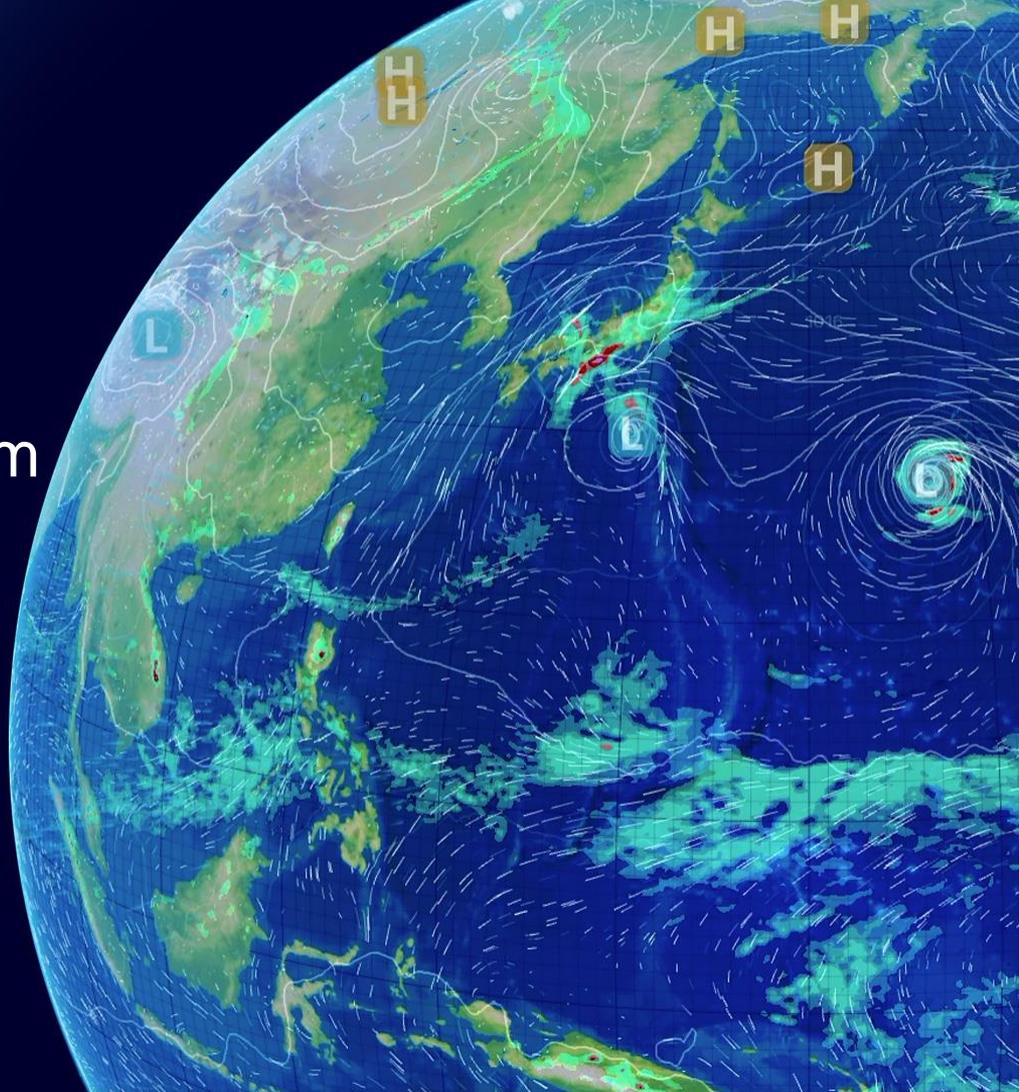
The application of interest is atmospheric data assimilation – focus on EDA;

A good ensemble needs to represent errors we want to correct – the quality of the ensemble is necessary to achieve it;

The inclusion of model error is essential to get a reliable ensemble;

Use DA technique to estimate model error.

- Motivation
- Model error calibration
- Application to an EDA system
- Comparison with stochastic physics schemes
- Summary



# Motivation

In an ensemble system we need to represent the correct statistics of model error so that the resulting ensemble is reliable.

The only available information on the truth comes from **observations**.

Let's regard the true evolution as a realisation of a stochastic model.

**Observations** measure (imperfectly) a single realisation of this stochastic model – so need large sample to get statistics.

DA techniques are a natural way to calibrate the model from **observations** because they allow for **observation error** in a systemic way and produce estimates in model space.

# Model error calibration

## Choice of model error

Using DA methods allow to exploit all available observations (taking into account their observation errors) to estimate model errors which represent all sources of errors.

stochastic model:

$dW$

$dW$  is t

el error

Stochastic methods are widely used but they only represent specific sources of errors.

The statistics of  $dW$  can be characterised by using **observations** (making stationary assumption) or alternatively using **stochastic schemes** that simulate model error within the model itself.

## Model error estimation using DA methods

We use DA methods as an inverse problem to fit a stochastic model to data by calculating increments which are realisations of the stochastic term.

We use cycled deterministic data assimilation to calculate the increments

- Since only a single true state is observed at each time, the statistics of the increments can only be inferred by accumulation over a large number of cases;
- Use same model that will be used in the EDA;
- This works if there are sufficient **observations** available (good enough in the atmosphere; not clear in the ocean).

Inverse methods need a prior pdf –  $\mathbf{Q}$  (initially use a typical background error covariance from standard data assimilation and then bootstrap).

## Calibration step

Assuming that the truth state evolution is given by:

$$\mathbf{x}_i = M_i(\mathbf{x}_{i-1}) + \eta_i$$

We use a reduced version of the cost function from Trémolet (2007) :

$$J(\eta) = \frac{1}{2} \sum_{i=1}^n \eta_i^T \mathbf{Q}^{-1} \eta_i + \frac{1}{2} \sum_{i=1}^n (H_i(\mathbf{x}_i) - \mathbf{y}_i)^T \mathbf{R}^{-1} (H_i(\mathbf{x}_i) - \mathbf{y}_i)$$

where:

$$\eta = \mathbf{QH}^T (\mathbf{R} + \mathbf{HQH}^T)^{-1} (H(\mathbf{x}_0) - \mathbf{y})$$

where  $\mathbf{H}$  includes the evolution of the deterministic model from the times the model error are added up to the observation times.

## Archive of analysis increments

(Ideally weak constraint) 4d-Var can be used to fit a stochastic model to **observations** over a training period of time to generate an archive of **analysis increments**.

Variational methods provide a minimum variance estimation - the **analysis increments** are the minimum required increments to allow the model to fit the **observations** within the observation error over a long period.

## Limitations and assumptions

We use the best available **observations** and the best available DA system (as in reanalysis) to minimise the uncertainty in estimating the increments.

The usefulness of this estimate is limited by the accuracy and completeness of the **observations**.

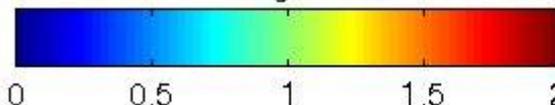
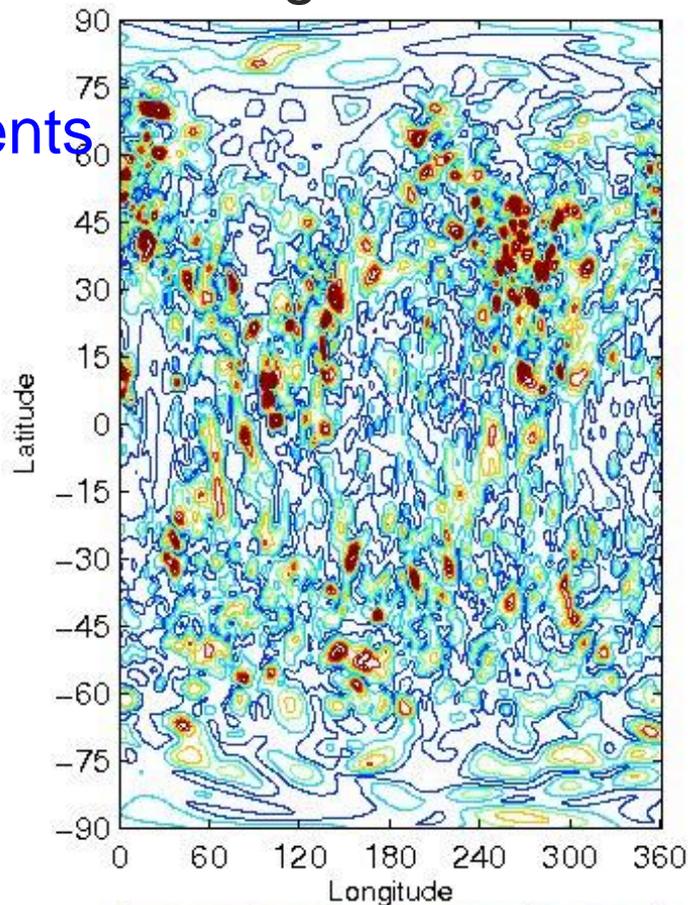
It is important to calculate the **increments** consistently with their use as forcing term in the ensemble - comparison between weak-constraint and strong-constraint **analysis increments**.

Probably the time correlation of the **analysis increments** should be allowed for (instead of using random forcing term every 6 hours).

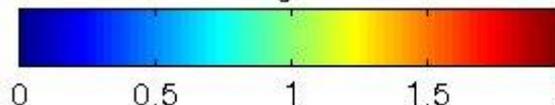
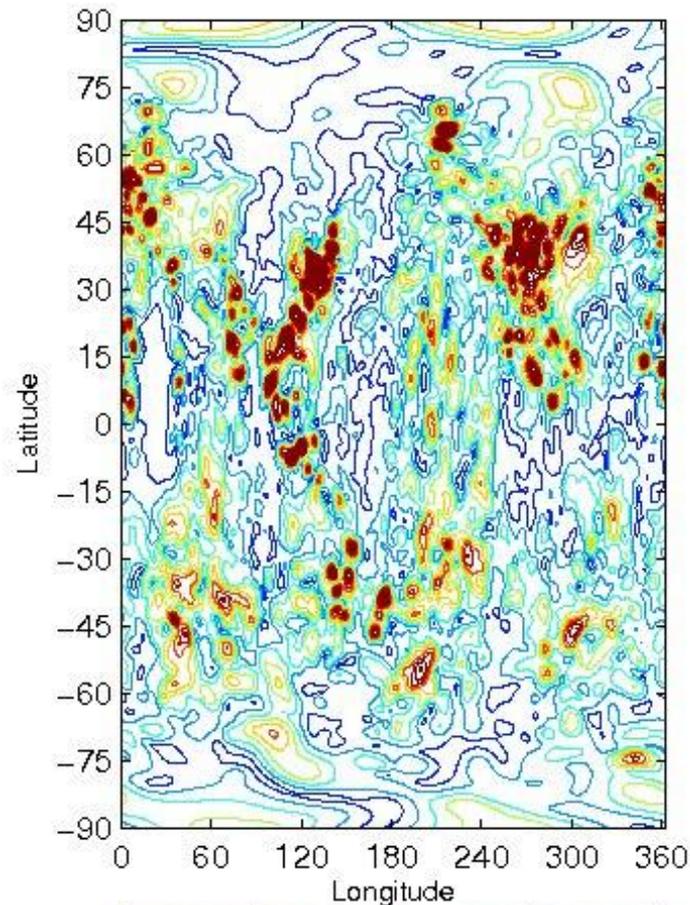
# Analysis increments for u at 850 hPa

More  
variance and  
larger scale  
when using  
weak  
constraint  
4d-Var.

## Strong constraint



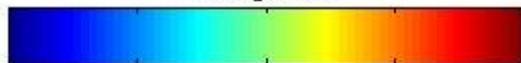
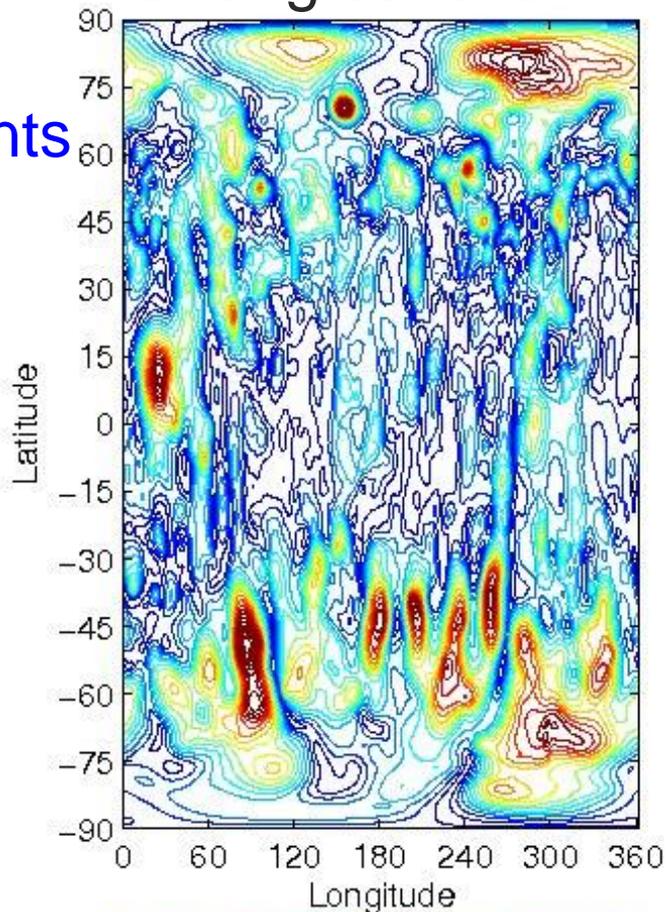
## Weak constraint



# Analysis increments for $\Theta$ at 850 hPa

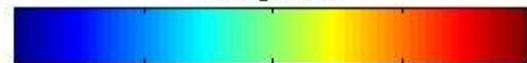
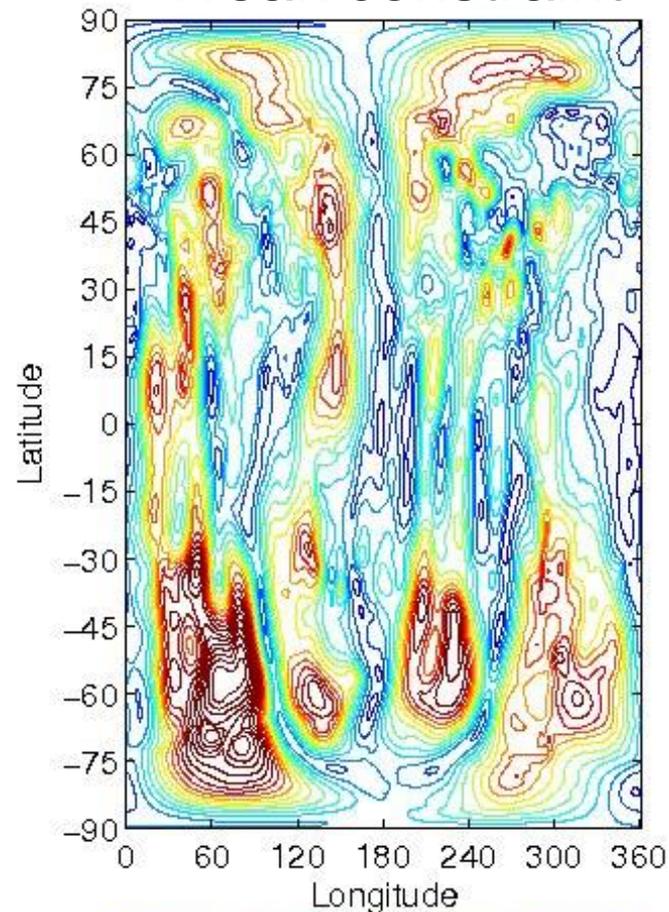
More  
variance and  
larger scale  
when using  
weak  
constraint  
4d-Var:  
bigger effect!

## Strong constraint



0 0.5 1 1.5 2

## Weak constraint



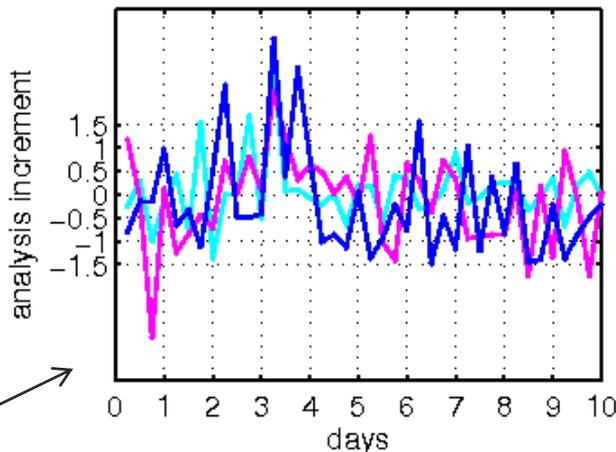
0 0.5 1 1.5 2

# Time correlation of analysis increments (NH)

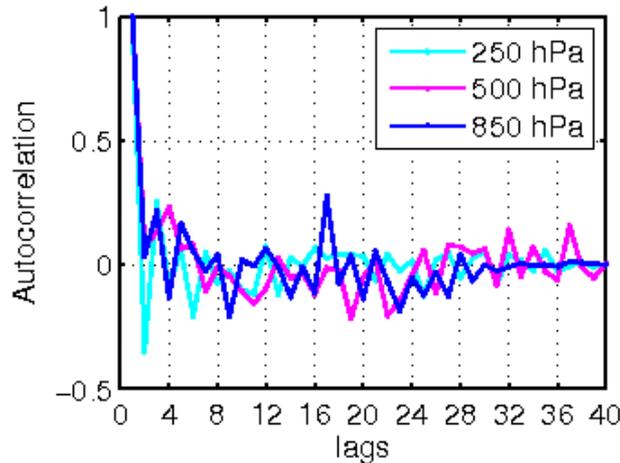
Diurnal correlation for u wind?

Strong semi-diurnal correlation for  $\Theta$ .

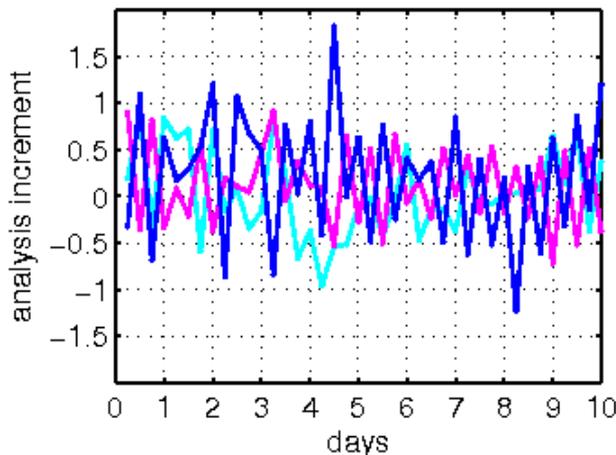
Time Series: u wind



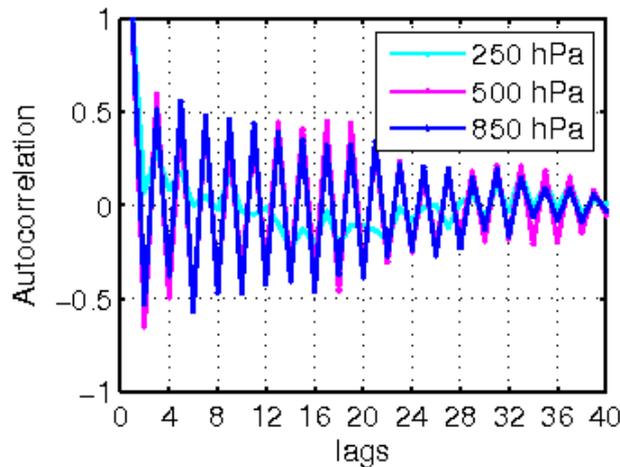
Autocorrelation: u wind



Time Series: theta



Autocorrelation: theta

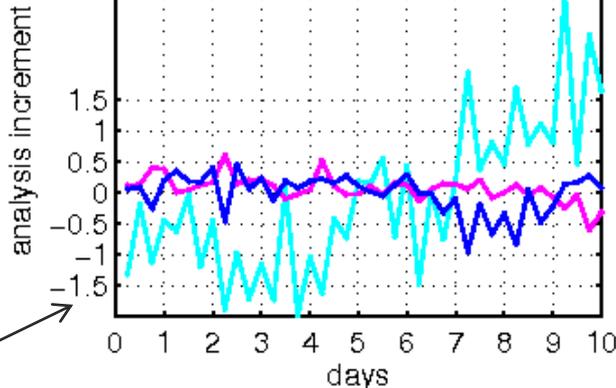


# Time correlation of analysis increments (EQU)

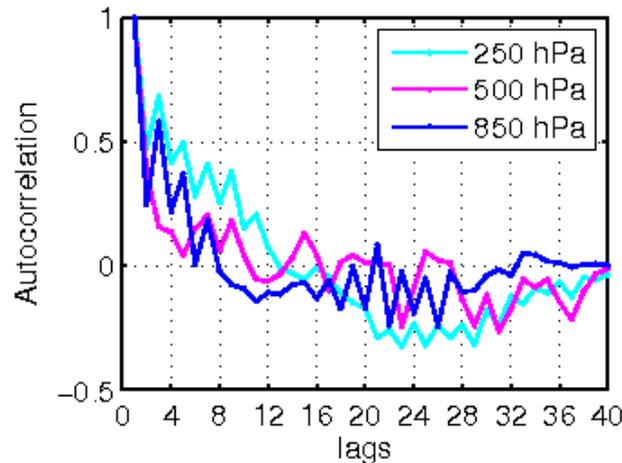
Significant  
longer time  
correlation  
for u wind

Diurnal  
correlation  
for  $\Theta$ .

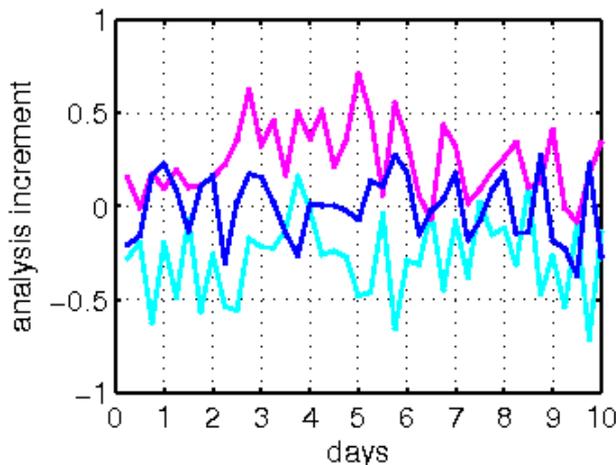
Time Series: u wind



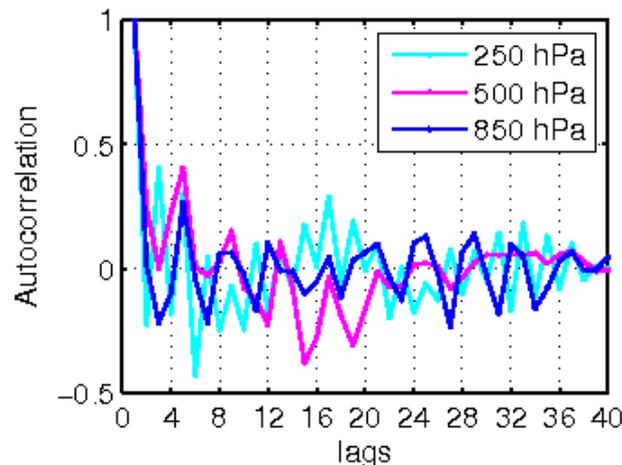
Autocorrelation: u wind



Time Series: theta



Autocorrelation: theta



# Application to an EDA system

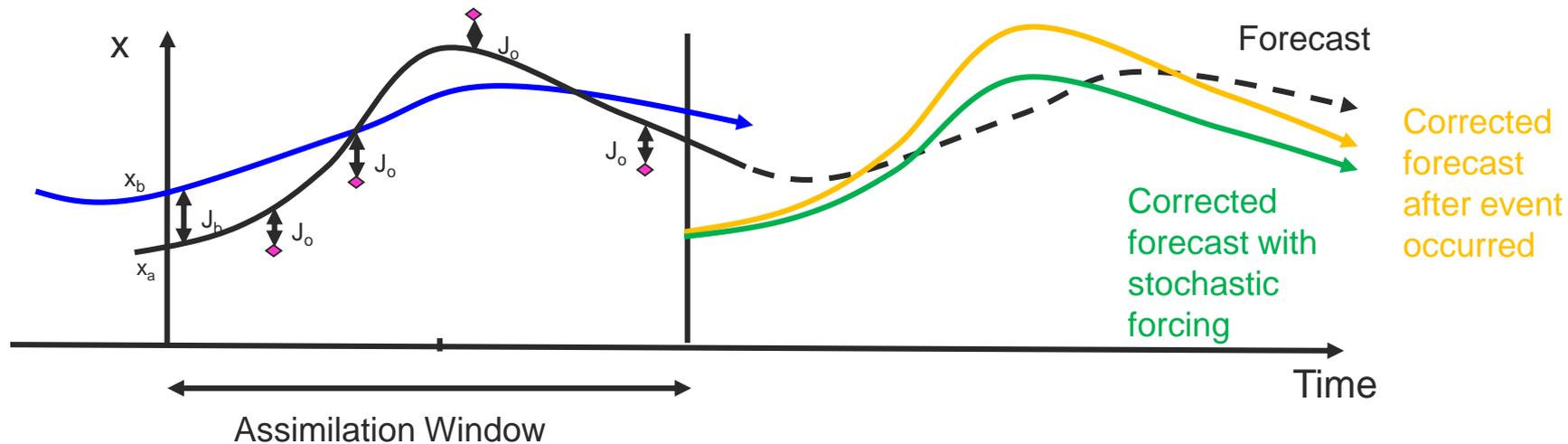
- random assumption of analysis increments
- EDA set-up
- performance at longer lead times

An ensemble is reliable if the truth is statistically indistinguishable from a randomly chosen ensemble member at any time.

This is measured by comparing the ensemble spread and the RMSE of the ensemble mean at all lead times.

A major assumption of this method is that a stochastic model forced with randomly chosen **analysis increments** does indeed deliver a reliable ensemble.

# As reanalysis ...



If the **analysis increments** can be considered as a random draw from an archive with stationary statistics, **a reanalysis trajectory** will be statistically indistinguishable from a random realisation of the model with the stochastic forcing.

## Random assumption of analysis increments ( $u@850\text{hPa}$ )

To test this assumption, we compare the T+6 hours ensemble spread with the RMSE of the ensemble mean measured against a random analysis member as the truth (Bowler et al. 2015).

	RMSE T+6 h	Spread T+6 h	Rel. Diff (%)
NH	1.98	1.93	2.40+/-1.87
Tropics	2.09	2.15	-2.42+/-1.67
SH	2.67	2.74	-2.68+/-2.02

+/- indicates 95% confidence interval.

Difference between spread and RMSE are not statistically different from zero.

## Ensemble DA set-up

Use the model error statistics to generate a stochastic forcing term in an EDA system:

- Random **analysis increments** drawn from an archive are used to force each member of the ensemble forecast.
- Minimise error of ensemble mean by using the best available deterministic model in the calibration step.

Ensemble DA system:

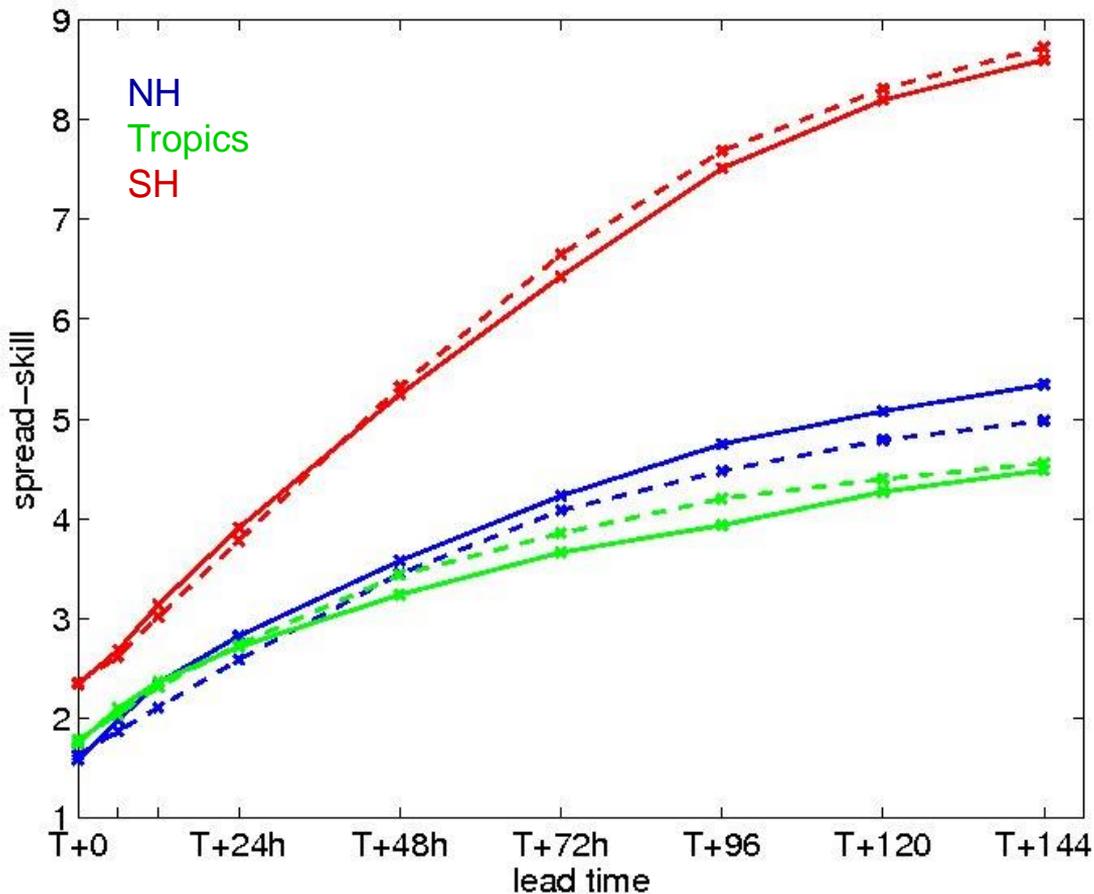
1. Use an ensemble of 10 independent 4dVars with **perturbed observations** and SSTs;
2. Draw every 6 hours random **analysis increments** from the archive;
3. Add at each time step over a window of 6 hours (time-window of DA system) perturbations consistent with the statistics of the **analysis increments**, over the overall period of forecast integration.

## RMSE vs spread

Solid: RMSE  
Dash: spread

**u@850 hPa**

Met Office N320L70 UM, i.e.  
40km horizontal resolution and  
70 levels (80 km model top).



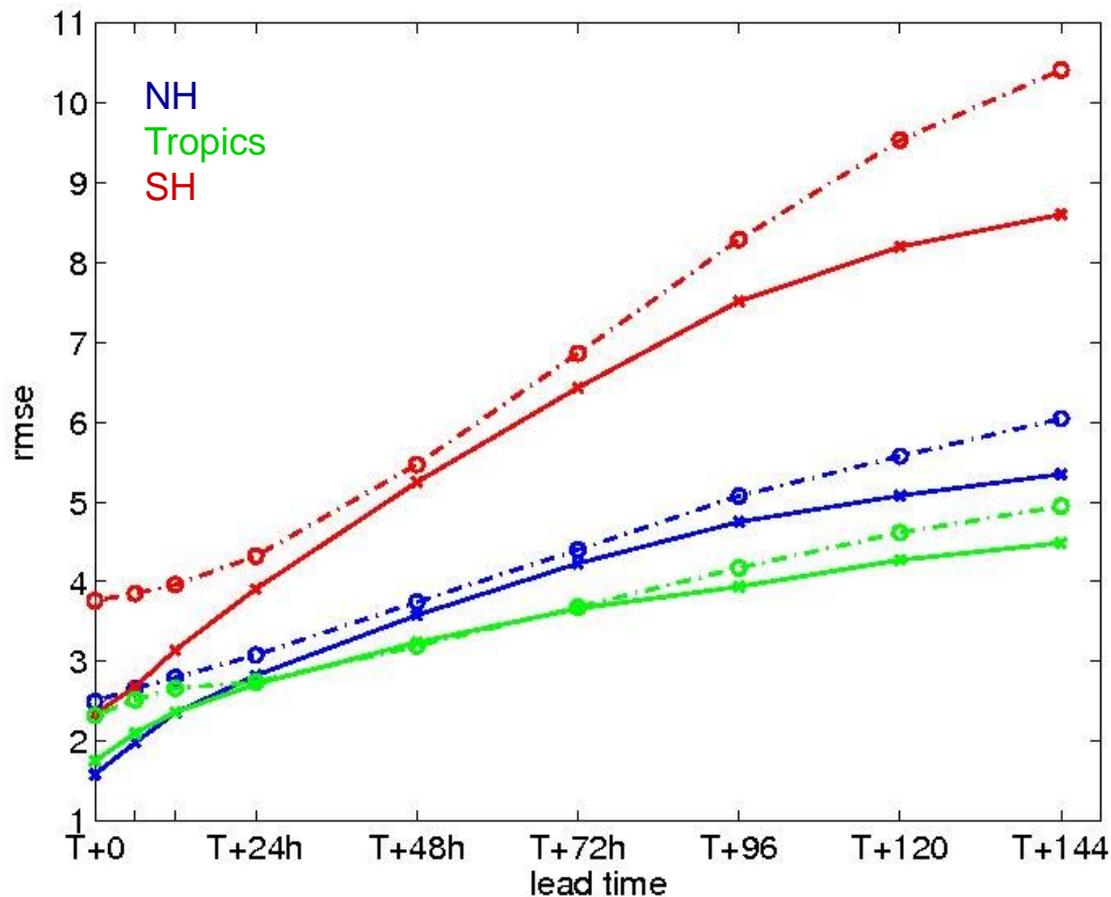
## Ens mean vs det RMSE

Solid:  
ensemble  
mean

Dash-dot:  
control

**u@850 hPa**

Met Office N320L70 UM, i.e.  
40km horizontal resolution and  
70 levels (80 km model top).



# Comparison with stochastic schemes

## Stochastic schemes

There are various stochastic schemes to simulate the model error within the model itself.

The initial conditions are generated by an ETKF (Ensemble Transform Kalman Filter) and they are centered around the deterministic 4d-Var analysis.

Operational MOGREPS uses:

- Random perturbations to physical parameters (RP)
- Stochastic kinetic energy backscatter (SKEB)

**CNT**

Alternative methods (e.g. used at ECMWF) use:

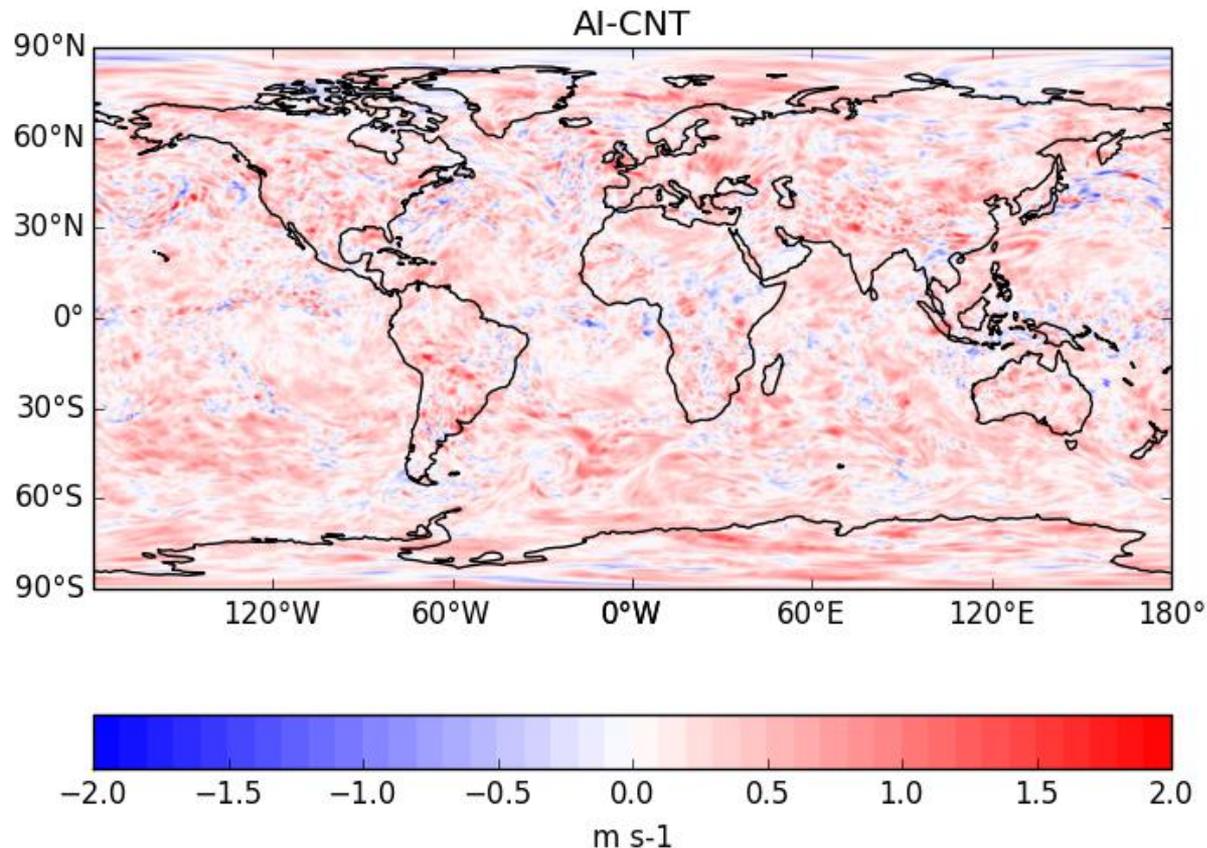
- Stochastic Perturbation of Tendencies (SPT)
- Stochastic kinetic energy backscatter (SKEB)

**SPT**

How do these schemes compare with **analysis increments** forcing derived from data assimilation?

**AI**

## Geographical variation of spread at T+6h



**CNT** picks up sources of model error mainly in the NH storm track.

**SPT** shows localised increase of spread in the NH storm track and tropics.

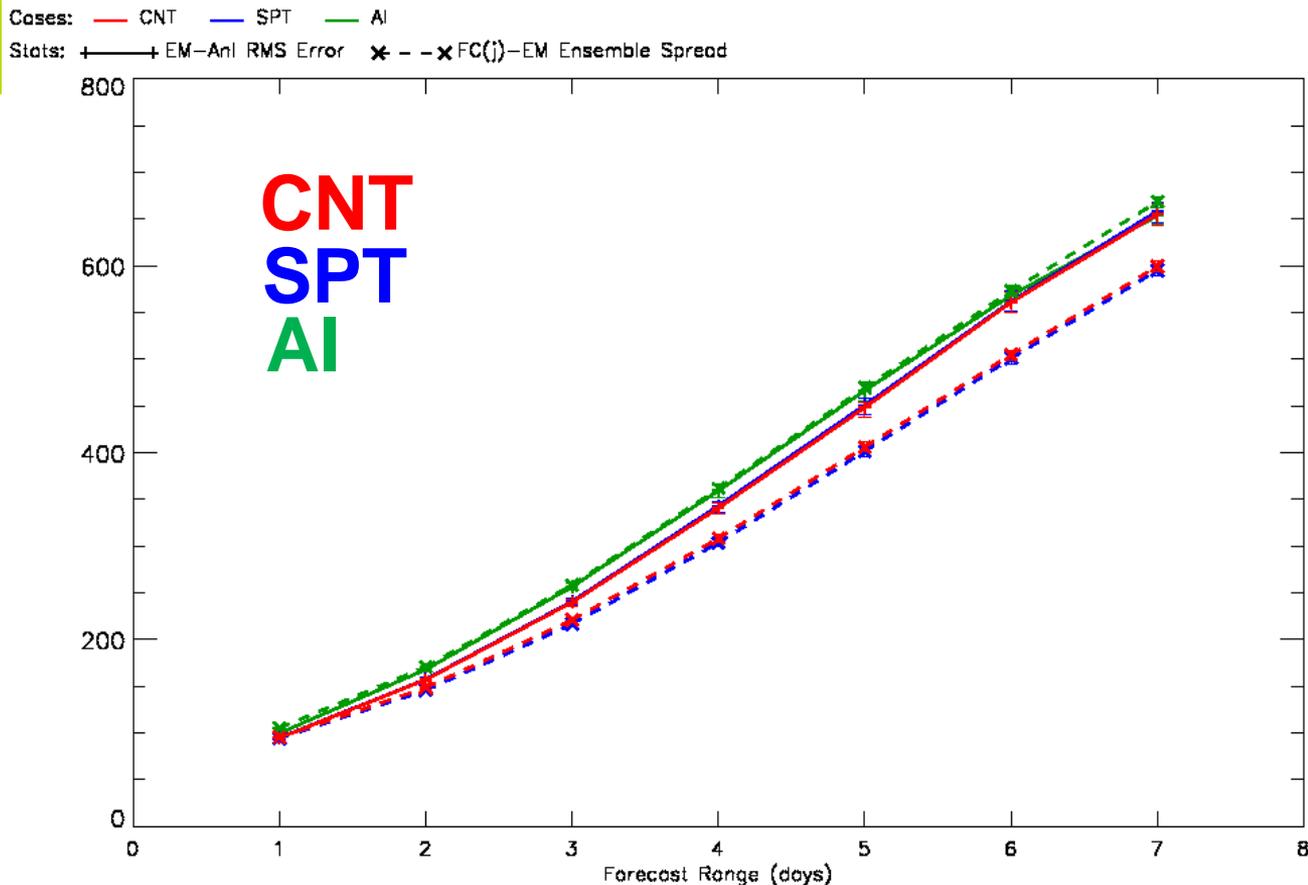
**AI** introduces more large scale spread across all regions but lacks flow-dependency. It also better represents the error in the SH and tropics.

## RMSE vs spread

MOGREPS  
Verification against  
analysis

Mean Sea Level  
Pressure (Pa) - NH

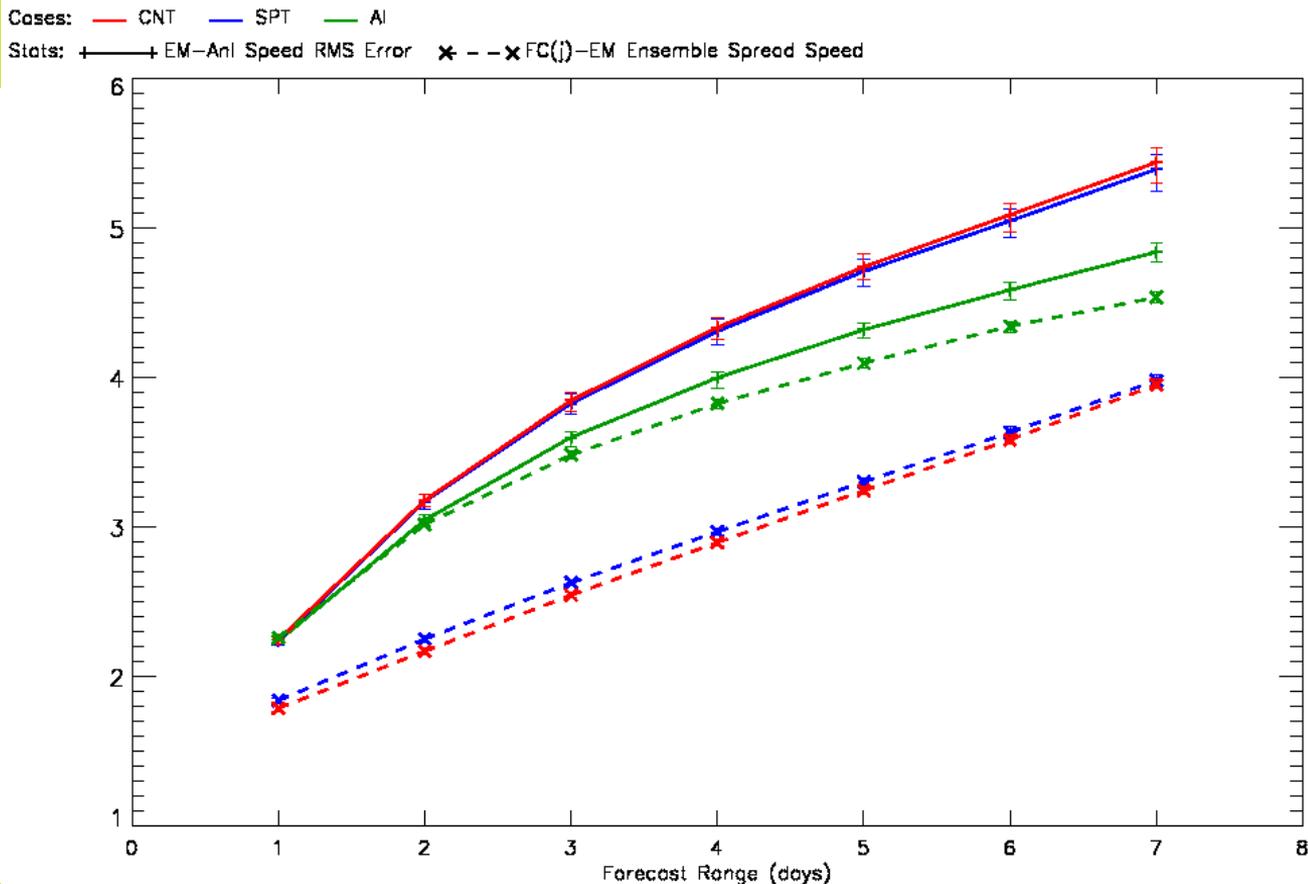
Solid: RMSE  
Dash: spread



## RMSE vs spread

**MOGREPS**  
**Verification against**  
**analysis**  
**250 hPa winds (m/s)**  
**Tropics**

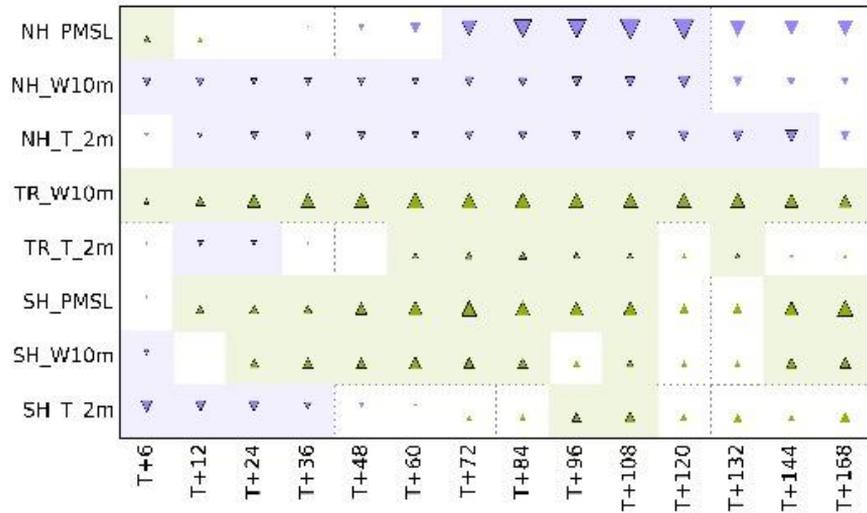
Solid: RMSE  
 Dash: spread



## Scorecard of changes in CRPS

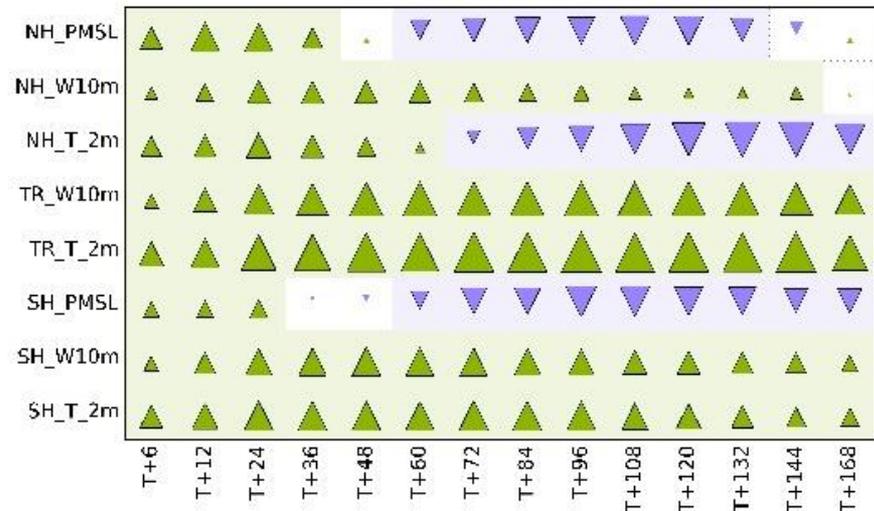
% Difference (SPT vs. CNT) - Overall 0.1%

Continuous Ranked Probability Score against observations for 20160201-20160315



% Difference (AI vs. CNT) - Overall 2.7%

Continuous Ranked Probability Score against observations for 20160201-20160315



**SPT - CNT**

▲ Better CRPS  
▼ Worse CRPS

**AI - CNT**

# Summary

- Need model error to get a useful ensemble, so include stochastic term in the model;
- Use DA techniques to calibrate the stochastic model against observations over a long period;
- Find minimum variance estimate of the stochastic term;
- Consider the **analysis increments** as a random draw from an archive with stationary statistics, a **reanalysis trajectory** should be statistically indistinguishable from a random realisation of this model;
- Test random assumption and ensemble performance at longer lead times;
- Compare this scheme with stochastic schemes that simulate the model error within the model itself.

Questions?

