Department of
Meteorology

University of Reading

Trapped lee waves: a currently neglected source of low-level orographic drag

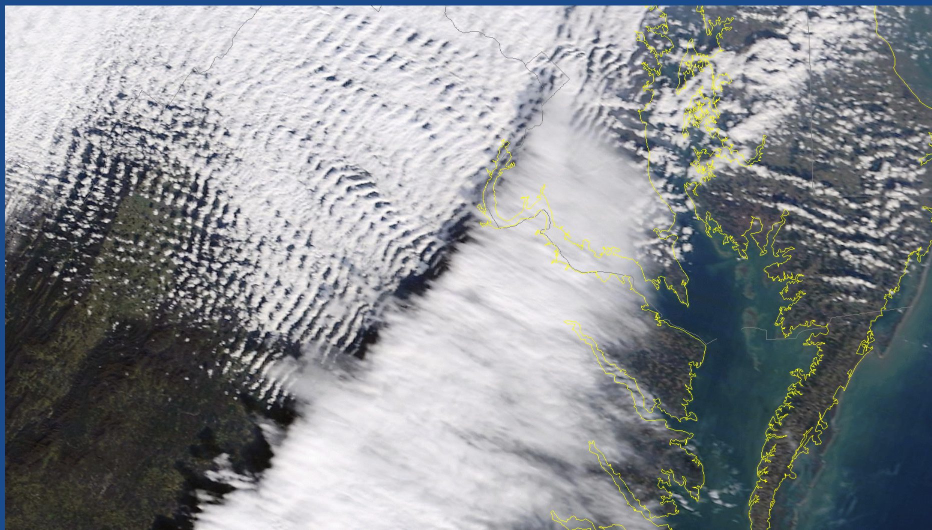
Miguel A. C. Teixeira[1], Jose L. Argain[2], Pedro M. A. Miranda[3]

[1]Department of Meteorology, University of Reading, Reading, UK

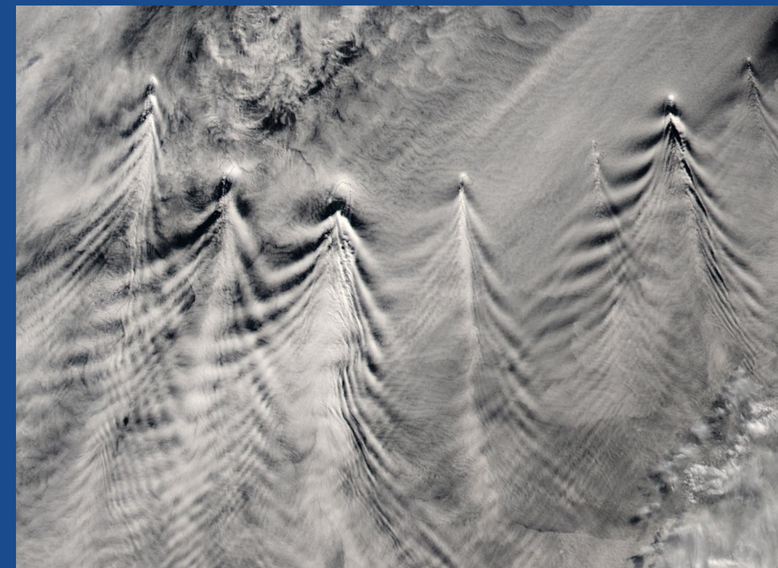
[2]Department of Physics, University of Algarve, Faro, Portugal

[3]Instituto Dom Luiz (IDL), University of Lisbon, Lisbon, Portugal

Trapped lee waves



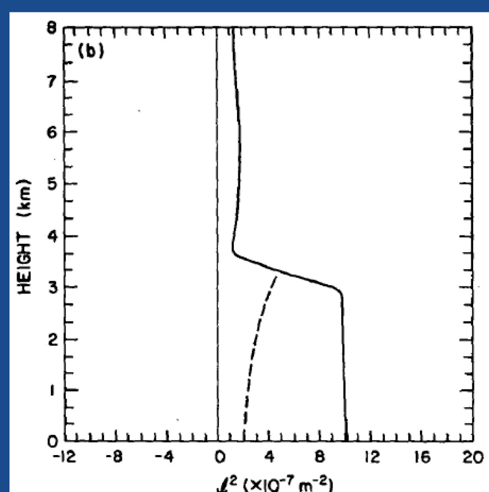
Non-hydrostatic

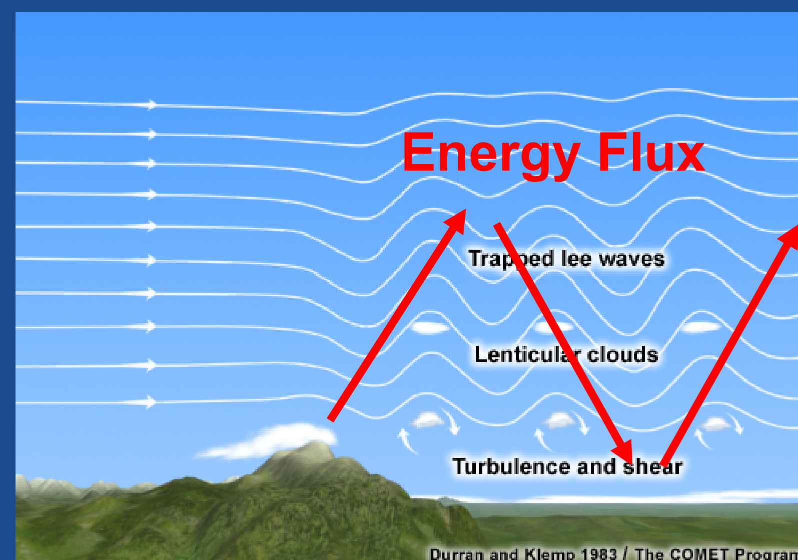


$$N^2 = \frac{g}{\theta_0}\frac{d\overline{\theta}}{dz}$$

Scorer parameter

$$l = \left(\frac{N^2}{U^2} - \frac{1}{U}\frac{d^2U}{dz^2} \right)^{1/2}$$





Mountain wave drag

Important for drag parametrization schemes in global climate and weather prediction models

It is known that drag decreases as flow becomes more nonhydrostatic (narrower obstacles) → this would suggest that trapped lee waves (highly nonhyrsotatic) would produce little drag

However, trapped lee waves exist due to energy trapping in a layer or interface: wave reflections and resonance → may lead to drag amplification

How is drag partitioned into trapped lee waves and vertically propagating (untrapped) mountain waves?

Bell-shaped 2D and 3D circular mountains

$$h = \frac{h_0}{1+(x/a)^2}$$

$$h = \frac{h_0}{\left[1+(x/a)^2+(y/a)^2\right]^{3/2}}$$

Linear, hydrostatic, non-rotating, constant l limit

$$D_0 = \frac{\pi}{4}\rho_0 U^2 l h_0^2$$

$$D_0 = \frac{\pi}{4}\rho_0 U^2 l a h_0^2$$

Department of
Meteorology

University of Reading

Linear theory

- Linearization, Boussinesq approximation
- Inviscid, nonrotating, stationary, uniform flow

$$w(x,y,z) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \hat{w}(k_1,k_2,z)e^{i(k_1x+k_2y)}dk_1dk_2$$

$$\frac{d^2\hat{w}}{dz^2} + \frac{k_1^2+k_2^2}{k_1^2}(l^2-k_1^2)\hat{w} = 0$$

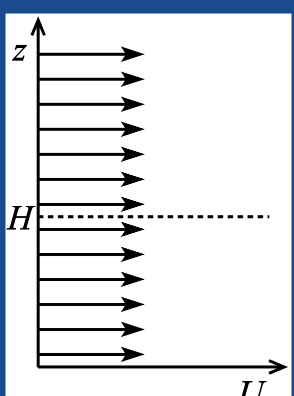
Taylor-Goldstein equation

Boundary conditions: $\hat{w}(z=0) = iUk_1\hat{h}$

$\hat{w}$ $\hat{p}$ continuous at $z=H$

Waves propagate energy upward or decay as $z\to\infty$

2-layer atmospheres

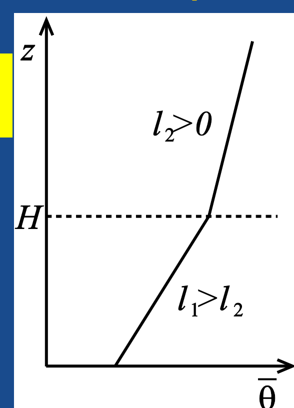


$$l_1 = \frac{N_1}{U}$$

$$l_2 = \frac{N_2}{U}$$

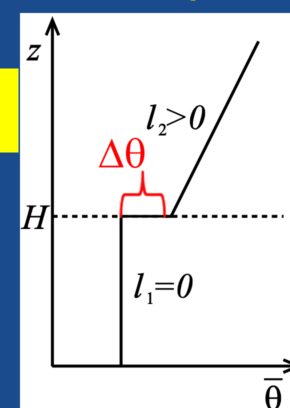
Case 1

Scorer (1949)



Case 2

Vosper (2004)



$$g' = g\frac{\Delta\theta}{\theta_0}$$

Gravity wave drag

$$D=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p(z=0)\frac{\partial h}{\partial x}\,dxdy = 8\pi^2\,\mathrm{Im}\left[\int_{-\infty}^{+\infty}\int_{0}^{+\infty} k_1\hat{p}(z=0)\hat{h}^*\,dk_1dk_2\right]$$

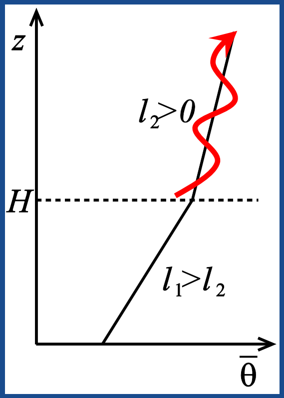
$\hat{p}$ determined from solutions for $\hat{w}$

Case 1

Propagating wave drag (2D)

$$D_1 = 4\pi\rho_0 U^2 \int_0^{l_2} \frac{k\,|\hat{h}|^2\, m_1^2 m_2}{m_1^2\cos^2(m_1H)+m_2^2\sin^2(m_1H)}\,dk$$

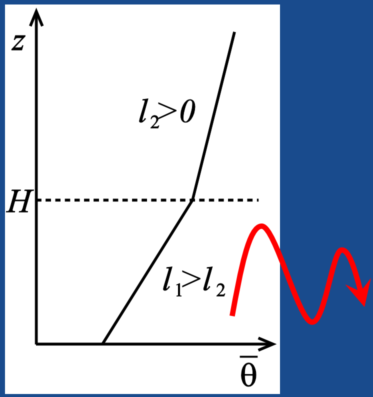
$|k|<l_2$



Trapped lee wave drag (2D)

$$D_2 = 4\pi^2\rho_0 U^2 \sum_j |\hat{h}(k_j)|^2 \frac{m_1^2(k_j)\,n_2(k_j)}{1+n_2(k_j)H}$$

$l_2<|k|<l_1$



Resonance condition (2D) $$\tan\left[m_1(k_j)H\right] = -\frac{m_1(k_j)}{n_2(k_j)}$$

Drag normalized by $D_0=\frac{\pi}{4}\rho_0U^2l_1h_0^2$ or $D_0=\frac{\pi}{4}\rho_0U^2l_1ah_0^2$

Depends on l_1H l_2/l_1 l_1a

Department of
Meteorology

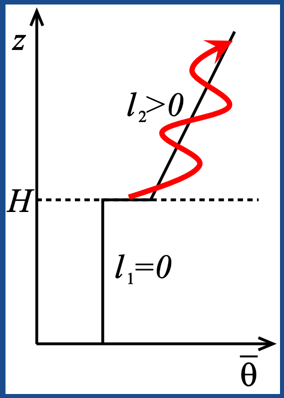
Case 2

University of Reading

Propagating wave drag (2D)

$$D_1 = 4\pi\rho_0 U^2 \int_0^{l_2} \frac{k^2\,|\hat{h}|^2\,(m_2 H)(kH)}{\left[kH\cosh(kH) - Fr^{-2}\sinh(kH)\right]^2 + (m_2 H)^2 \sinh^2(kH)}\,dk$$

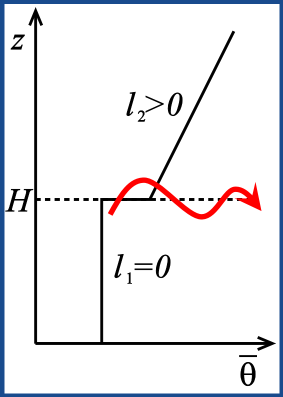
$|k|<l_2$



Trapped lee wave drag (2D)

$$D_2 = 4\pi^2\rho_0 U^2 \frac{k_L^2\left|\hat{h}(k_L)\right|^2\left\{\left[Fr^{-2} - n_2(k_L)H\right]^2 - (k_L H)^2\right\}}{(k_L H)^2\left[H + n_2^{-1}(k_L)\right] + H\left[1 + n_2(k_L)H - Fr^{-2}\right]\left[Fr^{-2} - n_2(k_L)H\right]}$$

$|k|>l_2$



Resonance condition (2D) $\tanh(k_L H) = \dfrac{k_L H}{Fr^{-2} - n_2(k_L)H}$

Drag normalized by $D_0 = \dfrac{\pi}{4}\rho_0 U^2 l_2 h_0^2$ or $D_0 = \dfrac{\pi}{4}\rho_0 U^2 l_2 a h_0^2$

Depends on

$Fr = \dfrac{U}{\sqrt{g'H}}$ $l_2 H$ $l_2 a$

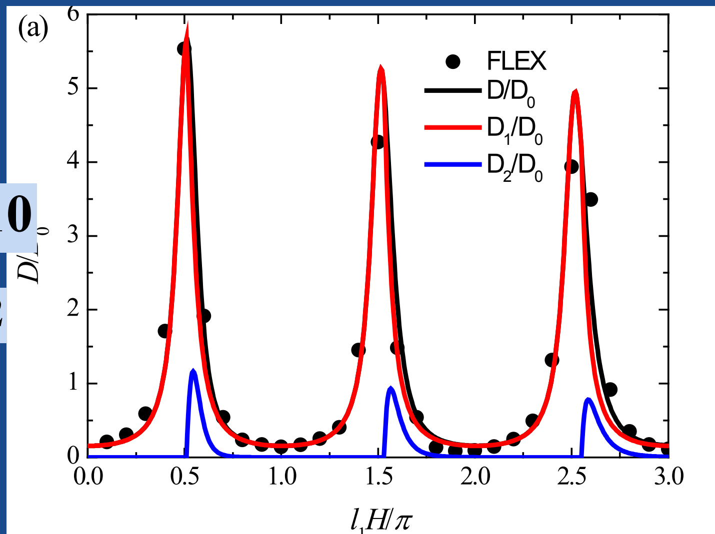
Case 1 (2D): Drag

Results for $l_2 / l_1 = 0.2$

Numerical simulations $l_1 h_0 = 0.02$

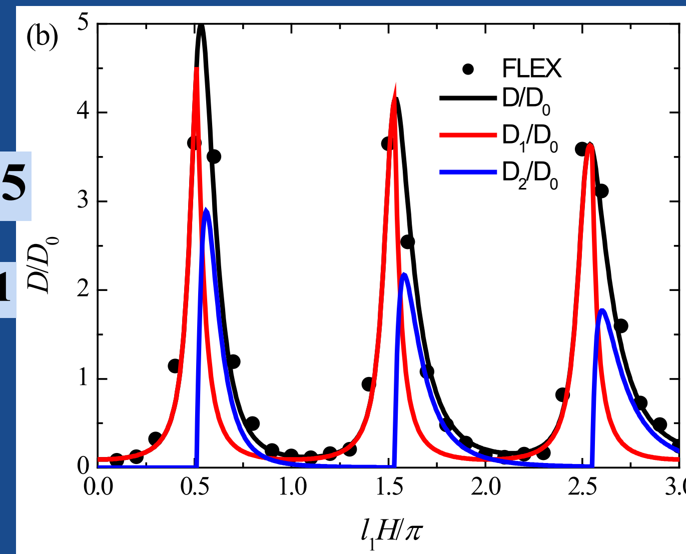
$l_1 a = 10$

$l_2 a = 2$



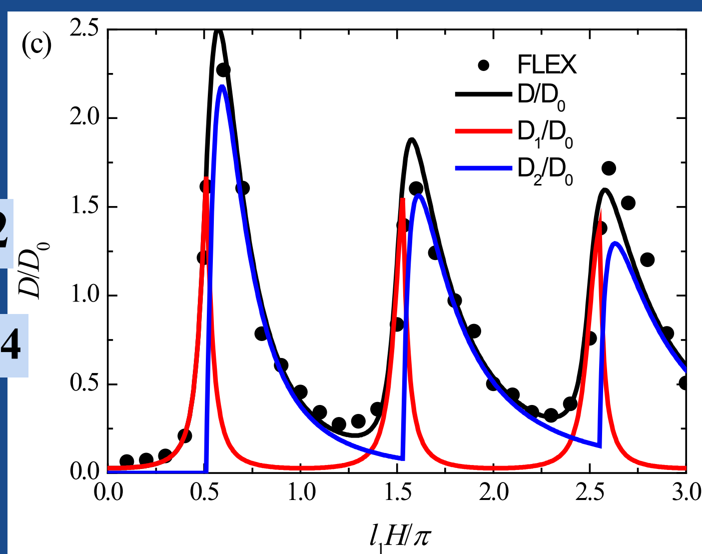
$l_1 a = 5$

$l_2 a = 1$



$l_1 a = 2$

$l_2 a = 0.4$

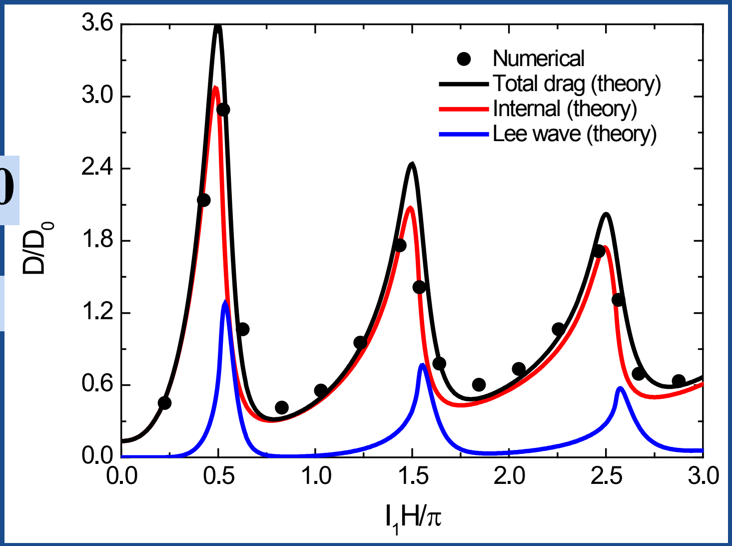


- D_2/D_0 may be large (~3)
- Drag maxima coincide with establishment of trapped lee wave modes
- Agreement with numerical simulations requires considering both D_1 and D_2
- D_2/D_1 increases as $l_2 a$ decreases

Case 1 (3D): Drag

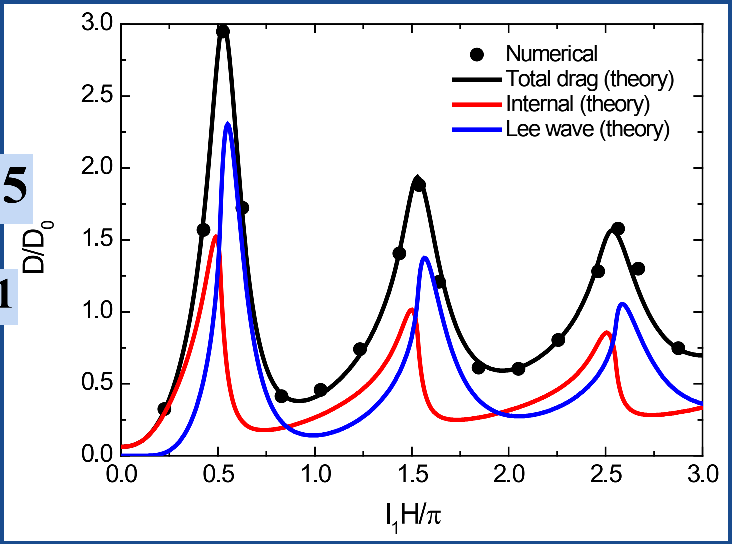
$l_1 a = 10$

$l_2 a = 2$



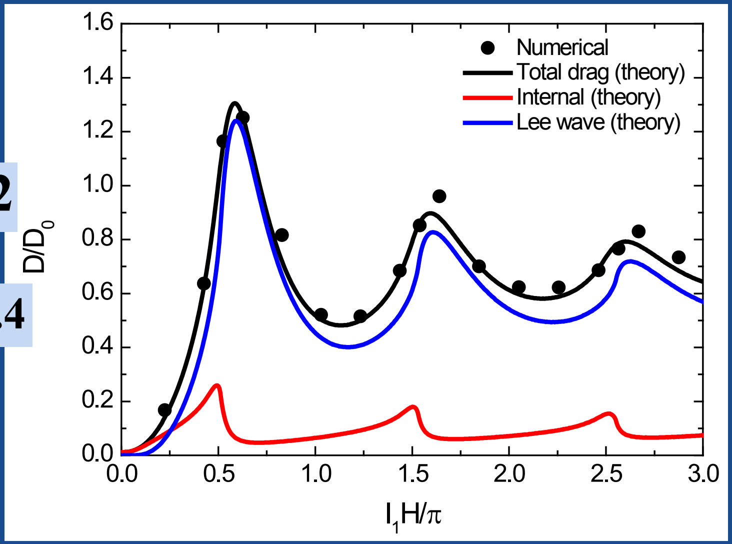
$l_1 a = 5$

$l_2 a = 1$



$l_1 a = 2$

$l_2 a = 0.4$



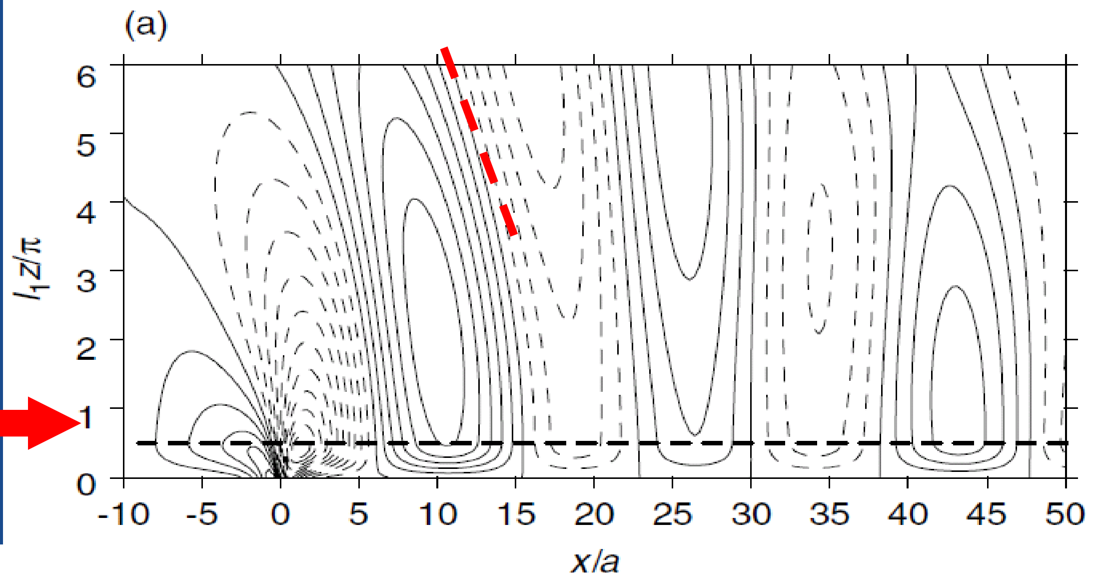
- D_2/D_0 may be large (~2) → some directional wave dispersion
- Drag maxima lower and wider than in 2D: → continuous spectrum, even for trapped lee waves
- Agreement with numerical simulations requires considering both D_1 and D_2
- D_2/D_1 substantially higher than in 2D → non-hydrostatic effects more important

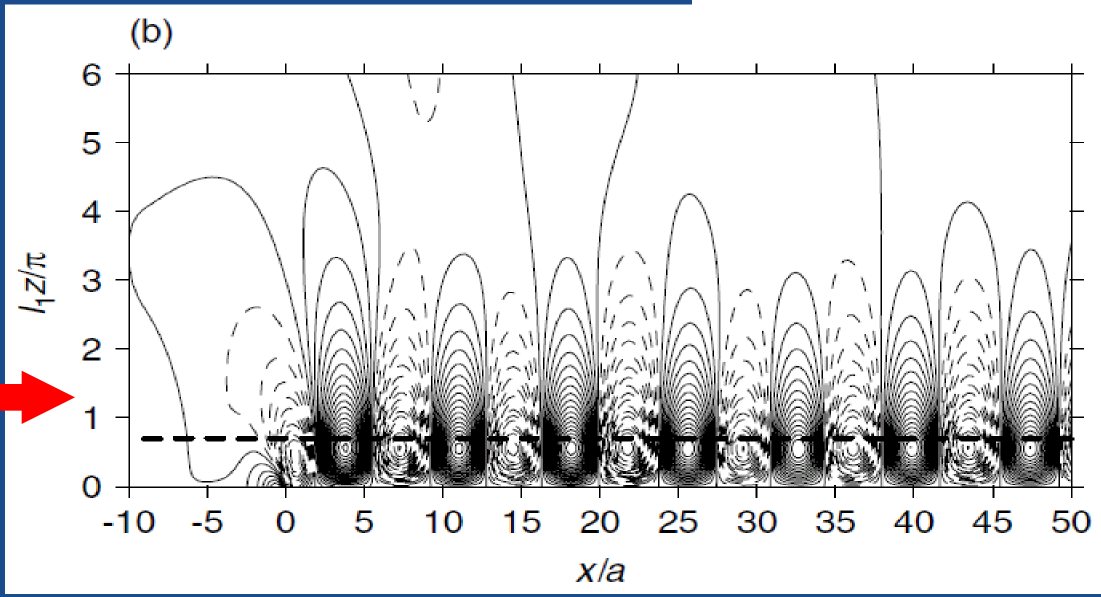
Case 1 (2D): Flow field

$w/(Uh_0/a)$ for $l_2/l_1 = 0.2$ $\boldsymbol{l_1 a = 2}$

$l_1 H/\pi = 0.5$ $D_2/D_1 = 0.08$

Propagating waves dominate





$l_1 H/\pi = 0.7$ $D_1/D_2 = 0.06$

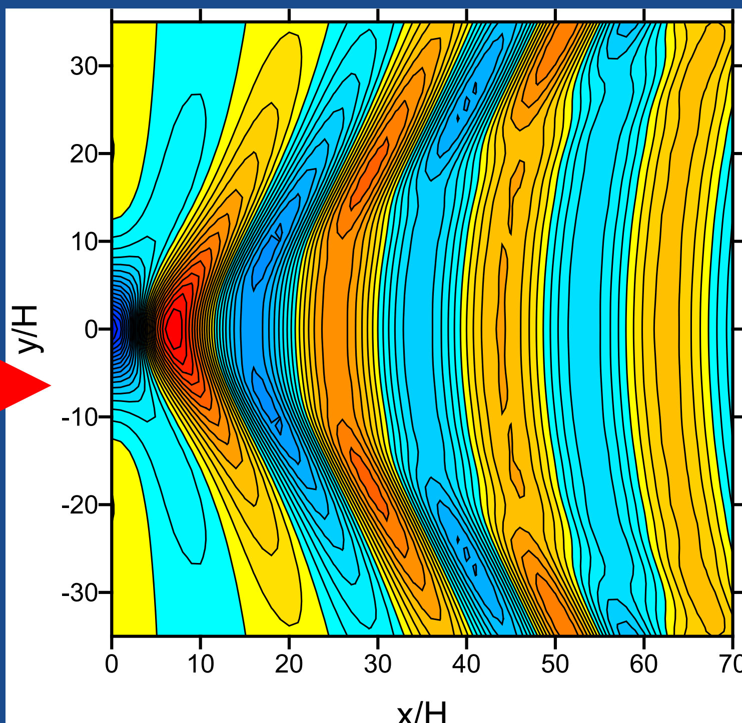
Trapped lee waves dominate

Case 1 (3D): Resonant trapped lee wave field

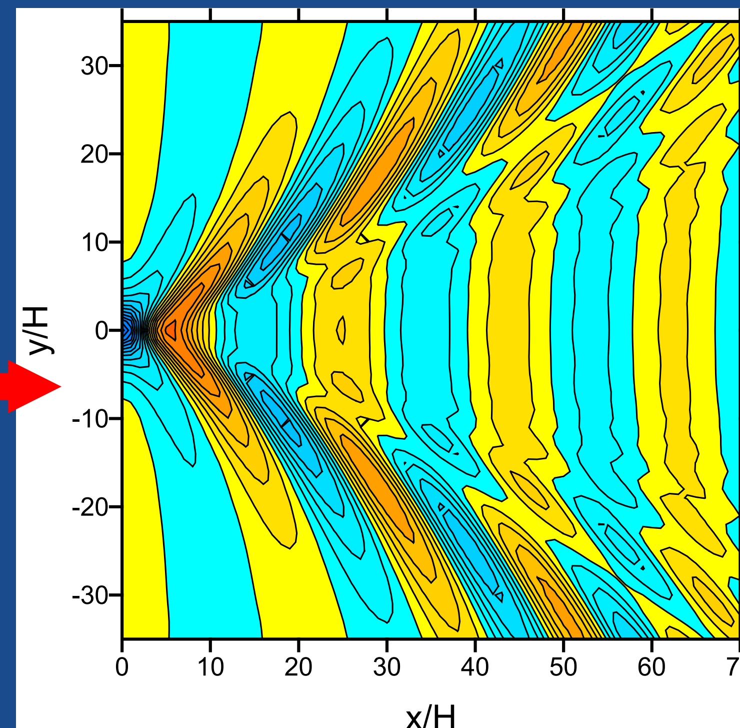
$w/(Uh_0/a)$ at $z=H/2$

for $l_2 / l_1 = 0.2$ $l_1 H / \pi = 0.5$

$l_1 a = 5$



$l_1 a = 2$



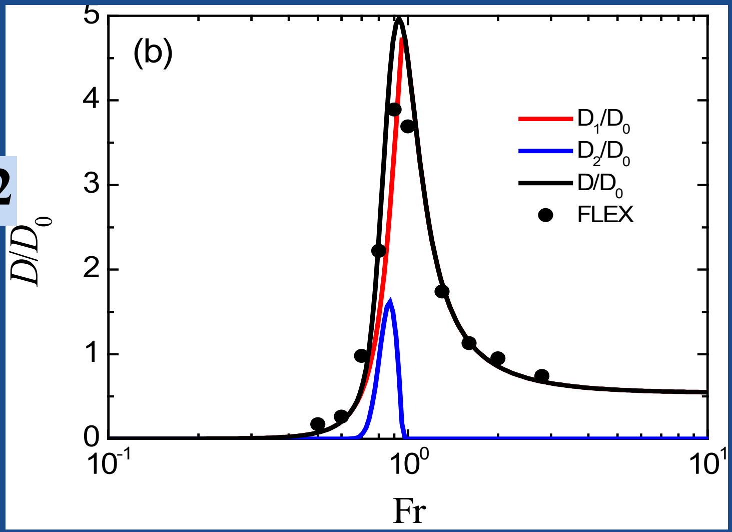
"Ship-wave" pattern

Case 2 (2D): Drag

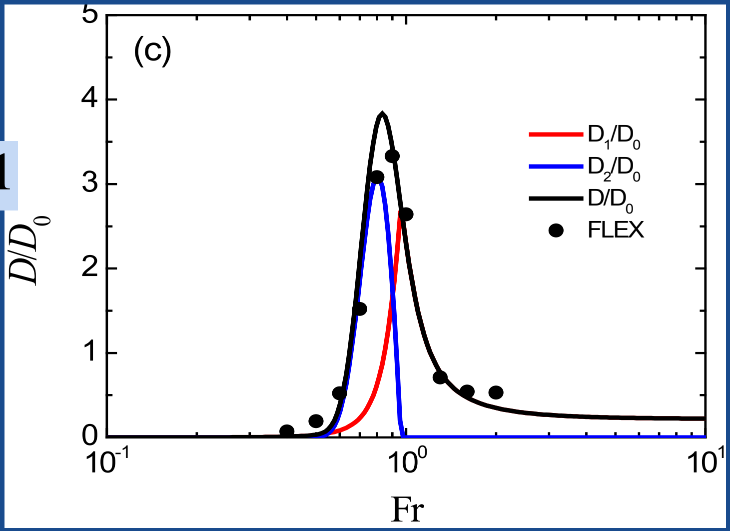
Results for $l_2 H = 0.5$

Numerical simulations $l_2 h_0 = 0.01$

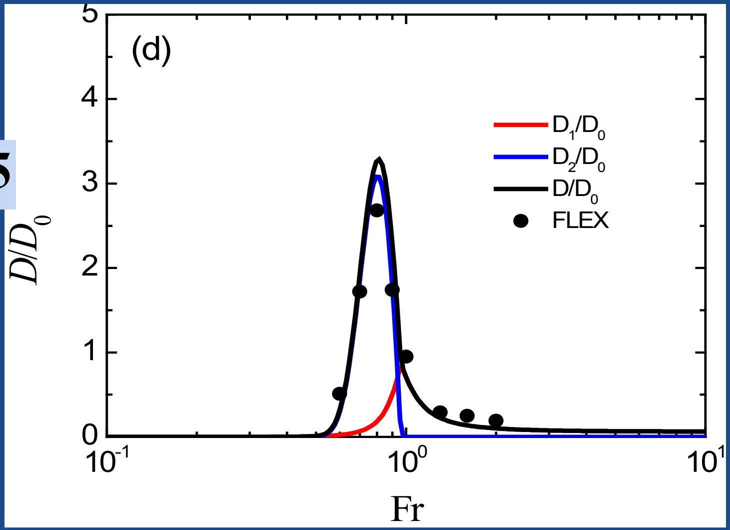
$l_2 a = 2$



$l_2 a = 1$



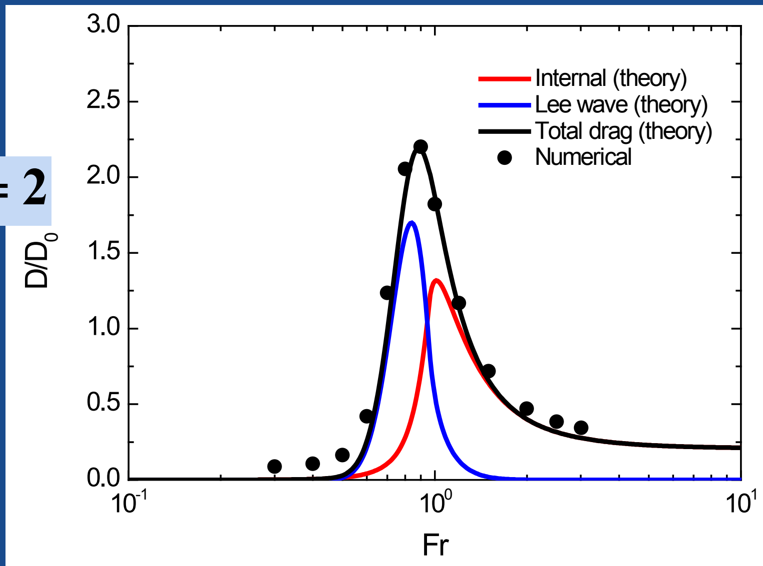
$l_2 a = 0.5$



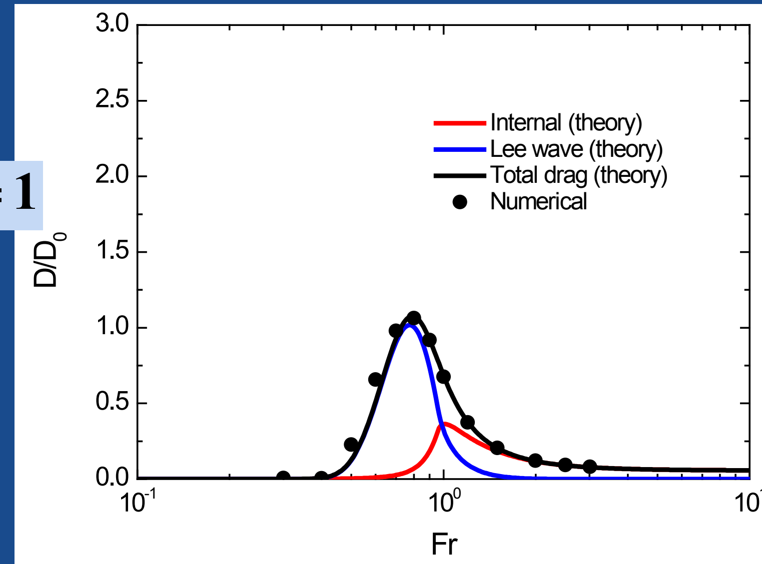
- D_2/D_0 may be large (~3)
- Single drag maximum exists at $Fr \approx 1$
- Agreement with numerical simulations requires considering both D_1 and D_2
- D_2/D_1 increases as $l_2 a$ decreases

Case 2 (3D): Drag

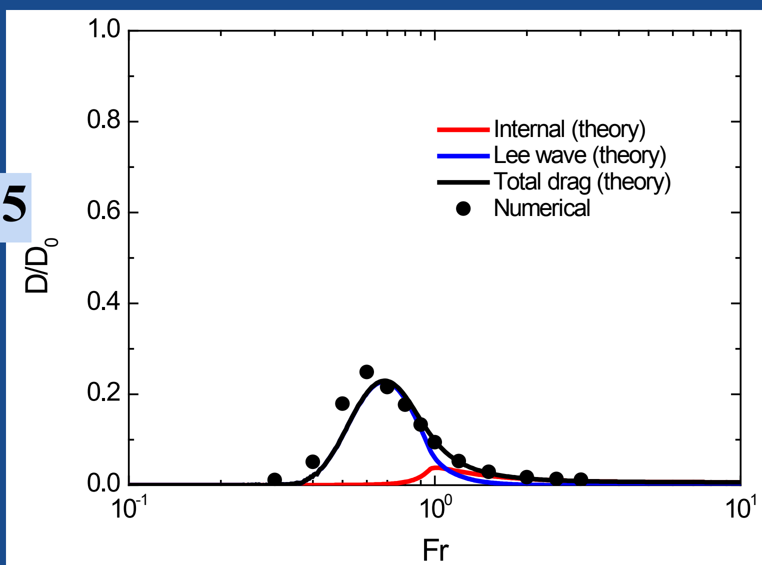
$l_2 a = 2$



$l_2 a = 1$



$l_2 a = 0.5$



- D_2/D_0 may be large (~1.5) → some directional wave dispersion
- Drag maximum lower and wider than in 2D → continuous spectrum of trapped lee waves
- Agreement with numerical simulations requires considering both D_1 and D_2
- D_2/D_1 substantially larger than in 2D and occur for lower $l_2 a$ → more non-hydrosatatic flow.

Department of
Meteorology

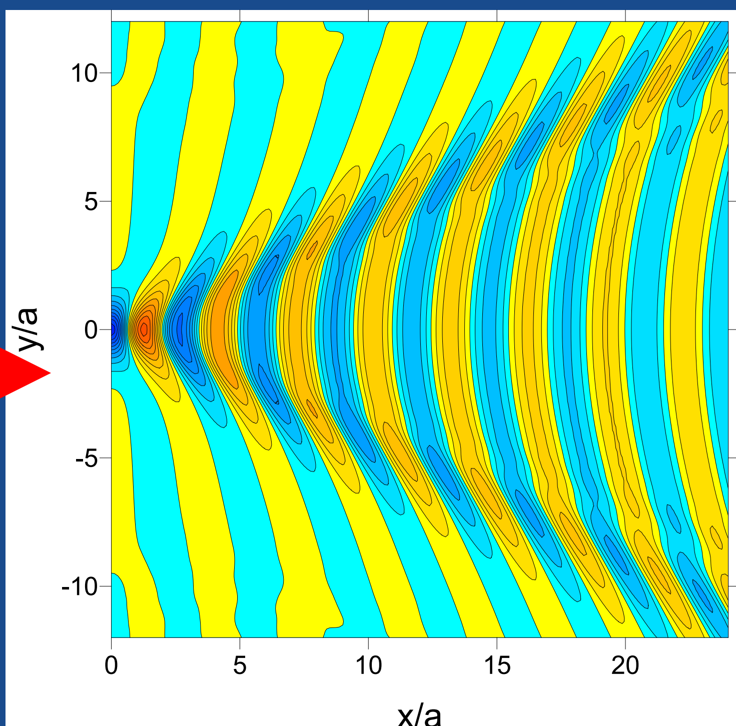
Case 2 (3D): Resonant trapped lee wave field

University of Reading

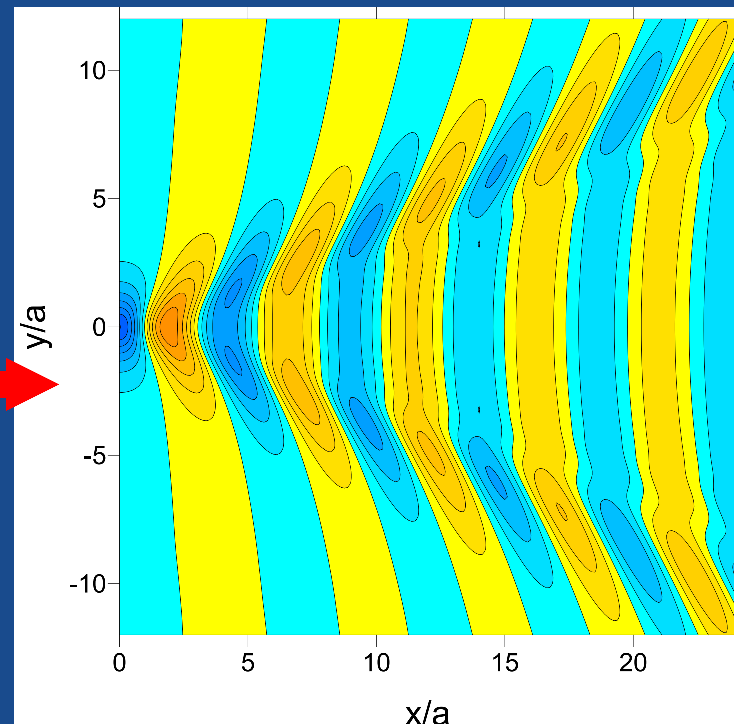
$w/(Uh_0/a)$ at $z=H$

for $Fr = 0.85$ $l_2 H / \pi = 0.5$

$l_2 a = 2$



$l_2 a = 1$



"Ship wave" pattern

Drag coefficient

2D obstacle

$$h = \frac{h_0}{1+(x/a)^2}$$

$$c_D = \frac{D}{(1/2)\rho_0 U^2 A_{length}} = \frac{D}{D_0}\frac{\pi}{2} l h_0$$

3D obstacle

$$h = \frac{h_0}{\left[1+(x/a)^2+(y/a)^2\right]^{3/2}}$$

$$c_D = \frac{D}{(1/2)\rho_0 U^2 A} = \frac{D}{D_0}\frac{\pi}{4} l h_0$$

Since for realistic atmospheric and orographic parameters, lh_0=0.1~0.5, multiplying factor relating D/D_0 and c_D is typically 0.1~0.8

c_D may easily be of O(1), especially for 2D mountains.

This is comparable to turbulent form drag on obstacles in non-stratified flow.

More details

Teixeira, Argain and Miranda (2013a), QJRMS, 139, 964-981

Teixeira, Argain and Miranda (2013b), JAS, 70, 2930-2947

Acknowledgements

- European Commission, through Marie Curie Career Integration Grant GLIMFLO, contract PCIG13-GA-2013-618016

Special Issue of Frontiers in Earth Science

"The Atmosphere over Mountainous Regions"

http://journal.frontiersin.org/researchtopic/3327/the-atmosphere-over-mountainous-regions

Summary

- 2D waves trapped in a layer may have multiple modes, waves trapped at temperature inversion may only have single mode
- Due to resonant amplification, trapped lee wave drag may be comparable to drag associated with waves propagating in stable upper layer, higher than uniform-flow hydrostatic reference value
- D_2/D_1 increases as $l_2 a$ decreases and as mountain becomes more 3D – non-hydrostatic effects. Trapped lee wave drag maximized for $l_2 a = O(1)$: wavelength of trapped lee waves matches mountain width
- 3D trapped lee waves produce less drag, and drag maxima are lower and wider: continuous wave spectrum – "ship wave" pattern.
- Trapped lee waves give substantial contribution to low-level drag, may be counted mistakenly as blocking drag or turbulent form drag (different dependence)