The COSMO model: towards cloud-resolving NWP

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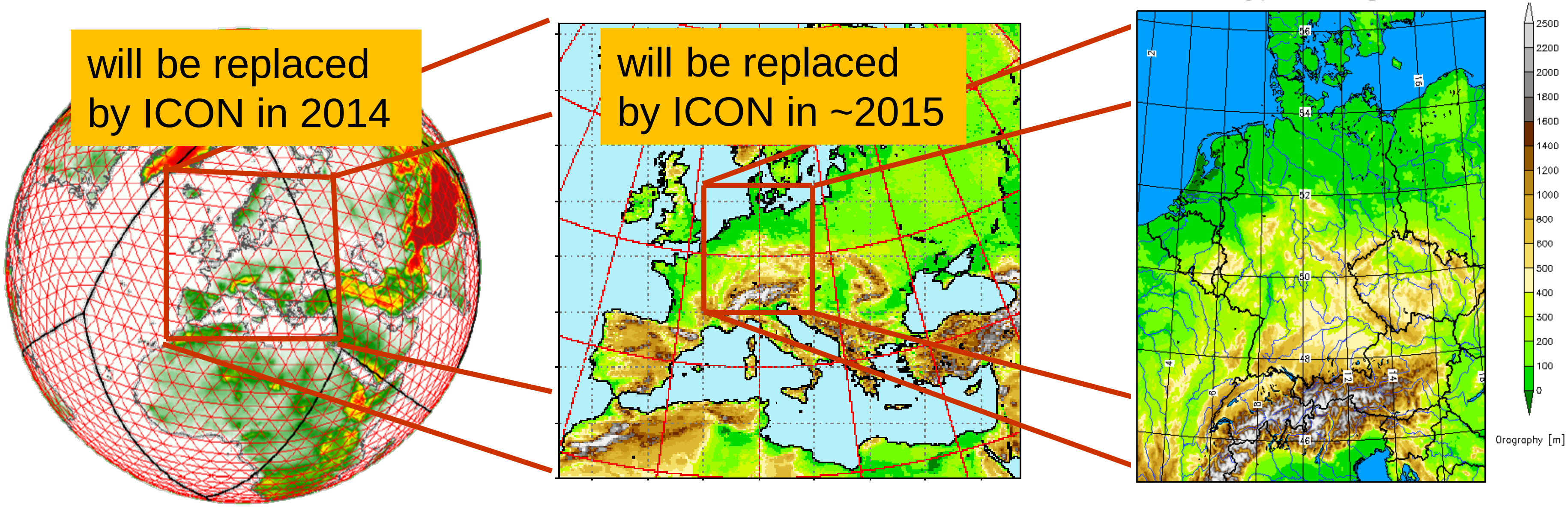
Outline

- The new fast waves solver of the COSMO model - consequences from the vertically stretched grid
- An new analytic solution to test LAM and global dynamical cores
- Influence of the water loading in strong convective situations
- Staggered vs. Unstaggered grids
  … and what does this mean for discontinuous Galerkin methods

The operational Model Chain of DWD: GME, COSMO-EU and -DE

GME | **COSMO-EU** | **COSMO-DE & -EPS**



GME	COSMO-EU	COSMO-DE & -EPS
hydrostatic	non-hydrostatic	non-hydrostatic
parameterised convection	parameterised convection	convection-permitting
$\Delta x \approx 20$ km	$\Delta x = 7$ km	$\Delta x = 2.8$ km
1482250 * 60 GP	665 * 657 * 40 GP	421 * 461 * 50 GP
$\Delta t = 66.7$ sec., T = 7 days	$\Delta t = 66$ sec., T = 78 h	$\Delta t = 25$ sec., T = 27 h

Revision of the current dynamical core

- redesign of the fast waves solver

Time integration scheme of COSMO dynamical core:

Wicker, Skamarock (2002) MWR:
split-explicit 3-stage Runge-Kutta
→ stable integration of 5th order upwind advection (large ΔT);
these tendencies are added in each fast waves step (small Δt)

‚Fast waves‘ processes (p'T'-dynamics):

$$
\begin{aligned}
\frac{\partial u}{\partial t} &= -\frac{1}{\rho}\,\frac{1}{r\cos\phi}\left(\frac{\partial p'}{\partial \lambda} + \frac{\partial \zeta}{\partial \lambda}\frac{\partial p'}{\partial \zeta}\right) + \frac{1}{\rho}\,\frac{1}{r\cos\phi}\left(\frac{\partial\, \alpha^h_{div}\,\rho\, D}{\partial \lambda} + \frac{\partial \zeta}{\partial \lambda}\frac{\partial\, \alpha^h_{div}\,\rho\, D}{\partial \zeta}\right) + f_u \\
\frac{\partial v}{\partial t} &= -\frac{1}{\rho}\,\frac{1}{r}\left(\frac{\partial p'}{\partial \phi} + \frac{\partial \zeta}{\partial \phi}\frac{\partial p'}{\partial \zeta}\right) + \frac{1}{\rho}\,\frac{1}{r}\left(\frac{\partial\, \alpha^h_{div}\,\rho\, D}{\partial \phi} + \frac{\partial \zeta}{\partial \phi}\frac{\partial\, \alpha^h_{div}\,\rho\, D}{\partial \zeta}\right) + f_v \\
\frac{\partial w}{\partial t} &= -\frac{1}{\rho}\left(\frac{\partial \zeta}{\partial z}\frac{\partial p'}{\partial \zeta}\right) + g\left(\frac{p_0}{p}\frac{T'}{T_0} - \frac{p'}{p} + \frac{p_0}{p}\frac{T}{T_0}\,q_x\right) + \frac{1}{\rho}\frac{\partial \zeta}{\partial z}\frac{\partial\, \alpha^v_{div}\,\rho\, D}{\partial \zeta} + f_w \\
\frac{\partial p'}{\partial t} &= -\frac{c_p}{c_V}\,p\,D + g\rho_0 w + f_p \\
\frac{\partial T'}{\partial t} &= -\frac{R}{c_V}\,T\,D - \frac{\partial T_0}{\partial z}\,w + f_T
\end{aligned}
$$

sound | buoyancy | artificial divergence damping, stabil. whole RK-scheme

$D = \mathrm{div}\, \boldsymbol{v}$

f_u, f_v, ... denote advection, Coriolis force and all physical parameterizations

Spatial discretization: centered differences (2nd order)

Temporal discretization: horizontally forward-backward, vertically implicit

stability: *Skamarock, Klemp (1992) MWR, Baldauf (2010) MWR*

Main changes towards the old fast waves solver:

1. improvement of the vertical discretization:
   use of weighted averaging operators for all vertical operations
2. divergence in strong conservation form
3. optional: complete 3D (=isotropic) divergence damping
4. optional: Mahrer (1984) discretization of horizontal pressure gradients

additionally some 'technical' improvements;
hopefully a certain increase in code readability

overall goal: improve numerical stability of COSMO

→ new version `fast_waves_sc.f90` contained in official COSMO 4.24

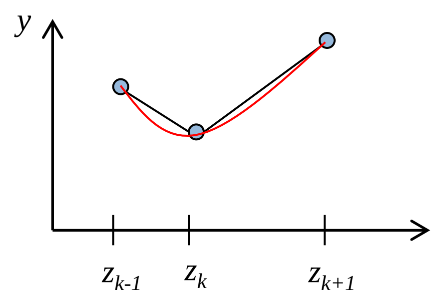
M. Baldauf (2013) COSMO Technical report No. 21 (www.cosmo-model.org)

Discretization in stretched grids

Example: calculate 1st derivative $\partial y / \partial z$ by an (at most) 3-point formula
(← tridiagonal solver)

Approach 1: by weightings in ‚original space‘

$$\left.\frac{dy}{dz}\right|_{z_k} = \frac{z_{k+1} - z_k}{z_{k+1} - z_{k-1}} \cdot \frac{y_k - y_{k-1}}{z_k - z_{k-1}} + \frac{z_k - z_{k-1}}{z_{k+1} - z_{k-1}} \cdot \frac{y_{k+1} - y_k}{z_{k+1} - z_k}$$



(e.g. *Ikeda, Durbin (2004) JCP*)

Approach 2: use of a coordinate transformation $z_k = f(\zeta_k)$, $\zeta_k = k\,\Delta\zeta$

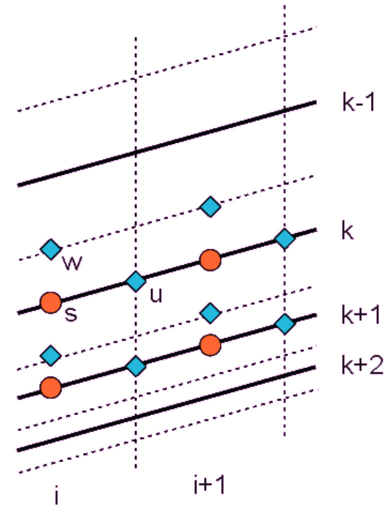
$$\frac{\partial y}{\partial z} = \frac{\partial \zeta}{\partial z} \frac{\partial y}{\partial \zeta}$$

↖ centered diff.

straightforward in unstaggered A-grid,
less clear in staggered C-grid

Improvement of the vertical discretization

COSMO: *Half levels* (w-positions) are defined by a stretching function $z_k = f(\zeta_k)$;
Main levels (p‘, T‘-pos.) lie in the middle of two half levels



Arithmetic average from half levels to main level:

$$\overline{\psi}^{\zeta} \equiv A_\zeta \psi|_{i,j,k} := \frac{1}{2}(\psi_{i,j,k-\frac{1}{2}} + \psi_{i,j,k+\frac{1}{2}})$$

Weighted average from main levels to half level

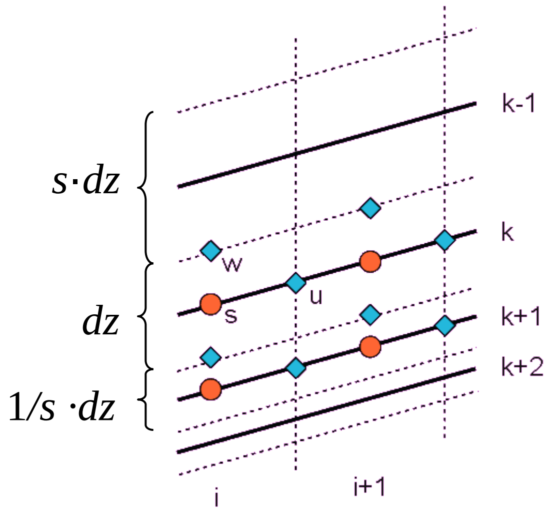
$$\overline{\psi}^{\zeta,N} \equiv A_\zeta^N \psi\Big|_{i,j,k-\frac{1}{2}} := g_{k-\frac{1}{2}}\psi_{i,j,k} + (1 - g_{k-\frac{1}{2}})\psi_{i,j,k-1}$$

Derivatives always by centered differences (appropriate average used before)

$$\delta_\zeta \psi|_{i,j,k} := \frac{\psi_{i,j,k+\frac{1}{2}} - \psi_{i,j,k-\frac{1}{2}}}{\Delta\zeta}$$

G. Zängl could show the advantages of weighted averages in the explicit parts of the fast waves solver.
New: application to all vertical operations (also the implicit ones)

How to inspect truncation errors in stretched grids?



… by Taylor expansion

Equidistant grids: let Δx, … → 0 (easy)

Non-equidistant grids: infinitely many possibilites to refine the grid!

Variant A. Define grid by a (fixed) stretching function

$z_k = f(\zeta_k), \quad \zeta_k = k \, \Delta\zeta,$

then let $\Delta\zeta$ → 0

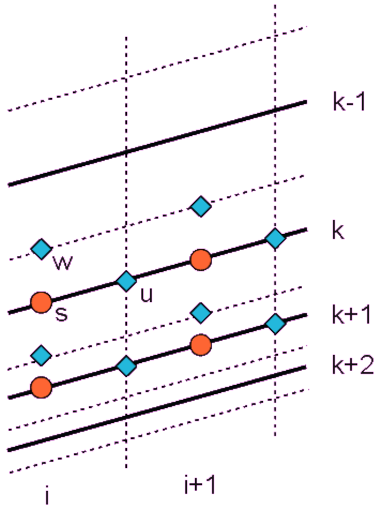
important: grid becomes locally increasingly linear (locally nearly non-stretched)

Variant B. Constant stretching ratio s between neighbouring grid cells:

$$\ldots, \quad z_{k+\frac{3}{2}} = \ldots, \quad z_{k+\frac{1}{2}} = z_{k+\frac{3}{2}} + \frac{1}{s}\Delta z, \quad z_{k-\frac{1}{2}} = z_{k+\frac{1}{2}} + \Delta z, \quad z_{k-\frac{3}{2}} = z_{k-\frac{1}{2}} + s\Delta z, \quad \ldots$$

then let Δz → 0

Buoyancy (~ $g\, T'/T_0$) – grid stretching variant A



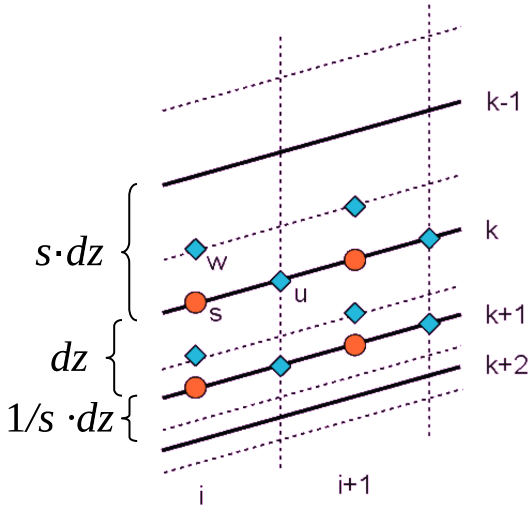
buoyancy term with weighted average of T' (T_0 exact):

$$\frac{1}{T_0} A_\zeta^N T' = \frac{T'}{T_0} + d\zeta^2 \frac{1}{T_0} \left[\frac{1}{8} \left(\frac{\partial z}{\partial \zeta} \right)^2 \frac{\partial^2 T'}{\partial z^2} \right] + O(d\zeta^4).$$

buoyancy term with arithmetic average of T' (T_0 exact):

$$\frac{1}{T_0} A_\zeta T' = \frac{T'}{T_0} + d\zeta^2 \frac{1}{T_0} \left[\frac{1}{8} \left(\frac{\partial z}{\partial \zeta} \right)^2 \frac{\partial^2 T'}{\partial z^2} + \boxed{\frac{1}{4} \frac{\partial^2 z}{\partial \zeta^2} \frac{\partial T'}{\partial z}} \right] + O(d\zeta^4).$$

Buoyancy (~ $g\ T'/T_0$) - grid stretching variant B



buoyancy term with weighted average of T' (T_0 exact):

$$\frac{1}{T_0} A^N_\zeta T' \quad = \quad \frac{T'}{T_0} + dz^2 \frac{1}{2} \frac{s}{(s+1)^2} \frac{1}{T_0} \frac{\partial^2 T'}{\partial z^2} + O(dz^3)$$

buoyancy term with arithmetic average of T' (T_0 exact):

$$\frac{1}{T_0} A_\zeta T' \quad = \quad \frac{T'}{T_0} + dz \frac{1}{2} \frac{s-1}{s+1} \frac{1}{T_0} \frac{\partial T'}{\partial z} + O(dz^2)$$

buoyancy term with weighted average for T' and T_0:

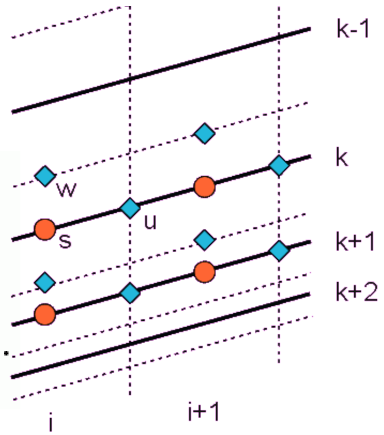
$$\frac{1}{A^N_\zeta T_0} A^N_\zeta T' \quad = \quad \frac{T'}{T_0} + dz^2 \frac{1}{2} \frac{s}{(s+1)^2} \left[\frac{1}{T_0} \frac{\partial^2 T'}{\partial z^2} - \frac{1}{T_0^2} \frac{\partial^2 T_0}{\partial z^2} T' \right] + O(dz^3)$$

Truncation error in stretched grids

Divergence – grid stretching variant A

$$\text{div}\,\mathbf{v} = \frac{1}{g}\left[\frac{\partial g u(x,\zeta)}{\partial x} + \frac{\partial}{\partial \zeta}\left(\frac{\partial z}{\partial x}u - w\right)\right]$$

Divergence with <u>weighted average</u> of u (and v) to the half levels:



$$\begin{aligned}\text{div}\mathbf{v} &= \frac{\partial u(x,z)}{\partial x} + \frac{\partial w(x,z)}{\partial z} + d\zeta^2\left(-\frac{1}{4}\frac{\partial^2 z}{\partial x \partial \zeta}\frac{\partial z}{\partial \zeta}\frac{\partial^2 u}{\partial z^2} - \frac{1}{4}\frac{\partial^2 z}{\partial \zeta^2}\frac{\partial z}{\partial x}\frac{\partial^2 u}{\partial z^2}\right.\\ &\left. -\frac{1}{6}\left(\frac{\partial z}{\partial \zeta}\right)^2\frac{\partial z}{\partial x}\frac{\partial^3 u}{\partial z^3} + \frac{1}{24}\left(\frac{\partial z}{\partial \zeta}\right)^2\frac{\partial^3 w}{\partial z^3}\right) + dx^2\,() + \ldots\end{aligned}$$

Divergence with <u>only arithmetic average</u> of u (and v) to the half levels:

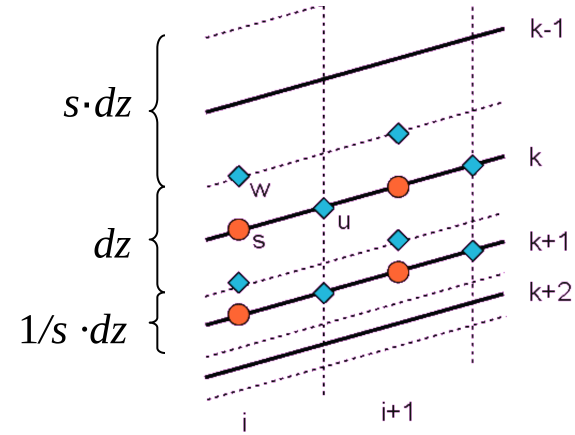
$$\begin{aligned}\text{div}\mathbf{v} &= \frac{\partial u(x,z)}{\partial x} + \frac{\partial w(x,z)}{\partial z} + d\zeta^2\left(-\frac{1}{4}\frac{\partial^2 z}{\partial x \partial \zeta}\frac{\partial z}{\partial \zeta}\frac{\partial^2 u}{\partial z^2} - \frac{1}{2}\frac{\partial^2 z}{\partial \zeta^2}\frac{\partial z}{\partial x}\frac{\partial^2 u}{\partial z^2}\right.\\ & -\frac{1}{6}\left(\frac{\partial z}{\partial \zeta}\right)^2\frac{\partial z}{\partial x}\frac{\partial^3 u}{\partial z^3} + \frac{1}{24}\left(\frac{\partial z}{\partial \zeta}\right)^2\frac{\partial^3 w}{\partial z^3}\\ &\left. -\frac{1}{4}\frac{\partial \zeta}{\partial z}\frac{\partial^2 z}{\partial x \partial \zeta}\frac{\partial^2 z}{\partial \zeta^2}\frac{\partial u}{\partial z} - \frac{1}{4}\frac{\partial \zeta}{\partial z}\frac{\partial^3 z}{\partial \zeta^3}\frac{\partial z}{\partial x}\frac{\partial u}{\partial z}\right) + dx^2\,() + \ldots\end{aligned}$$

$$\mathrm{div}\,\mathbf{v} = \frac{1}{g}\left[\frac{\partial g u(x,\zeta)}{\partial x} + \frac{\partial}{\partial \zeta}\left(\frac{\partial z}{\partial x} u - w\right)\right]$$

Divergence – grid stretching variant B

Divergence with <u>weighted average</u>
of u (and v) to the half levels:

$$\mathrm{div}\,\mathbf{v} = \left.\frac{\partial u}{\partial x}\right|_z + \left.\frac{\partial w}{\partial z}\right|_x + dz\,\frac{1}{8}\frac{\partial h}{\partial x}\frac{\partial^2 u}{\partial z^2}\left(\frac{1}{s} - s\right) + O(dz^2, dx^2)$$



Divergence with <u>only arithmetic average</u> of u (and v) to the half levels:

$$\begin{aligned}\mathrm{div}\,\mathbf{v} &= \left.\frac{\partial u}{\partial x}\right|_z + \frac{\partial h}{\partial x}\frac{\partial u}{\partial z}\left(\frac{1}{2} - \frac{1}{4}\left(s + \frac{1}{s}\right)\right) + \left.\frac{\partial w}{\partial z}\right|_x + \\ &+ dz\,\frac{1}{8}\frac{\partial h}{\partial x}\frac{\partial^2 u}{\partial z^2}\left(\left(\frac{1}{s} - s\right) + \frac{1}{2}\left(\frac{1}{s^2} - s^2\right)\right) + O(dz^2, dx^2)\end{aligned}$$

not a consistent discretization if $s \neq 1$ *!*

Summary

- New fast waves solver since COSMO 4.24
- in operational use at DWD since 16 Jan. 2013
- the higher numerical stability (in particular in steeper terrain) stems at least partly from a better and more consistent discretization in a vertically stretched grid
- Remind: in a stretched *and* staggered grid not every information is contained in the metric coefficients. The relations between main and half levels influence the discretization
- Proper derivation (use the exact positions of half and main levels!) of truncation errors helps in the decision in which way weightings should be used.

M. Baldauf (2013) COSMO Technical report No. 21 (www.cosmo-model.org)

How can we check the correctness of the previous considerations?

An analytic solution for linear gravity waves in a channel as a test case for solvers of the non-hydrostatic, compressible Euler equations

Motivation

For the development of dynamical cores (or numerical methods in general) idealized test cases are an important evaluation tool.

- Idealized standard test cases with (at least approximated) analytic solutions:
  - stationary flow over mountains
    linear: *Queney (1947, ...), Smith (1979, ...) Adv Geophys, Baldauf (2008) COSMO-Newsl.*
    non-linear: *Long (1955) Tellus* for Boussinesq-approx. Atmosphere
  - Balanced solutions on the sphere: *Staniforth, White (2011) ASL*
  - non-stationary, linear expansion of gravity waves in a channel
    Skamarock, Klemp (1994) MWR for Boussinesq-approx. atmosphere
- most of the other idealized tests only possess 'known solutions' gained from other numerical models.

There exist even fewer analytic solutions which use exactly the same equations as the numerical model used, i.e. in the sense that the numerical model converges to this solution. One exception is given here:

linear expansion of gravity/sound waves in a channel

Non-hydrostatic, compressible, 2D Euler equations in a flat channel (shallow atmosphere) on an f-plane

$$\frac{\partial u}{\partial t} + \mathbf{v} \cdot \nabla u = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv,$$

$$\frac{\partial v}{\partial t} + \mathbf{v} \cdot \nabla v = -fu,$$

$$\frac{\partial w}{\partial t} + \mathbf{v} \cdot \nabla w = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g,$$

$$\frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho = -\rho \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right),$$

$$\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p = c_s'^2 \left(\frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho \right),$$

$$T = \frac{p}{R\rho},$$

$$c_s' = \sqrt{\frac{c_p}{c_v} RT},$$

most LAMs using the compressible equations should be able to exactly use these equations in the dynamical core

For analytic solution only one further approximation is needed:
<u>linearisation</u> (= *controlled* approximation) around an **isothermal, steady, hydrostatic** atmosphere at rest (f≠0 possible) or with a constant basic flow U_0 (and f=0)

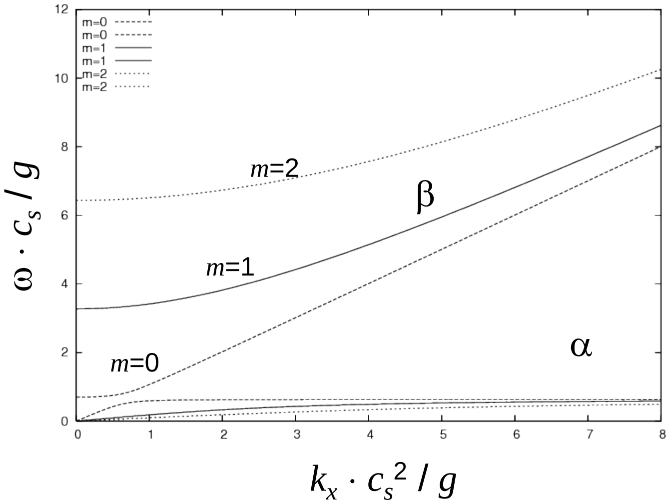
Bretherton-, Fourier- and Laplace-Transformation →

Analytic solution for the Fourier transformed vertical velocity w

$$\hat{w}_b(k_x, k_z, t) = -\frac{1}{\beta^2 - \alpha^2}\left[-\alpha \sin \alpha t + \beta \sin \beta t + (f^2 + c_s^2 k_x^2)\left(\frac{1}{\alpha}\sin \alpha t - \frac{1}{\beta}\sin \beta t\right)\right] g \frac{\hat{\rho}_b(k_x, k_z, t=0)}{\rho_s}$$

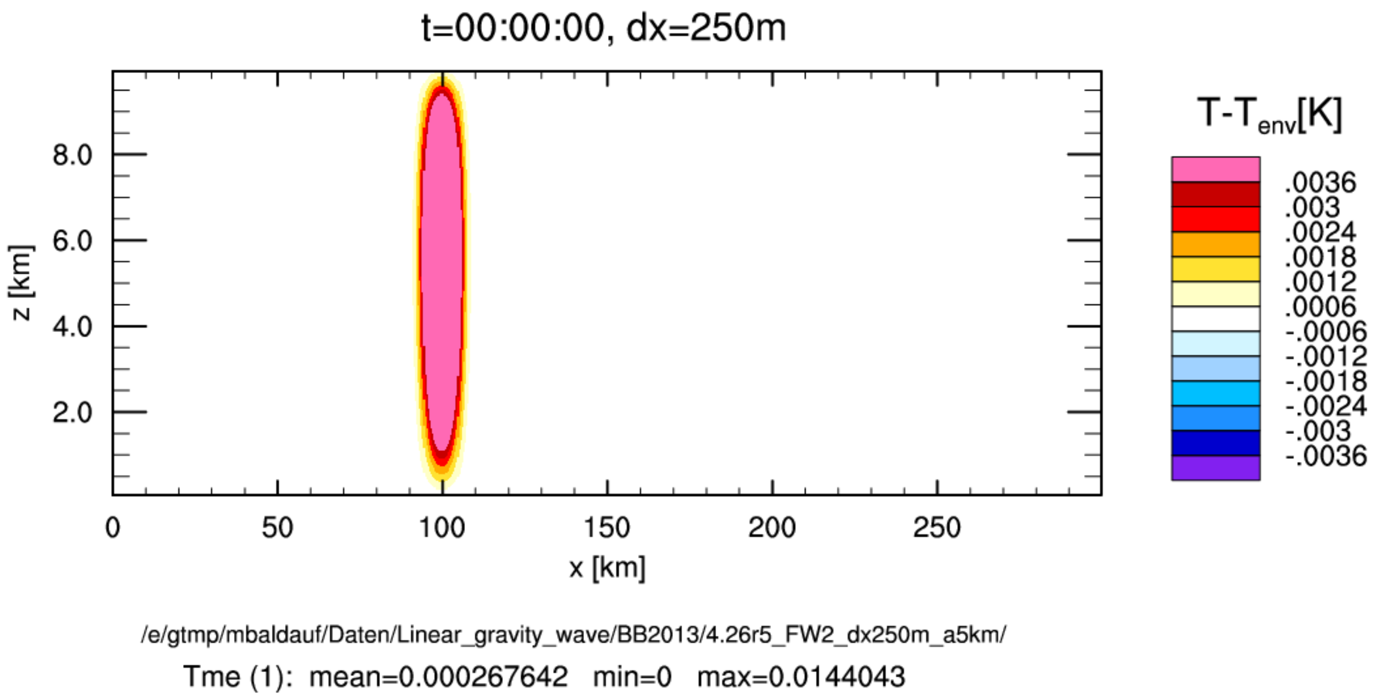
analogous expressions for $u_b(k_x, k_z, t)$, ...

The frequencies α, β are the gravity wave and acoustic branch, respectively, of the dispersion relation for compressible waves in a channel with height H;
$k_z = (\pi / H) \cdot m$



Baldauf, Brdar (2013) QJRMS

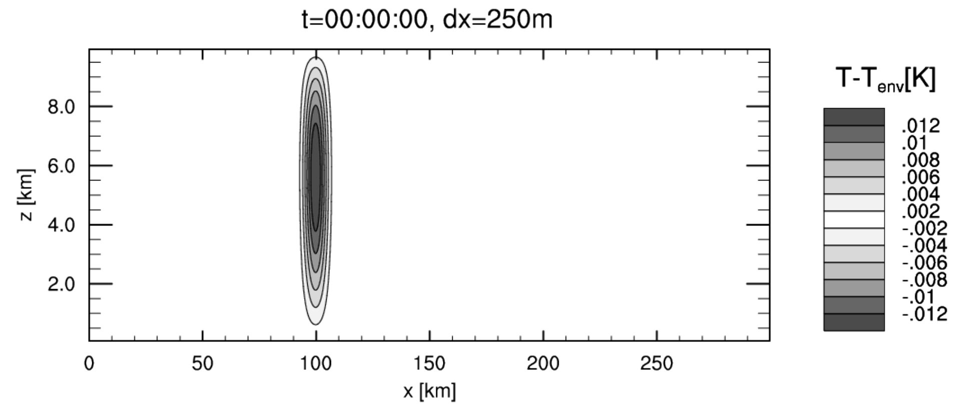
Linear, unsteady gravity wave
initialization similar to *Skamarock, Klemp (1994) MWR*



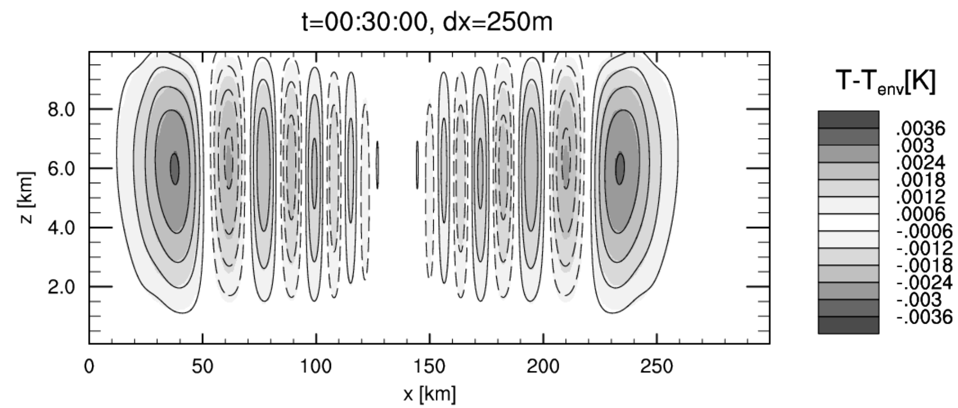
Initialization similar to
Skamarock, Klemp (1994)

$$T'(x,z,t=0) = \Delta T \cdot e^{\frac{1}{2}\delta z} \cdot e^{-\frac{(x-x_c)^2}{d^2}} \cdot \sin \pi \frac{z}{H}$$

$$p'(x,z,t=0) = 0$$

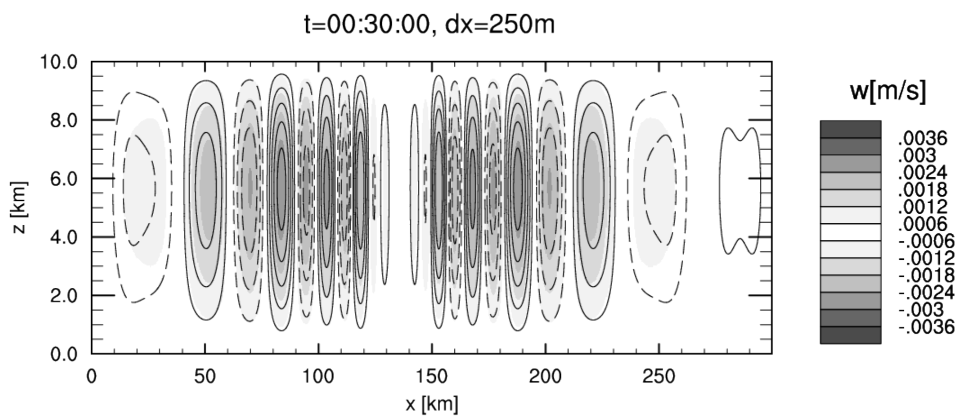


Small scale test
with a basic flow U_0=20 m/s
f=0



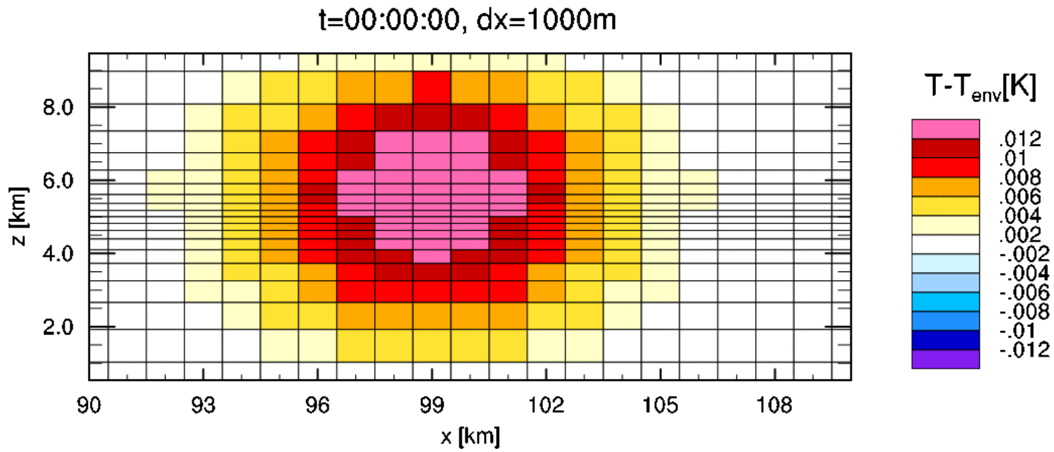
Black lines: analytic solution
(Baldauf, Brdar (2013) QJRMS)

Shaded: COSMO

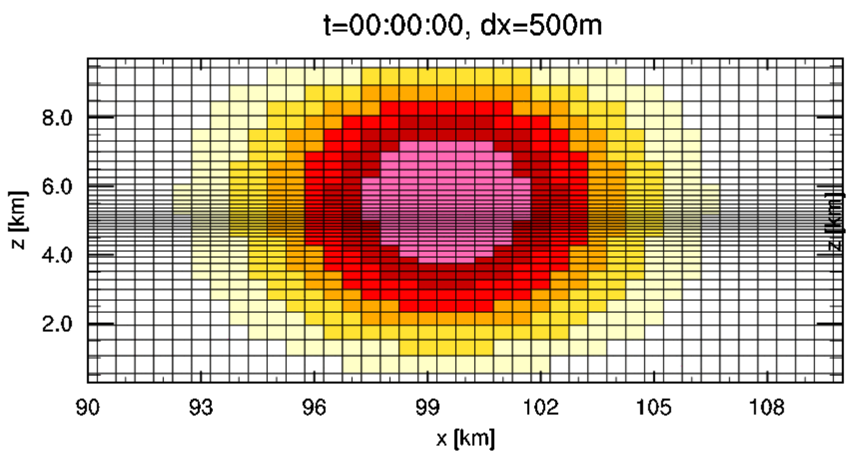


Convergence test with vertically stretched grid

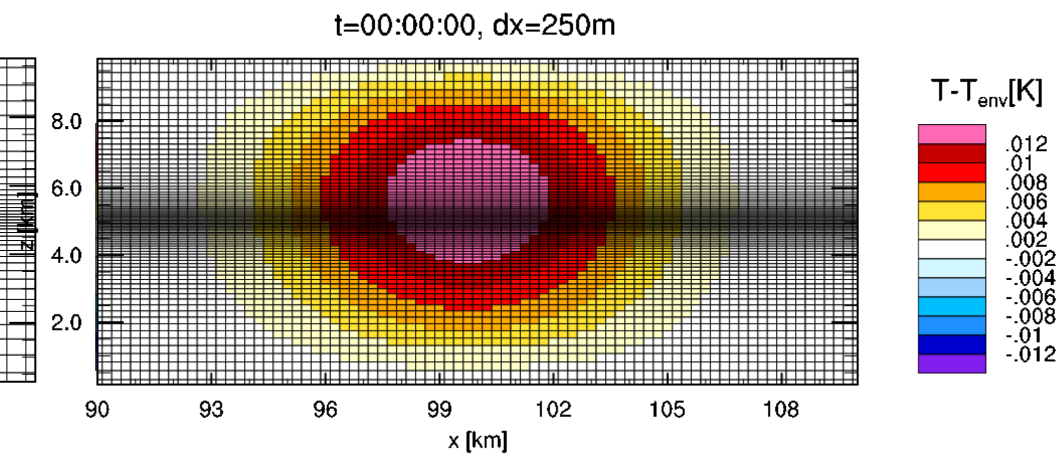
initial condition for T'
and grids
for the first 3 resolutions



/e/gtmp/mbaldauf/Daten/Linear_gravity_wave/BB2013/4.26r5_FW2_dx1000m_a5km/
Tme (1): mean=0.000330114 min=0 max=0.0144043



/e/gtmp/mbaldauf/Daten/Linear_gravity_wave/BB2013/4.26r5_FW2_dx500m_a5km/
Tme (1): mean=0.000330524 min=0 max=0.0144043

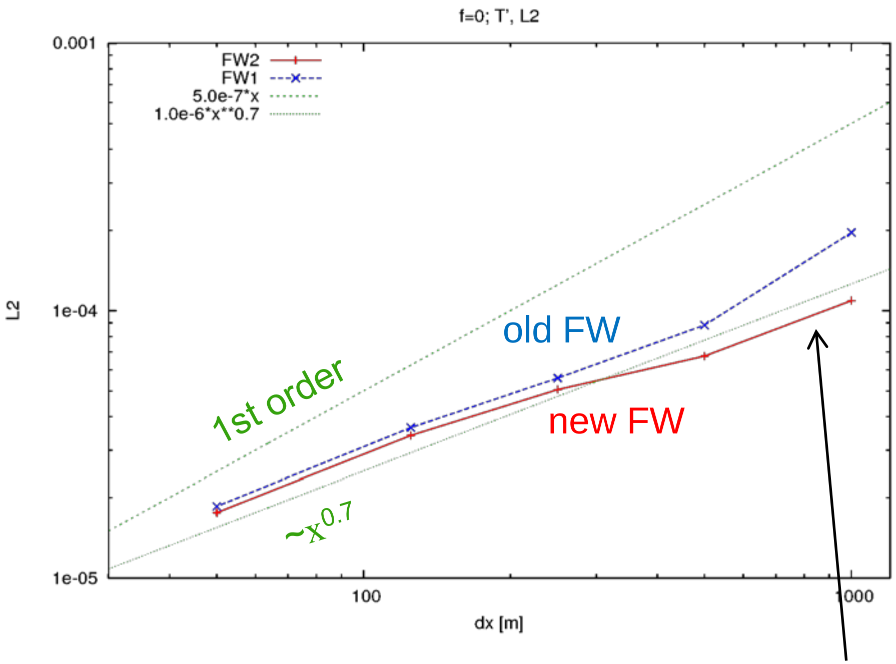


/e/gtmp/mbaldauf/Daten/Linear_gravity_wave/BB2013/4.26r5_FW2_dx250m_a5km/
Tme (1): mean=0.00033047 min=0 max=0.0144043

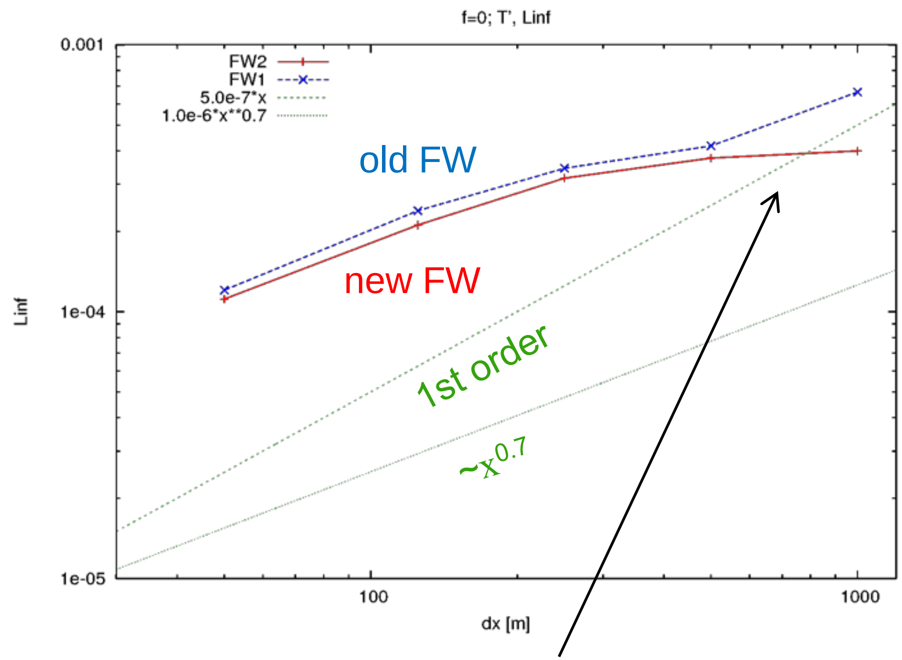
Convergence test with vertically stretched grid for old and new fast waves solver

T'

L_2-error



L_∞-error



the improvement is best for coarse resolutions, because here the highest relative stretching for neighbouring grid boxes occurs

The analogous linearized solution on the sphere …

Non-hydrostatic, compressible, shallow atmosphere, adiabatic, 3D Euler equations on a sphere with a rigid lid

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} = -\frac{1}{\rho}\nabla p - g\mathbf{e}_z - 2\mathbf{\Omega}\times\mathbf{v}$$

$$\frac{\partial \rho}{\partial t} + \mathbf{v}\cdot\nabla\rho = -\rho\nabla\cdot\mathbf{v}$$

$$\frac{\partial p}{\partial t} + \mathbf{v}\cdot\nabla p = c_s^2\left(\frac{\partial \rho}{\partial t} + \mathbf{v}\cdot\nabla\rho\right)$$

$$c_s = \sqrt{\frac{c_p}{c_v}\frac{p}{\rho}}$$

most global models using the compressible equations should be able to exactly use these equations in the dynamical core for testing.

Boundary conditions:
$w(r=r_s) = 0$
$w(r=r_s+H) = 0$

For an analytic solution only one further approximation is needed:
linearisation (= *controlled* approximation) around an
isothermal, steady, hydrostatic atmosphere

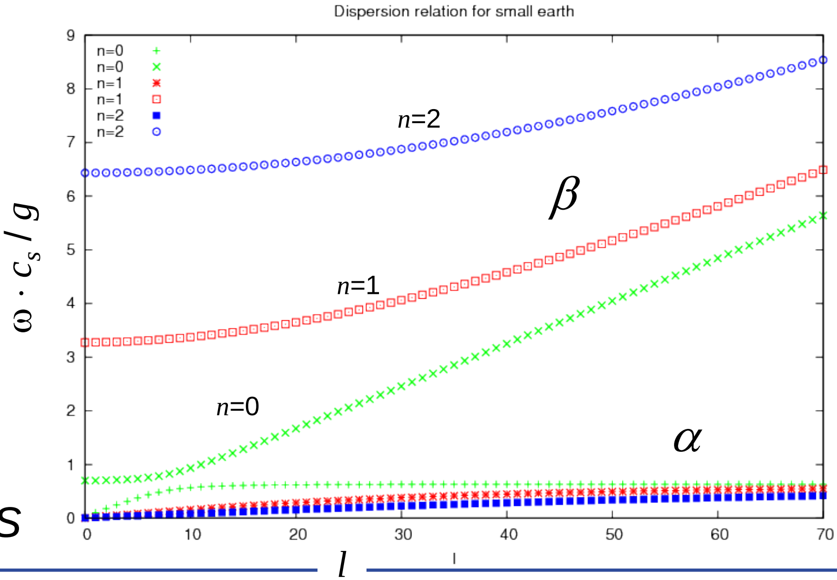
Analytic solution

for the vertical velocity w (Fourier component with k_z, spherical harmonic with l,m)

$$\hat{w}_{lm}(k_z,t) = -\frac{1}{\beta^2-\alpha^2}\left[-\alpha\sin\alpha t + \beta\sin\beta t + \left(f^2 + c_s^2\frac{l(l+1)}{r_s^2}\right)\left(\frac{1}{\alpha}\sin\alpha t - \frac{1}{\beta}\sin\beta t\right)\right] g\frac{\hat{\rho}_{lm}(k_z,t=0)}{\rho_s}$$

analogous expressions for $\hat{u}_{lm}(k_z, t)$, ...

The frequencies α, β are the gravity wave and acoustic branch, respectively, of the dispersion relation for compressible waves in a spherical channel of height H; $k_z = (\pi / H) \cdot n$



Baldauf, Reinert, Zängl (2013) acc. by QJRMS

Test scenarios

(A) Only gravity wave and sound wave expansion

(B) Additional Coriolis force (‚global f-plane approx. on a sphere')

$$2\boldsymbol{\Omega}(\lambda,\phi) = f \cdot \boldsymbol{e}_r(\lambda,\phi), \quad f=\text{const.} \qquad \text{(and } \boldsymbol{v}_0 = 0\text{)}$$

→ test proper discretization of inertia-gravity modes, e.g. in a C-grid discretization.

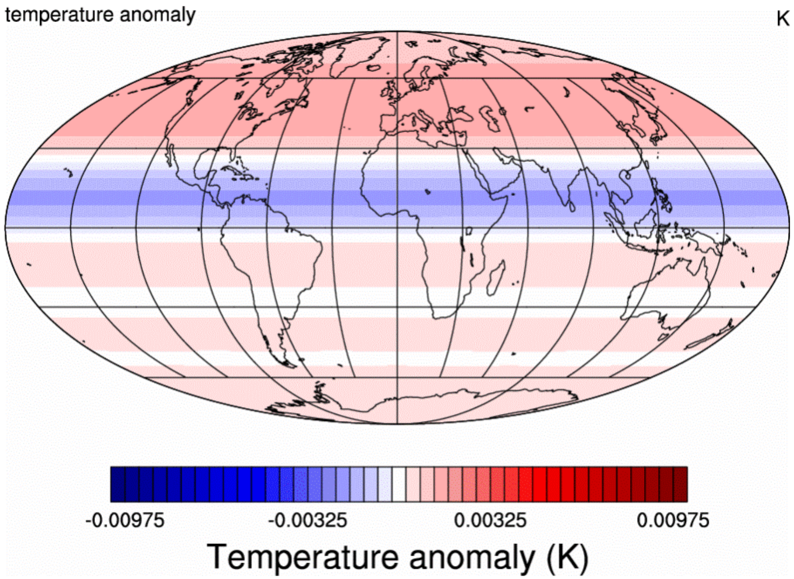
For problems with C-grid discretizations on non-quadrilateral grids see
Nickowicz, Gavrilov, Tosic (2002) MWR,
Thuburn, Ringler, Skamarock, Klemp (2009) JCP,
Gassmann (2011) JCP

(C) Additional advection by a solid body rotation velocity field $\boldsymbol{v}_0 = \boldsymbol{Q} \times \boldsymbol{r}$
→ test the coupling of fast (buoyancy, sound) and slow (advection, Coriolis) processes
Problem: solid body rotation field generates centrifugal forces!
Solution: $\boldsymbol{Q} = -\boldsymbol{\Omega}$ → similar to (A) in the absolute system
(analogous to *Läuter et al. (2005) JCP*)

ICON (joint development of DWD/MPI-M) **simulation**

→ Talk by G. Zängl

in z=5 km (f=0)



Small earth simulations

Wedi, Smolarkiewicz (2009) QJRMS

- $r_s = r_{earth} / 50 \sim 127$ km
  simulations with $\Delta\varphi \sim 1°... 0.0625°$
  → $\Delta x \sim 2.2$ km ... 0.14 km
  → non-hydrostatic regime
- for runs *with* Coriolis force:
  $f = f_{earth} \cdot 10 \sim 10^{-3}$ 1/s
  → dimensionless numbers

$$Ro = 0.2 \cdot Ro_{earth}$$

$$f / N = 10 \cdot f_{earth} / N \sim 0.05$$

Time evolution of T‘

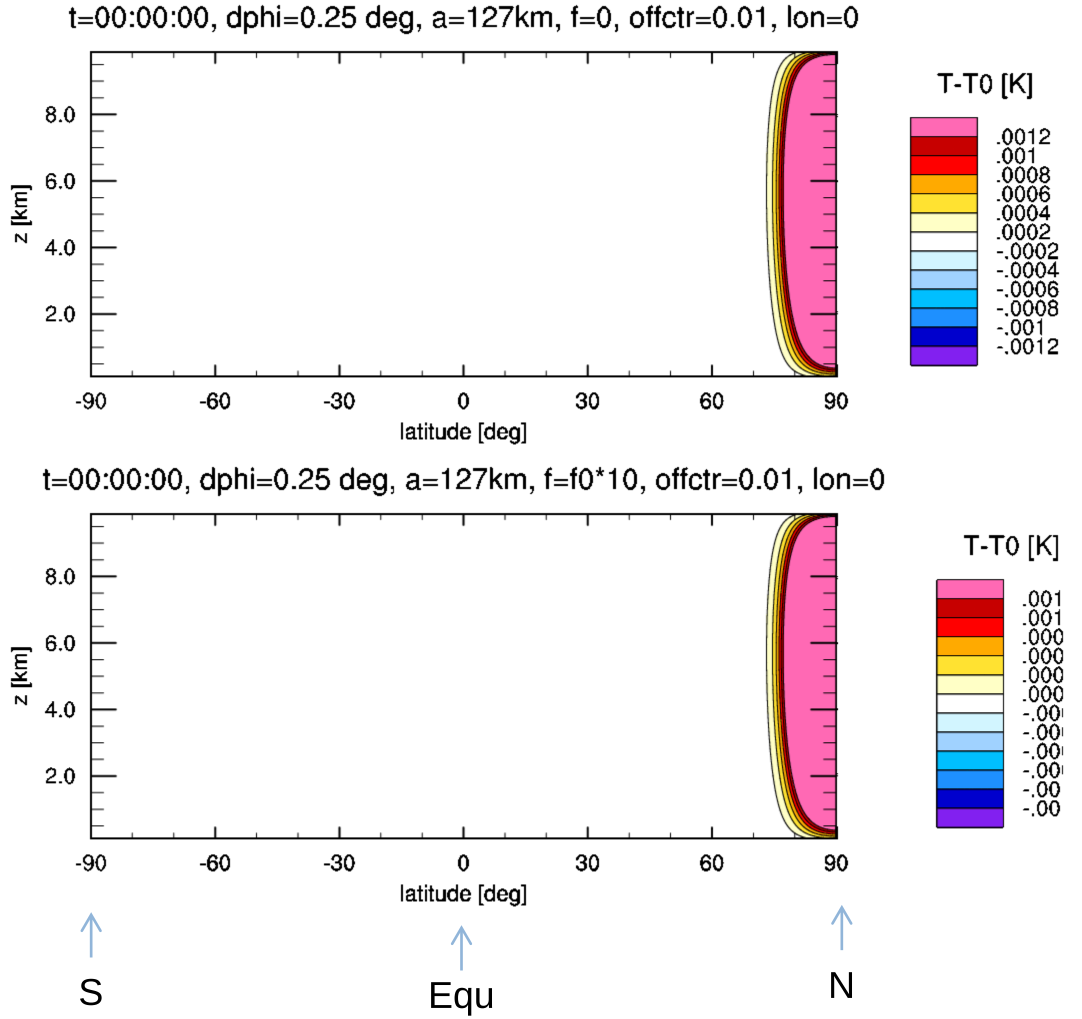
f=0

test scenario (A)

f≠0

test scenario (B)

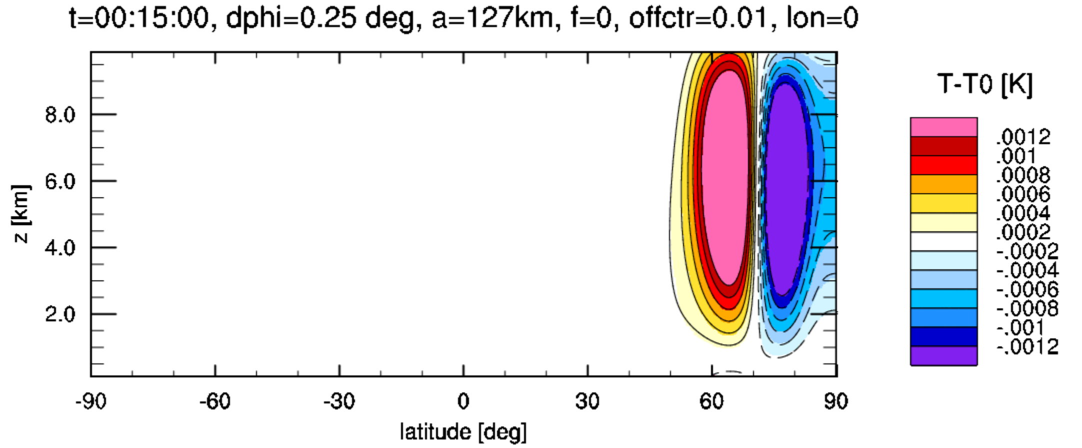
Black lines: analytic solution
Colours: ICON simulation



Time evolution of T‘

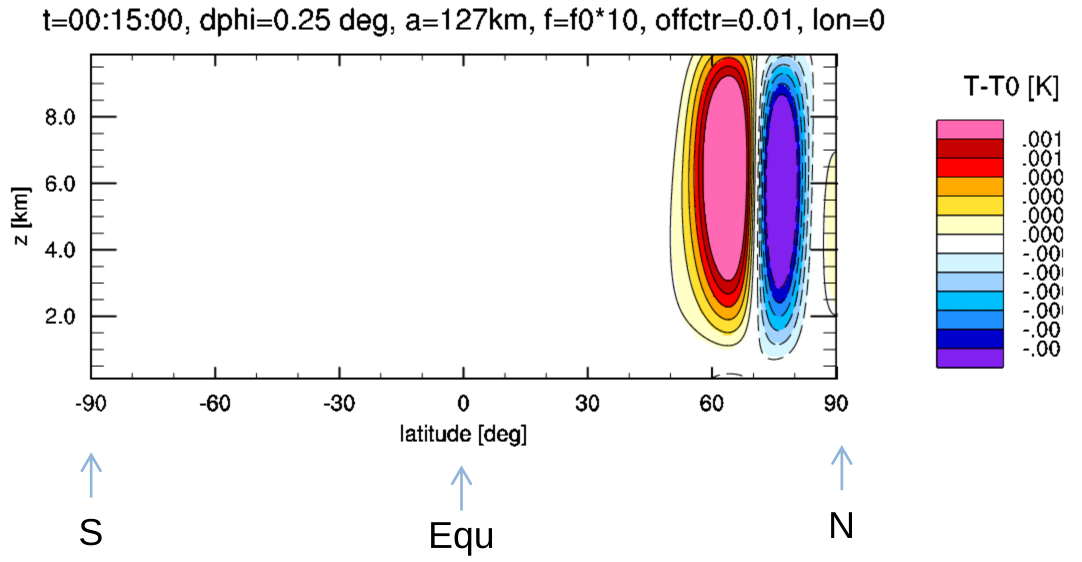
f=0

test scenario (A)



$f\neq0$

test scenario (B)



Black lines: analytic solution

Colours: ICON simulation

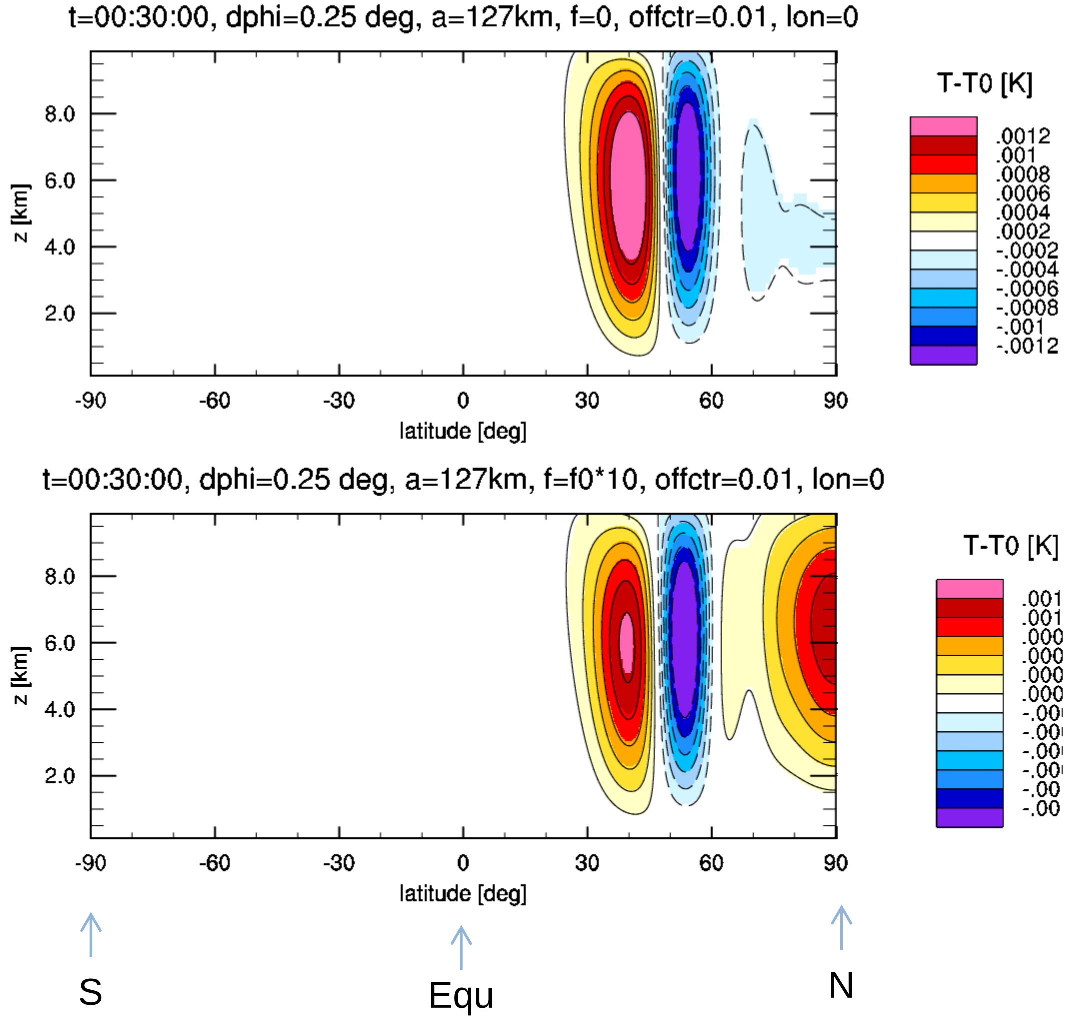
Time evolution of T'

$f=0$

test scenario (A)

$f\neq 0$

test scenario (B)

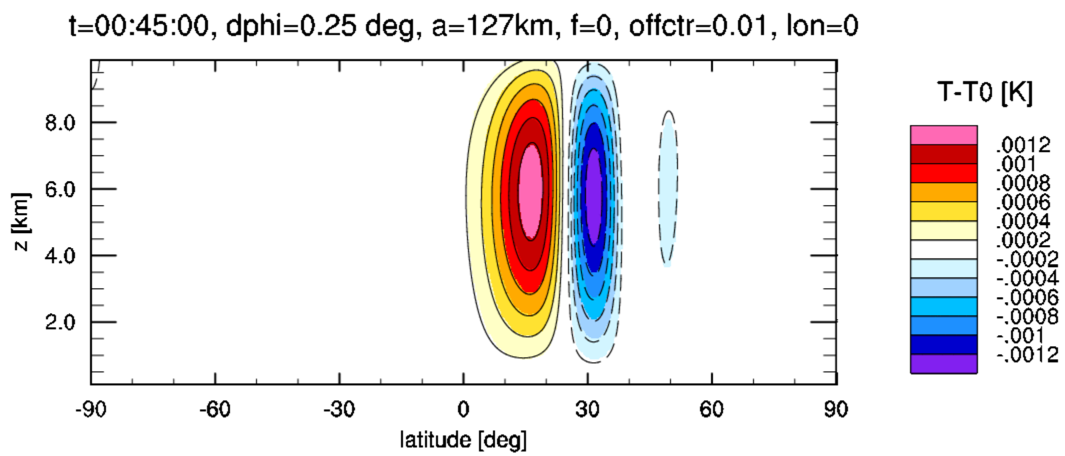


Black lines: analytic solution
Colours: ICON simulation

Time evolution of T‘

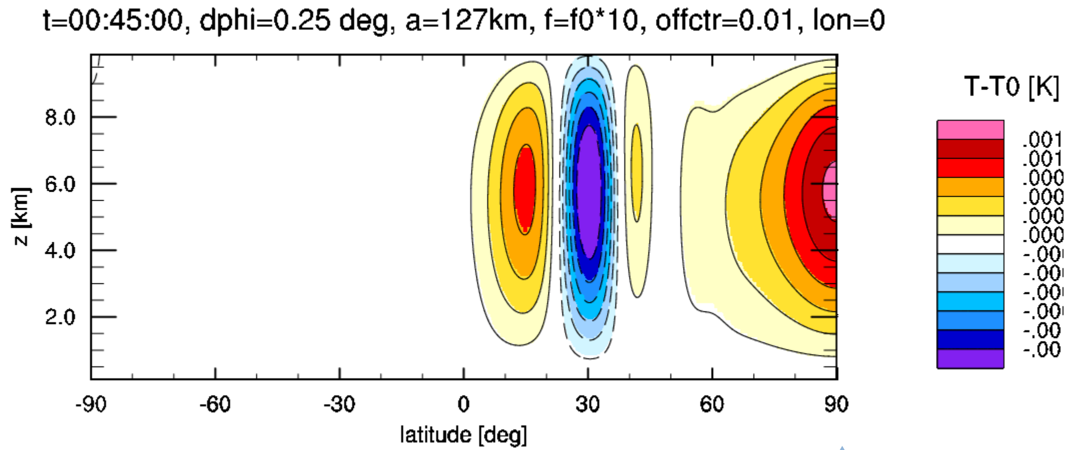
f=0

test scenario (A)



f≠0

test scenario (B)



S

Equ

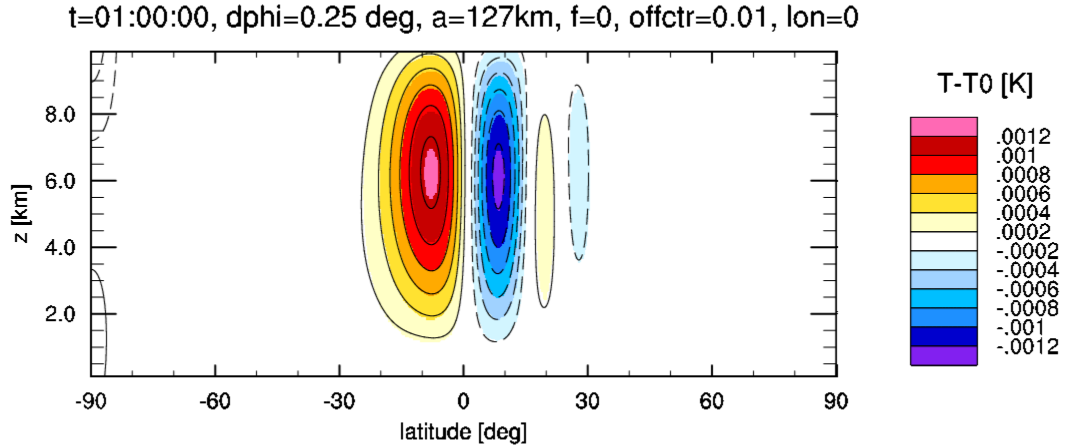
N

Black lines: analytic solution
Colours: ICON simulation

Time evolution of T‘

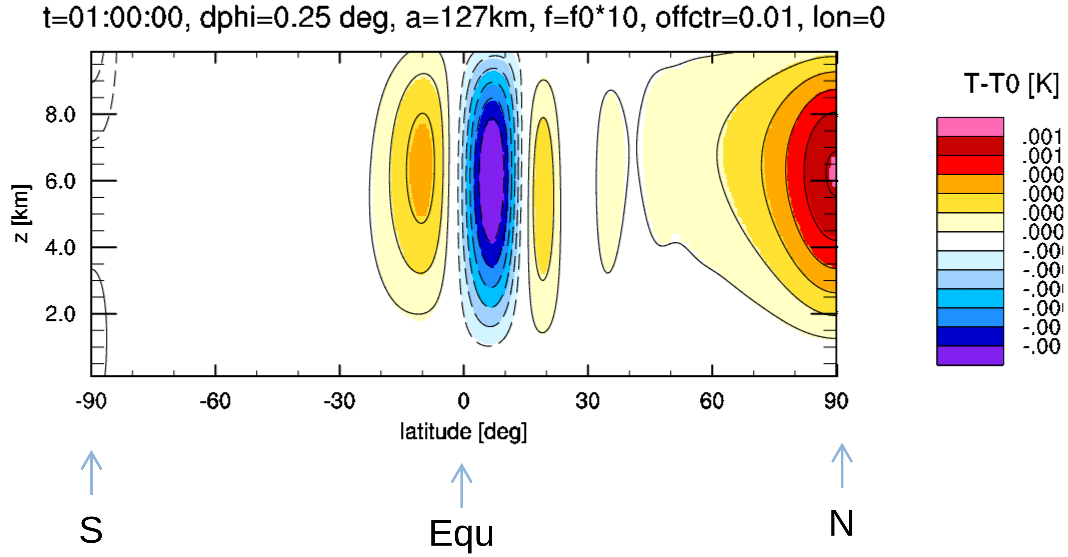
f=0

test scenario (A)



f≠0

test scenario (B)

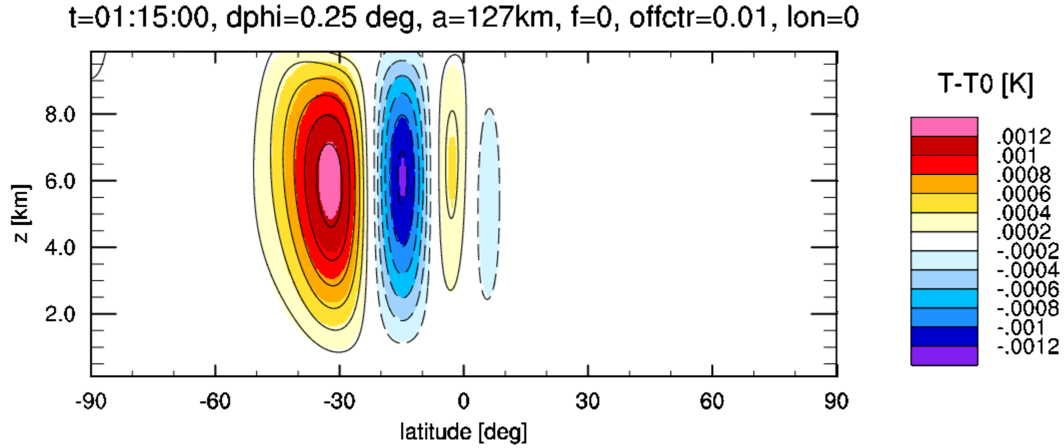


Black lines: analytic solution
Colours: ICON simulation

Time evolution of T‘

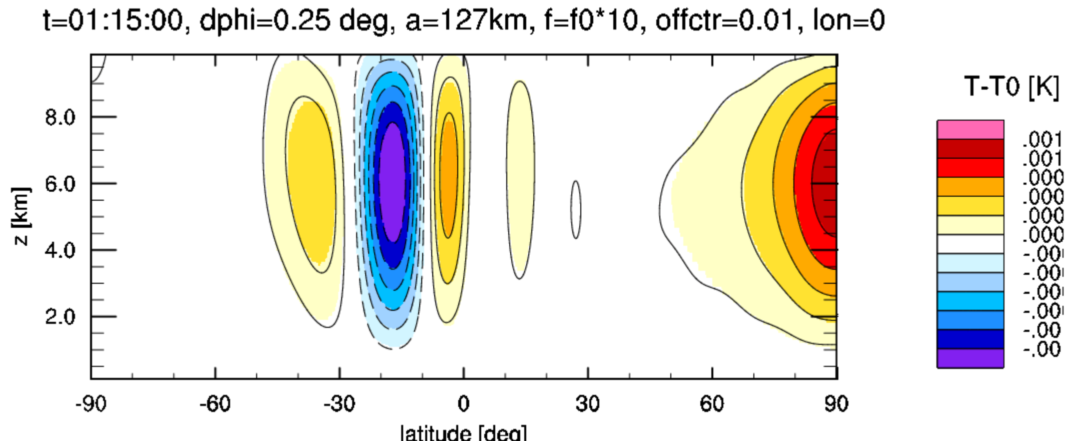
f=0

test scenario (A)



f≠0

test scenario (B)



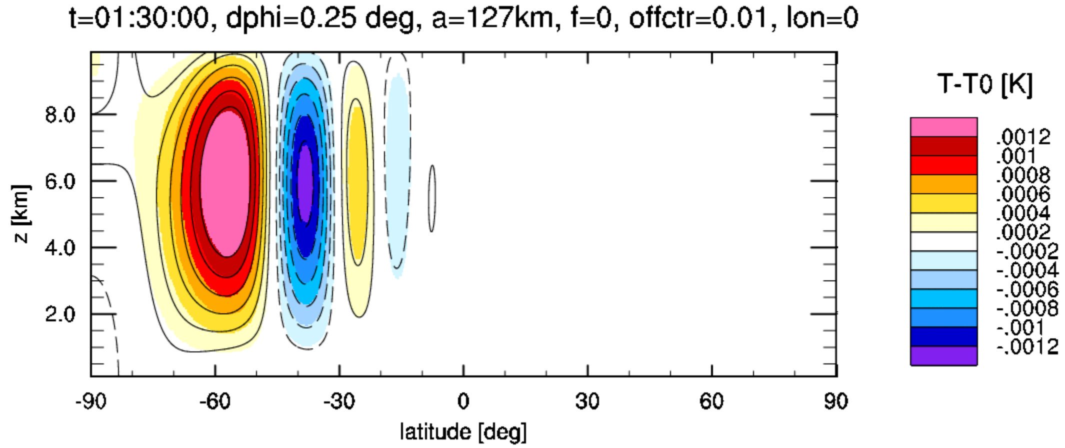
S Equ N

Black lines: analytic solution
Colours: ICON simulation

Time evolution of T'

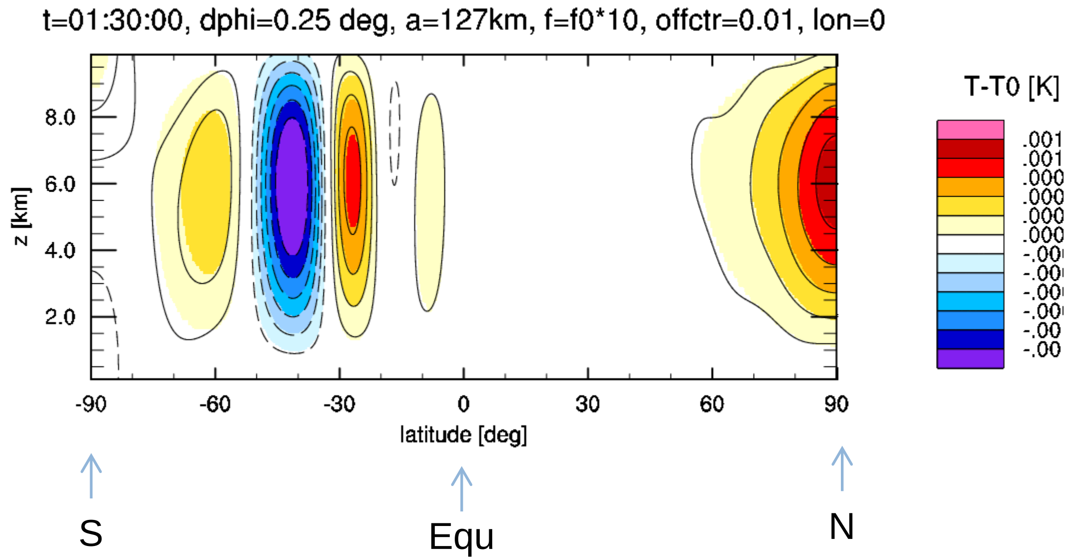
$f=0$

test scenario (A)



$f \neq 0$

test scenario (B)



Black lines: analytic solution
Colours: ICON simulation

Time evolution of T‘

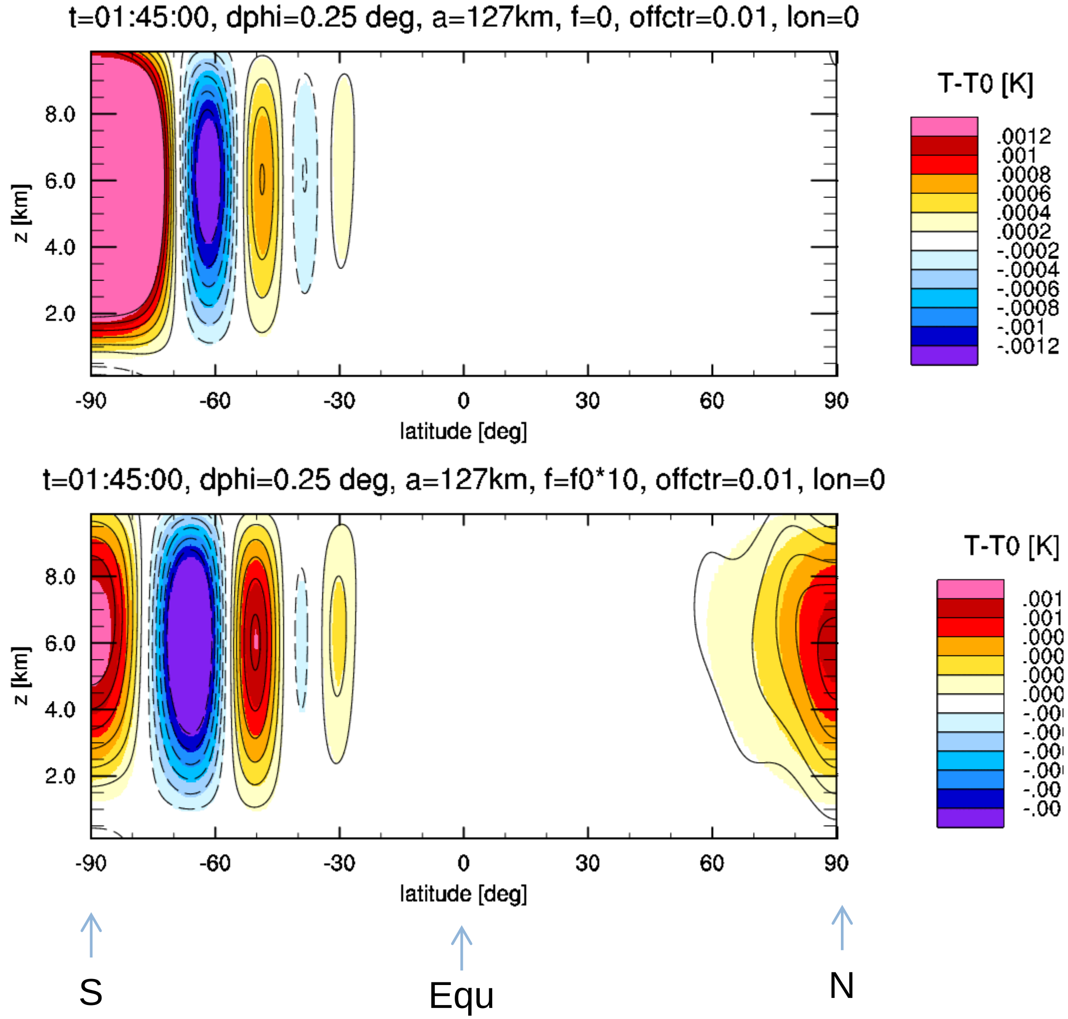
f=0

test scenario (A)

$f \neq 0$

test scenario (B)

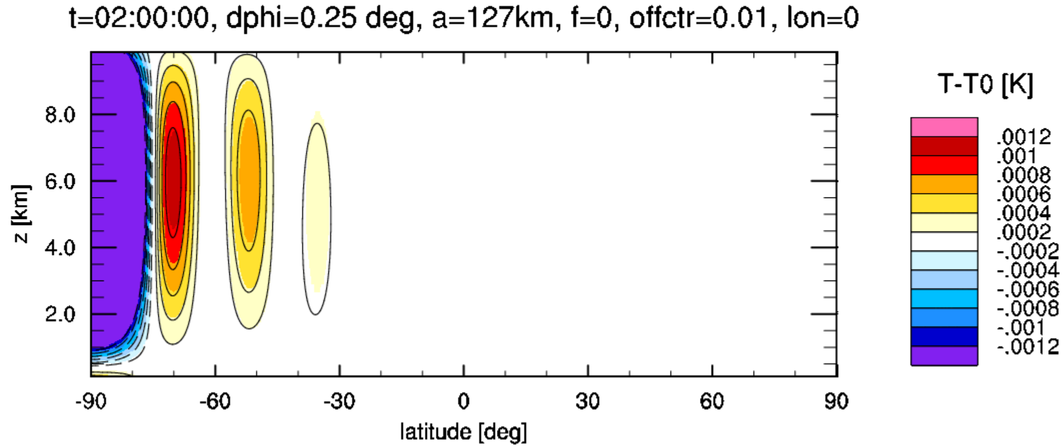
Black lines: analytic solution
Colours: ICON simulation



Time evolution of T‘

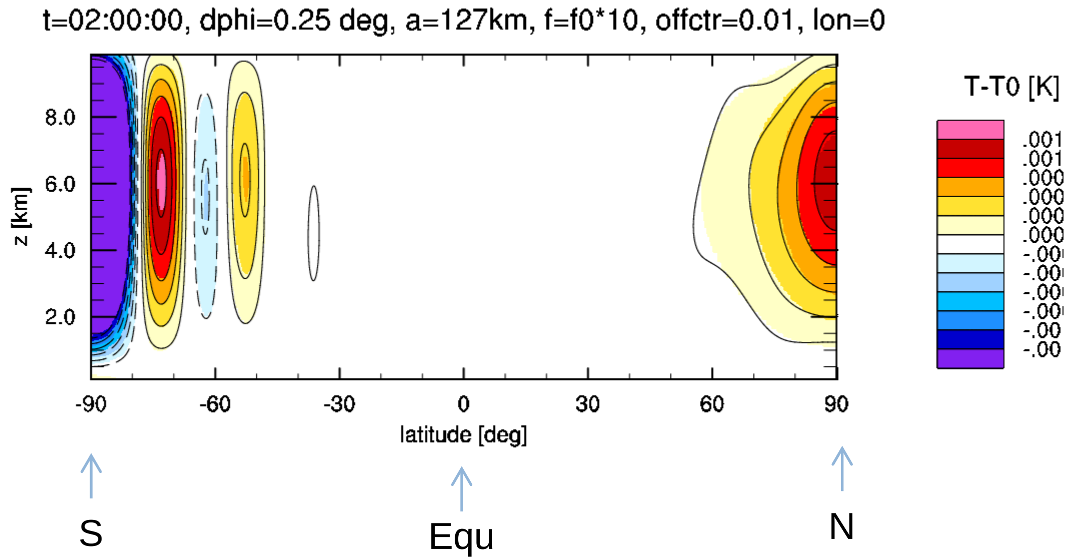
$f=0$

test scenario (A)



$f\neq0$

test scenario (B)

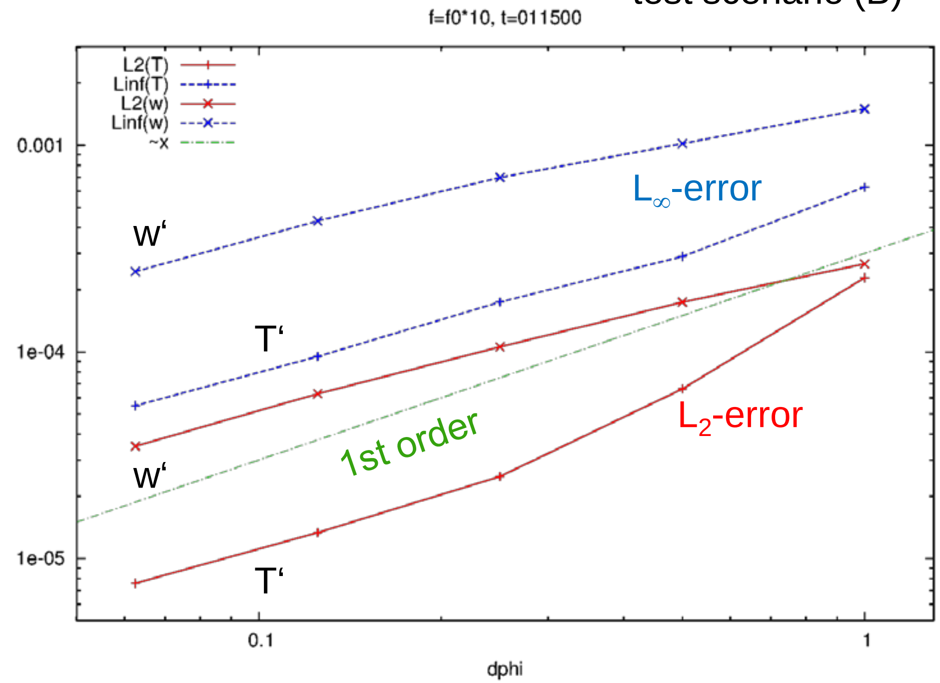


Black lines: analytic solution
Colours: ICON simulation

Convergence rate of the ICON model

- The ICON simulation with/without Coriolis force produces almost similar L_2, L_∞ errors
- Spatial-temporal convergence order of ICON is ~ 1

test scenario (B)



Test scenarios

(A) Only gravity wave and sound wave expansion

(B) …

(C) Additional advection by a solid body rotation velocity field $\mathbf{v}_0 = \mathbf{Q} \times \mathbf{r}$
→ test the coupling of fast (buoyancy, sound) and slow (advection) processes
Problem: solid body rotation field generates centrifugal forces!
Solution: $\mathbf{Q} = -\mathbf{\Omega}$ → similar to (A) in the absolute system
(analogous to *Läuter et al. (2005) JCP*)

Euler equations in spherical coordinates

$$\frac{\partial u}{\partial t} + \mathbf{v} \cdot \nabla u - \frac{\tan\phi}{r} uv + \boxed{\frac{1}{r} uw} = - \frac{1}{\rho r \cos\phi} \frac{\partial p}{\partial \lambda} + 2\Omega (v \sin\phi \boxed{- w \cos\phi)}$$

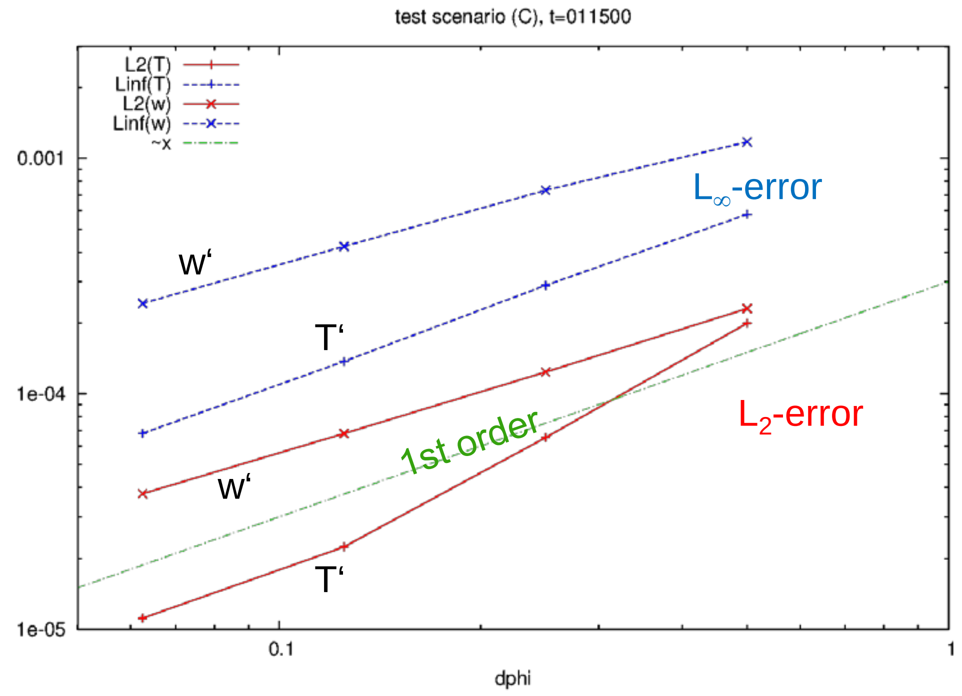
$$\frac{\partial v}{\partial t} + \mathbf{v} \cdot \nabla v + \frac{\tan\phi}{r} u^2 + \frac{1}{r} vw = - \frac{1}{\rho r} \frac{\partial p}{\partial \phi} - 2\Omega u \sin\phi - \Omega^2 r \cos\phi \sin\phi$$

$$\frac{\partial w}{\partial t} + \mathbf{v} \cdot \nabla w - \frac{u^2 + v^2}{r} = - \frac{1}{\rho} \frac{\partial p}{\partial r} - g + 2\Omega u \cos\phi + \Omega^2 r \cos^2\phi$$

Most deep terms are needed now for the analytic solution!
… but not all are contained in ICON

Test scenario (C) with the ICON model

Nevertheless:
The missing deep terms in the horizontal equations are not visible until 0.0625°: ICON converges ~ 1st order



Summary

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

- Analytic solution of the linearized, compressible, non-hydrostatic Euler equations *on the sphere* (for global models) and *on a* plane (for LAM's) have been derived
  → a reliable solution for a well known test exists and can be used not only for qualitative comparisons but even as a reference solution for convergence tests
- This solution/test exercises several important processes/terms and the time integration scheme of the numerical model
- On the sphere the test setup is quite similar to one of the DCMIP 2012 test cases
- 'standard' approximations used: shallow atmosphere,
  ‚global f-plane approx.' can be easily realised in every atmospheric model
- only one further approximation: linearisation (=controlled approx.)
- For fine enough resolutions ICON has a spatial-temporal convergence rate of about 1, no drawbacks visible
- Such tests can be used to evaluate improved discretizations.
  Example: vertical discretizations in the new fast waves solver in COSMO

References:

Baldauf, Brdar (2013) QJRMS (DOI:10.1002/qj.2105)
Baldauf, Reinert, Zängl (2013) accepted by QJRMS

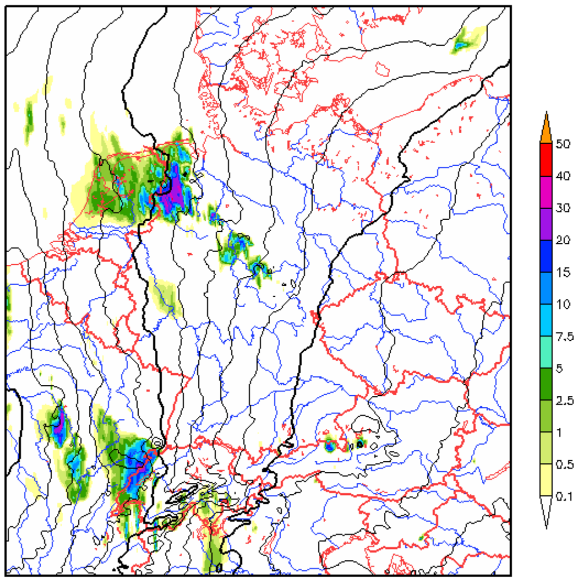
partly financed by MetStröm

Influence of the water loading in strong convective simulations

Motivation: a bad forecast quality of COSMO-DE at 20 June 2013

,nnew‘

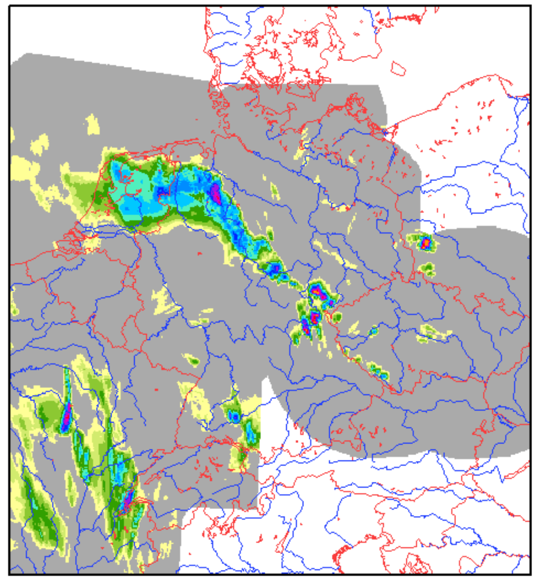
Start time: 20.06.2013 12:00 UTC 4.26r14_FW2_MF
Forecast time: 20.06.2013 15:00 UTC
Total precipitation [mm/1h] (shaded) Geopot. at 700 hPa [gpdm] (dist. isol. 1gpdm



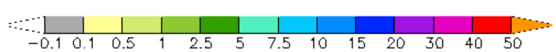
Totprec: Mean: 0.313589 Min: −0.000976562 Max: 41.9834 Sigma: 1.84055
FI700: Mean: 312.535 Min: 304.535 Max: 319.068 Sigma: 3.49868

Radar

RADAR COMPOSITE
valid: 20 JUN 2013 14 – 15 UTC
1h PRECIPITATION



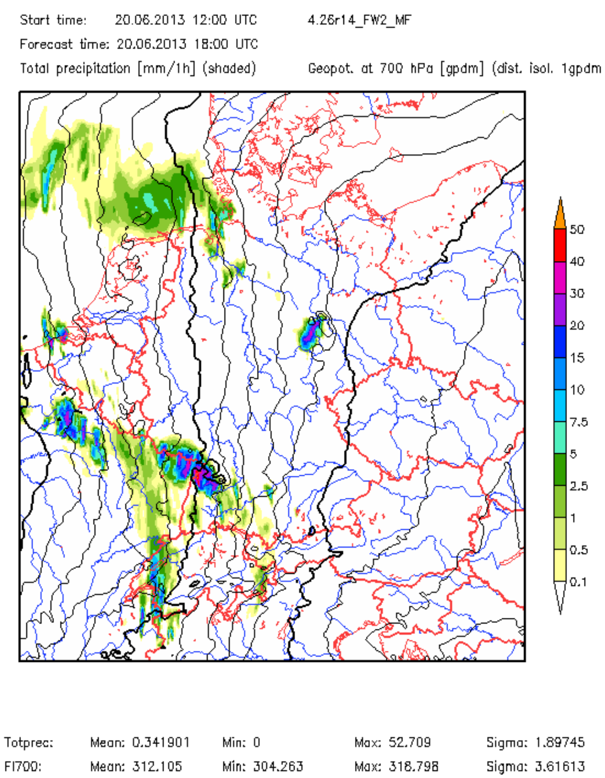
Mean: 0.622713 Min: 0 Max: 87.1358



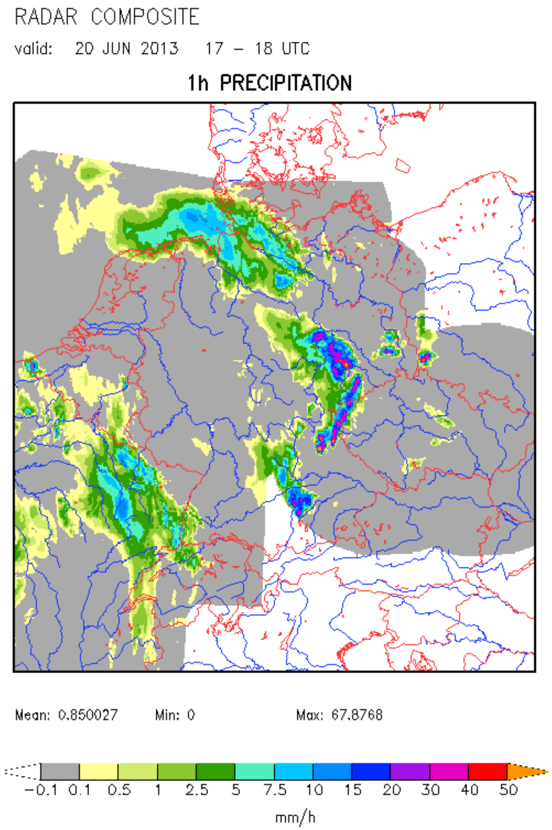
Front coming in at evening;
convergence line during afternoon with heavy precipitation

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

‚nnew'



Radar



Bug fix in the buoyancy term of COSMO:

$$\frac{\partial w}{\partial t} + \mathbf{v} \cdot \nabla w = -\frac{1}{\rho}\frac{\partial p'}{\partial z} + \underbrace{g\left(\frac{p_0}{p}\frac{T'}{T_0} - \frac{p'}{p} + \frac{p_0}{p}\frac{T}{T_0} q_x\right)}_{-g\frac{\rho'}{\rho}} + \dots$$

Moisture correction in the ideal gas law (water loading):

$$q_x := \left(\frac{R_v}{R_d} - 1\right) q_v - q_c - q_r - \dots$$

RK-scheme with new fast waves solver:

until COSMO 4.27: moisture variables q_v, q_c, … in q_x at time level `nnew`

bug fix : moisture variables q_v, q_c, … in q_x at time level `nnow`

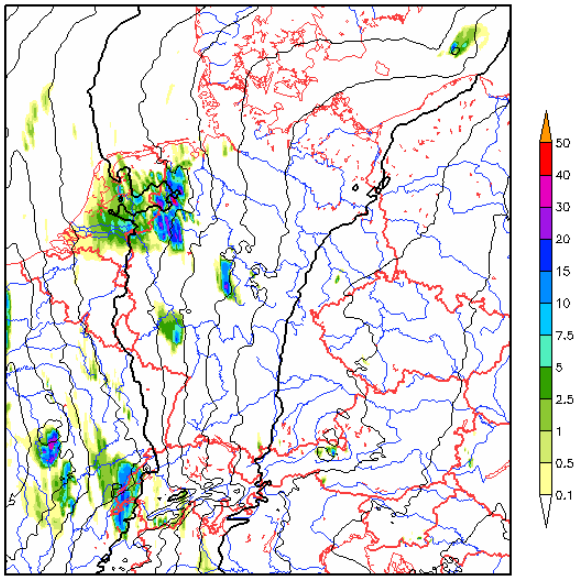
reason: during the RK scheme, `nnew` means ‚*old*' for the moisture variables!

COSMO-DE, 20 June 2013, 12 UTC run
1h precipitation sum

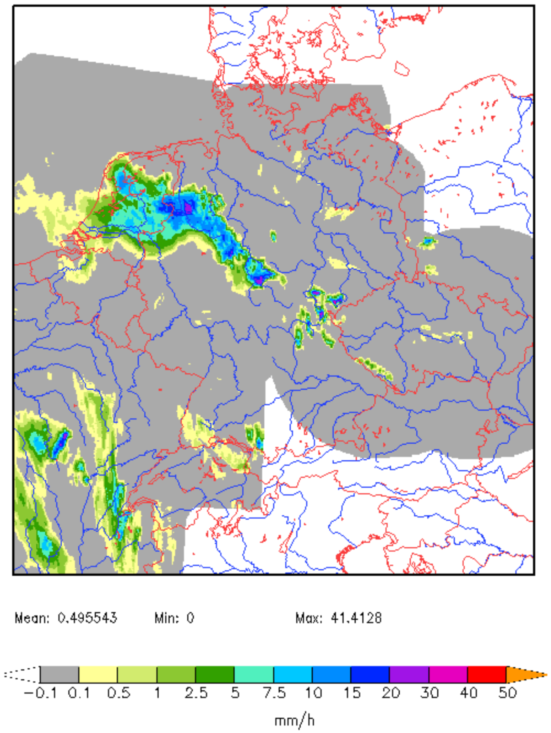
Deutscher Wetterdienst
Wetter und Klima aus einer Hand

DWD

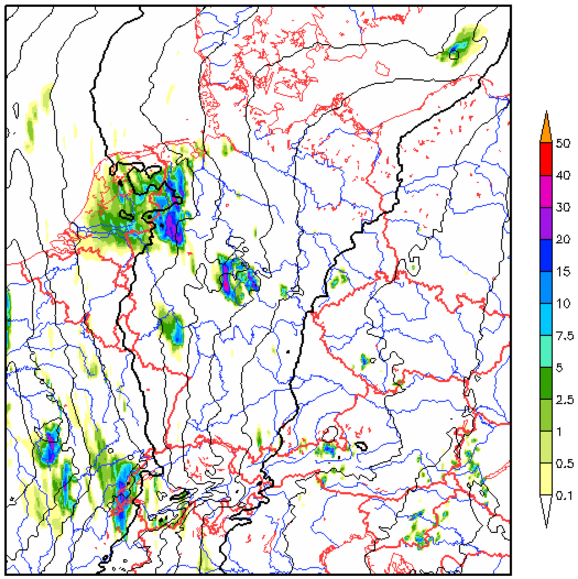
,nnew' = old



Radar



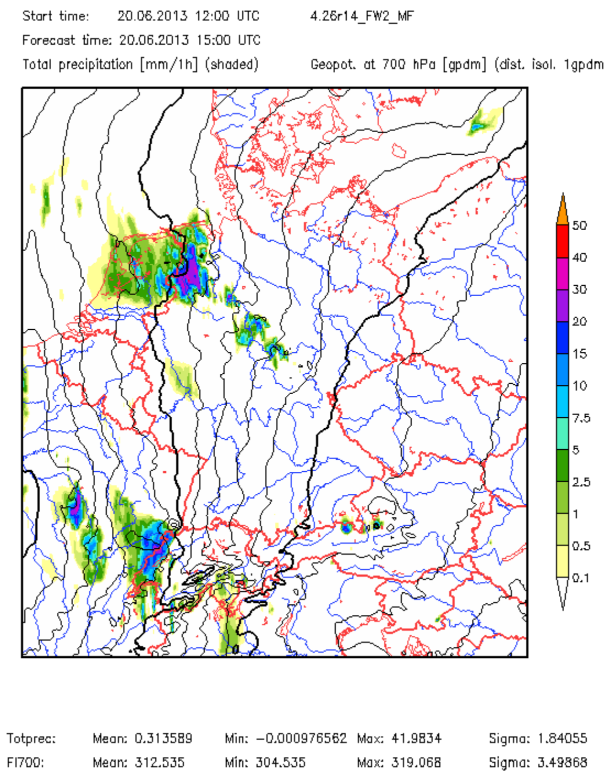
,nnow'



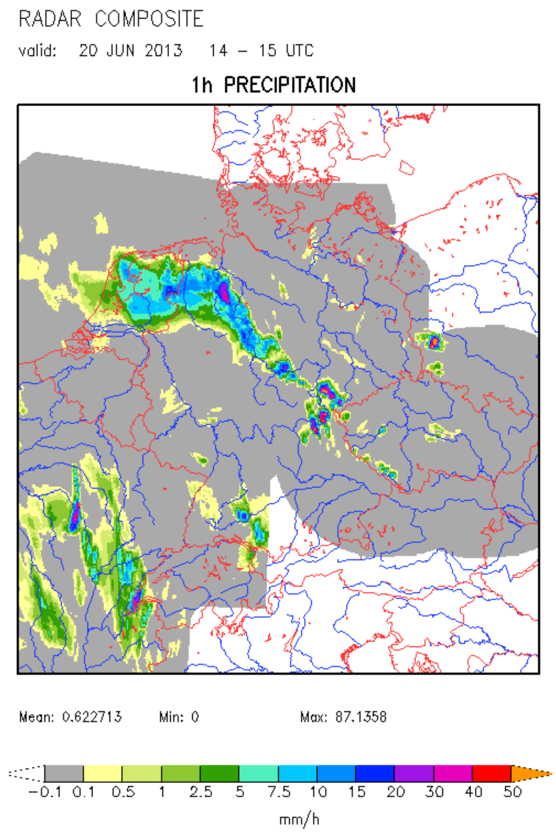
Front coming in at evening;
convergence line during afternoon with heavy precipitation

COSMO-DE, 20 June 2013, 12 UTC run
1h precipitation sum

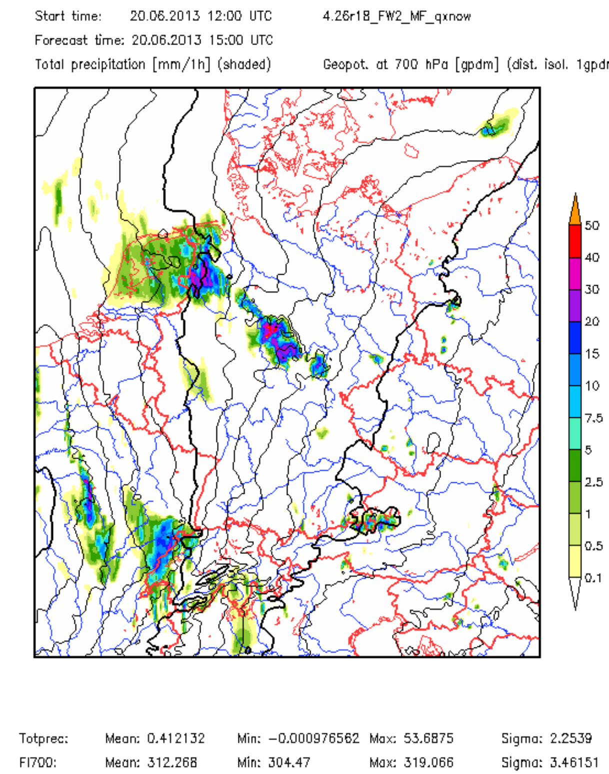
‚nnew‘ = old



Radar

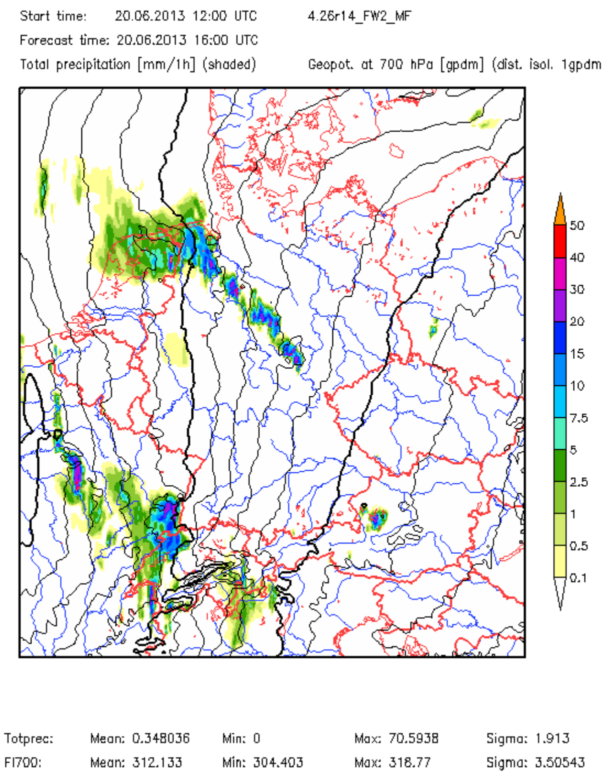


‚nnow‘

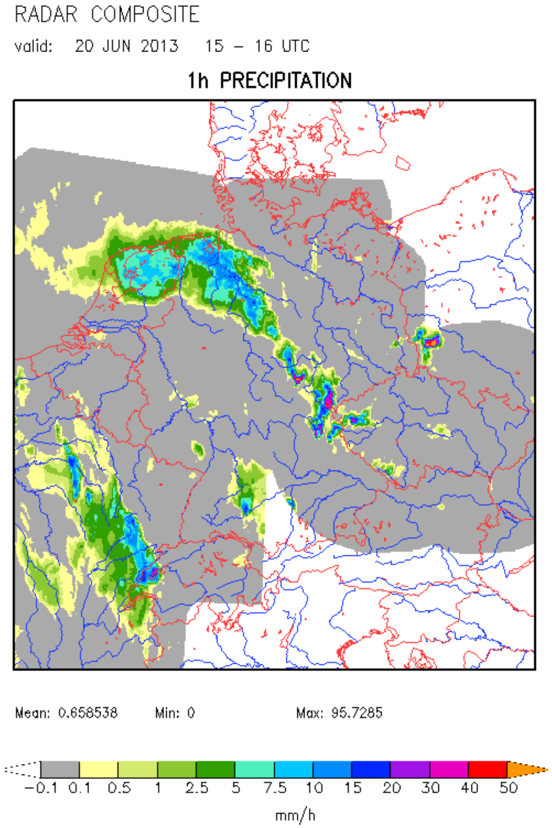


COSMO-DE, 20 June 2013, 12 UTC run
1h precipitation sum

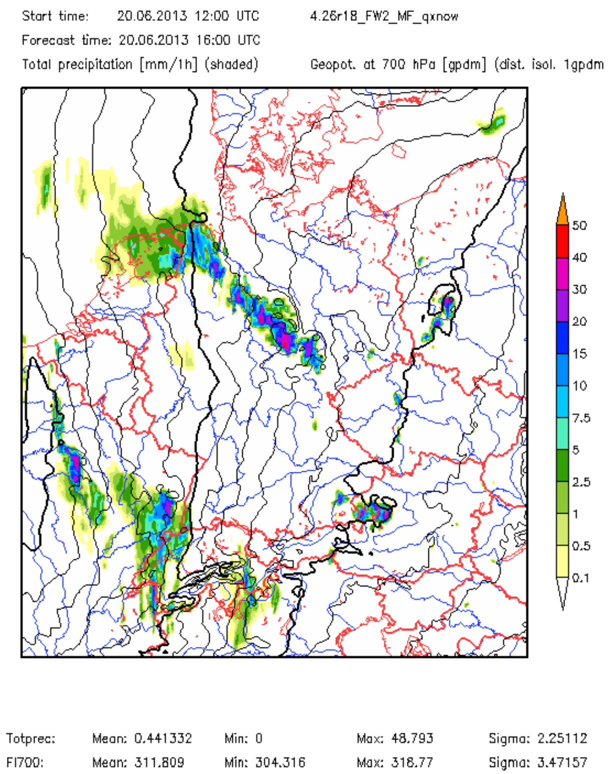
‚nnew' = old



Radar



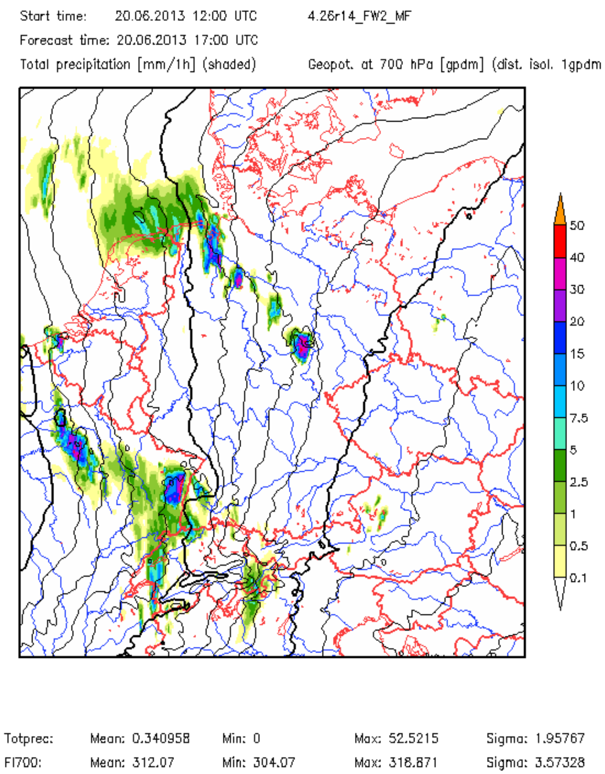
‚nnow'



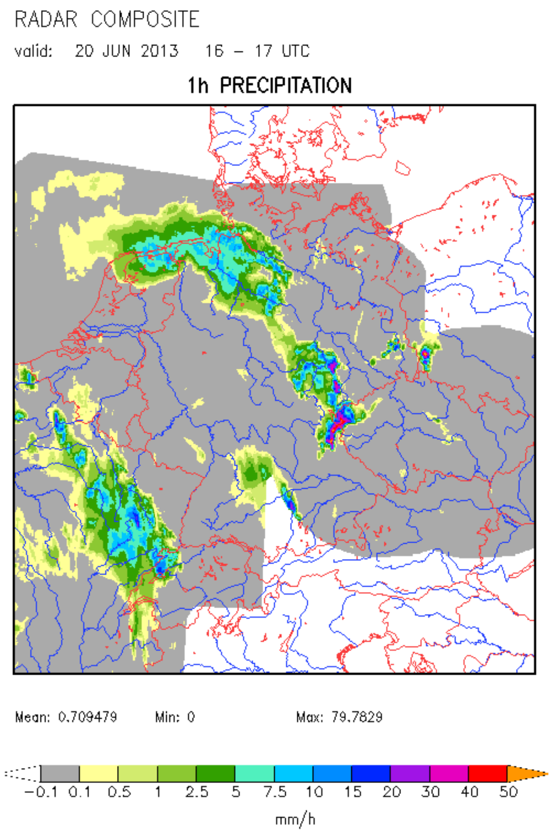
COSMO-DE, 20 June 2013, 12 UTC run
1h precipitation sum

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

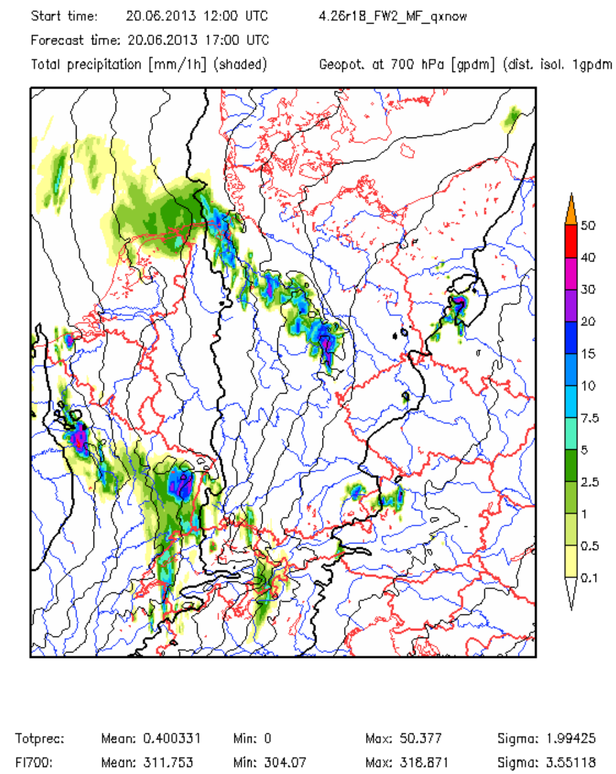
‚nnew' = old



Radar

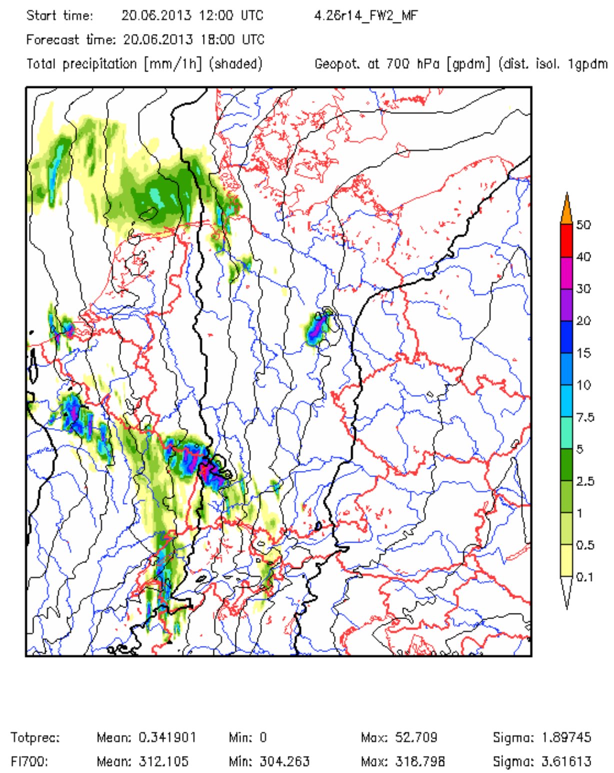


‚nnow'

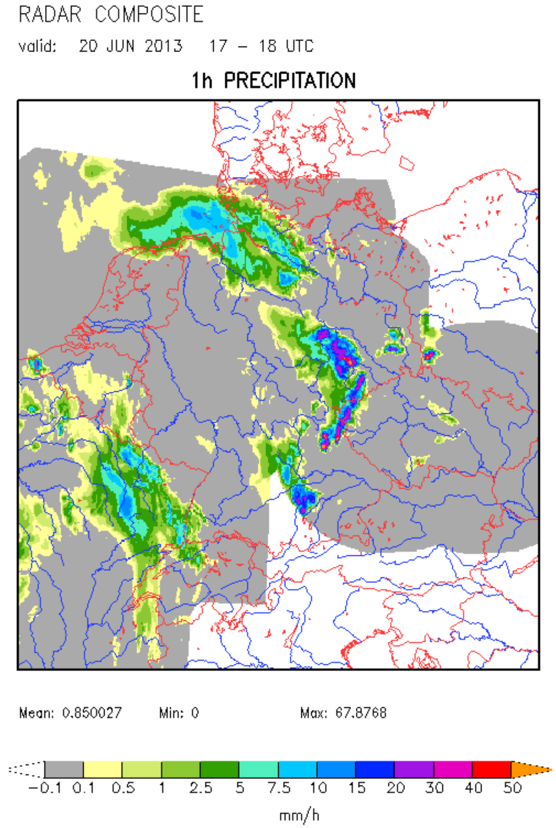


COSMO-DE, 20 June 2013, 12 UTC run
1h precipitation sum

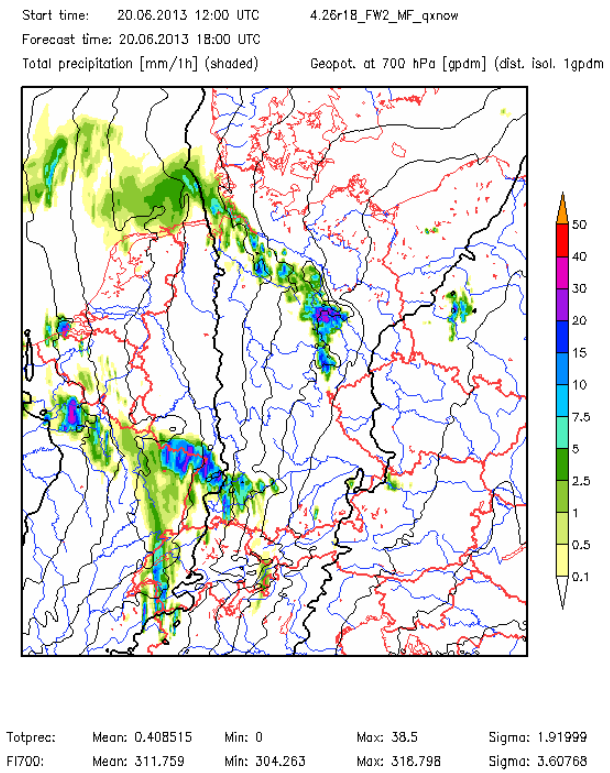
‚nnew‘ = old



Radar

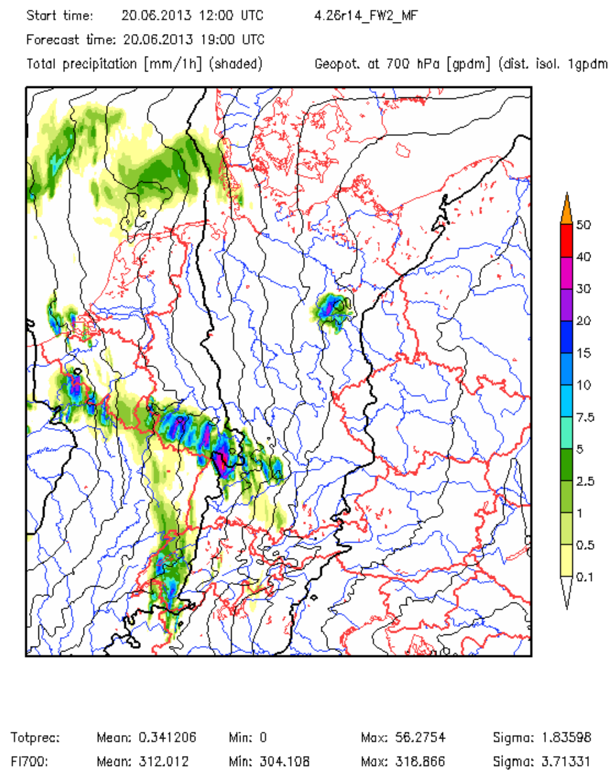


‚nnow‘

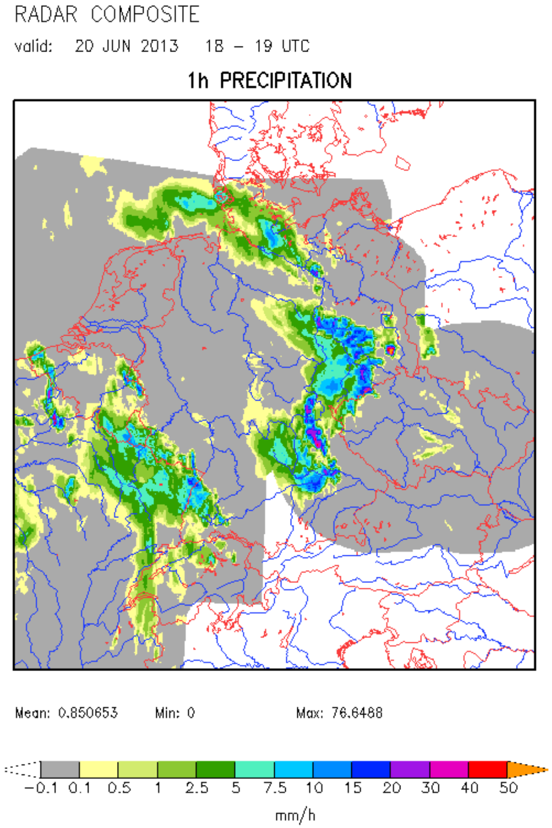


COSMO-DE, 20 June 2013, 12 UTC run
1h precipitation sum

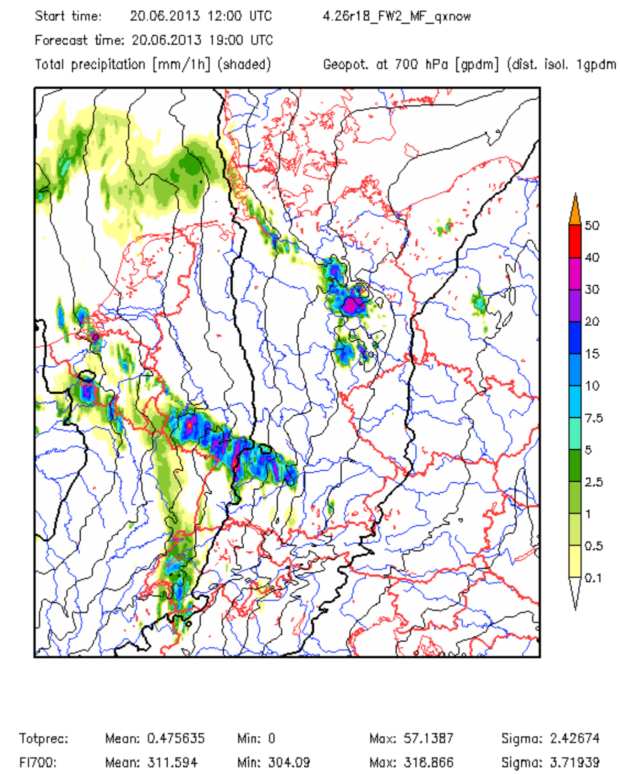
‚nnew‘ = old



Radar



‚nnow‘



COSMO-DE, 20 June 2013, 12 UTC run
1h precipitation sum

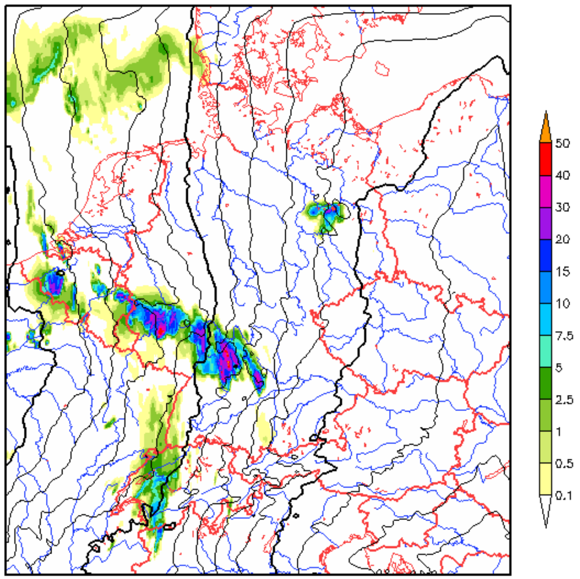
Deutscher Wetterdienst
Wetter und Klima aus einer Hand

‚nnew‘ = old

Radar

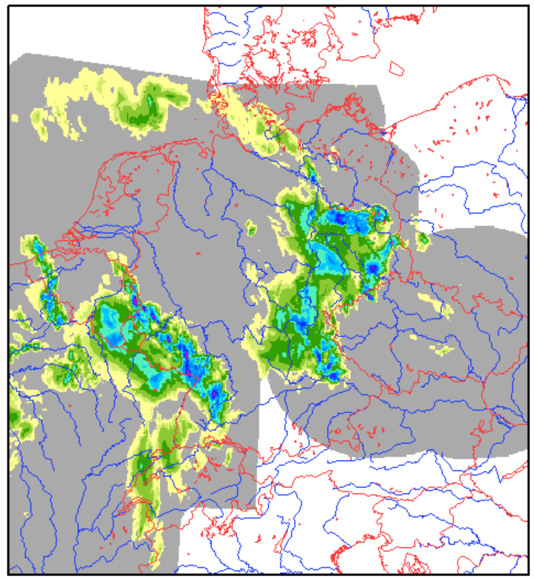
‚nnow‘

Start time: 20.06.2013 12:00 UTC 4.26r14_FW2_MF
Forecast time: 20.06.2013 20:00 UTC
Total precipitation [mm/1h] (shaded) Geopot. at 700 hPa [gpdm] (dist. isol. 1gpdm

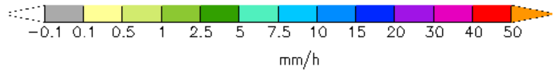


Totprec: Mean: 0.416513 Min: 0 Max: 63.959 Sigma: 2.36812
FI700: Mean: 311.838 Min: 304.03 Max: 319.038 Sigma: 3.76957

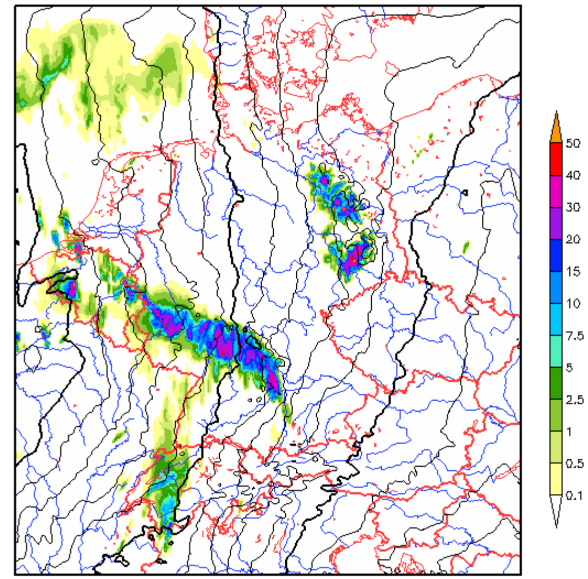
RADAR COMPOSITE
valid: 20 JUN 2013 19 – 20 UTC
1h PRECIPITATION



Mean: 0.725119 Min: 0 Max: 32.3805



Start time: 20.06.2013 12:00 UTC 4.26r18_FW2_MF_qxnow
Forecast time: 20.06.2013 20:00 UTC
Total precipitation [mm/1h] (shaded) Geopot. at 700 hPa [gpdm] (dist. isol. 1gpdm

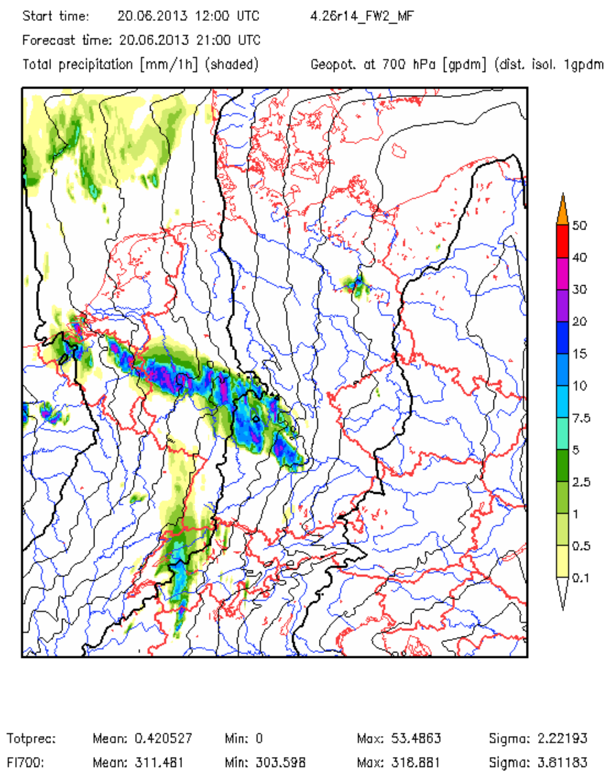


Totprec: Mean: 0.531288 Min: 0 Max: 56.5 Sigma: 2.77347
FI700: Mean: 311.373 Min: 303.817 Max: 319.038 Sigma: 3.7561

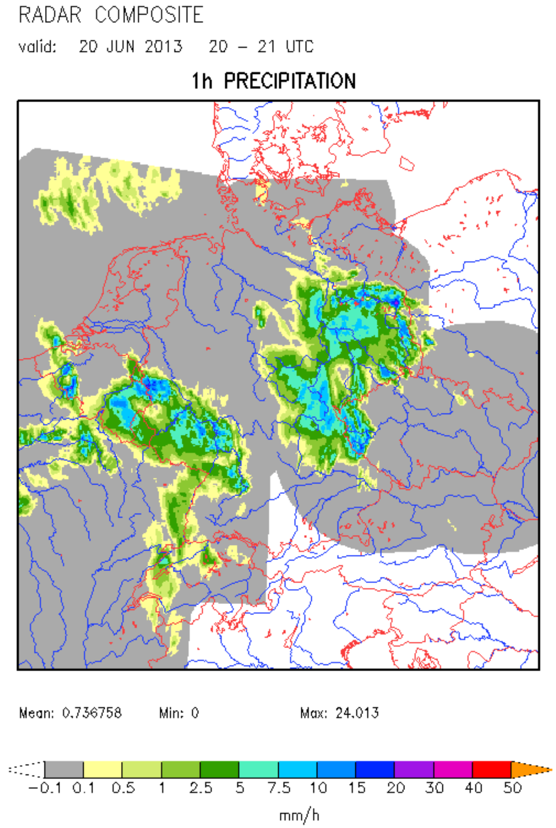
COSMO-DE, 20 June 2013, 12 UTC run
1h precipitation sum

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

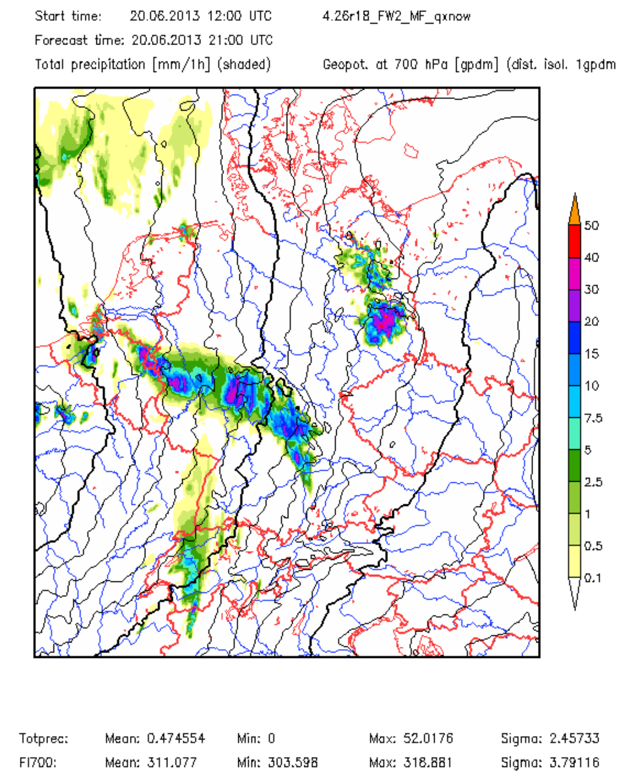
‚nnew' = old



Radar



‚nnow'

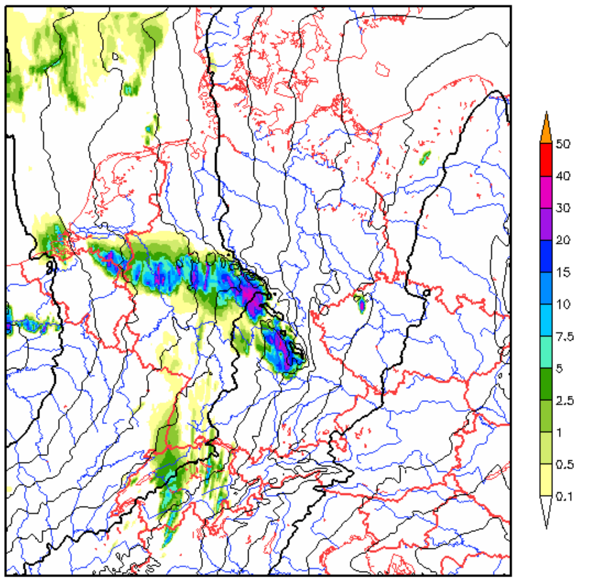


COSMO-DE, 20 June 2013, 12 UTC run
1h precipitation sum

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

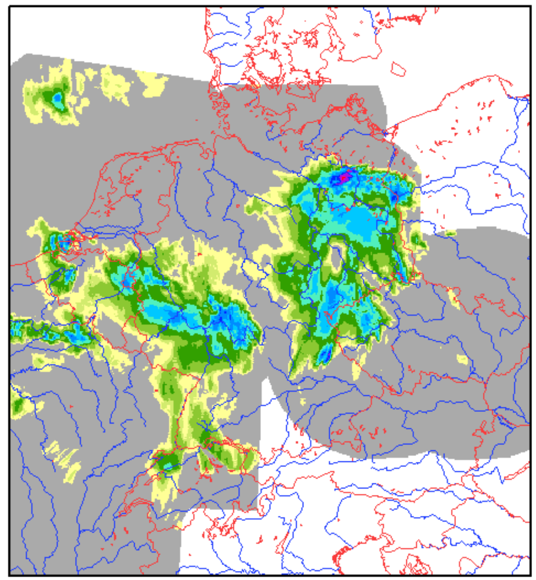
‚nnew‘ = old	Radar	‚nnow‘

Start time: 20.06.2013 12:00 UTC 4.26r14_FW2_MF
Forecast time: 20.06.2013 22:00 UTC
Total precipitation [mm/1h] (shaded) Geopot. at 700 hPa [gpdm] (dist. isol. 1gpdm



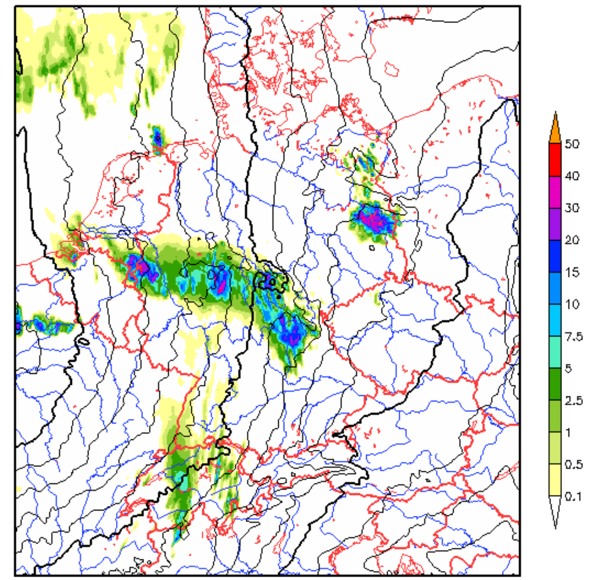
Totprec: Mean: 0.417346 Min: 0 Max: 45.7441 Sigma: 2.26247
FI700: Mean: 311.277 Min: 303.802 Max: 319.114 Sigma: 3.72163

RADAR COMPOSITE
valid: 20 JUN 2013 21 – 22 UTC
1h PRECIPITATION



Mean: 0.93343 Min: 0 Max: 42.3395

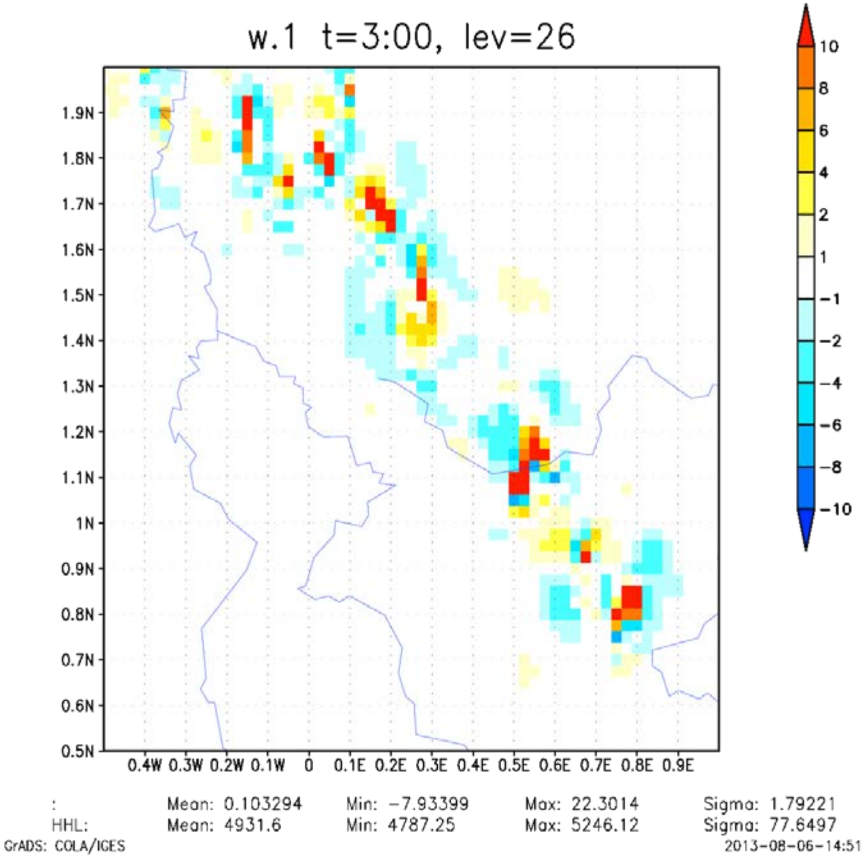
Start time: 20.06.2013 12:00 UTC 4.26r18_FW2_MF_qxnow
Forecast time: 20.06.2013 22:00 UTC
Total precipitation [mm/1h] (shaded) Geopot. at 700 hPa [gpdm] (dist. isol. 1gpdm



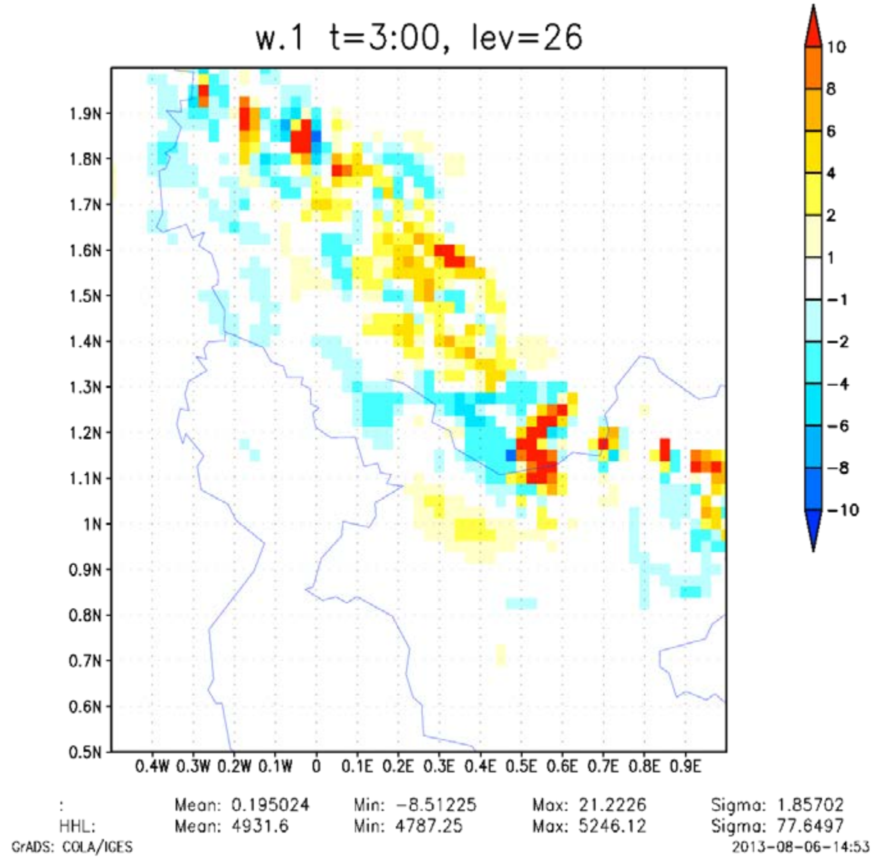
Totprec: Mean: 0.405306 Min: 0 Max: 50.3535 Sigma: 2.07876
FI700: Mean: 311.064 Min: 303.782 Max: 319.114 Sigma: 3.72039

vertical velocity w [m/s] in z ~ 5 km

‚nnew' = old

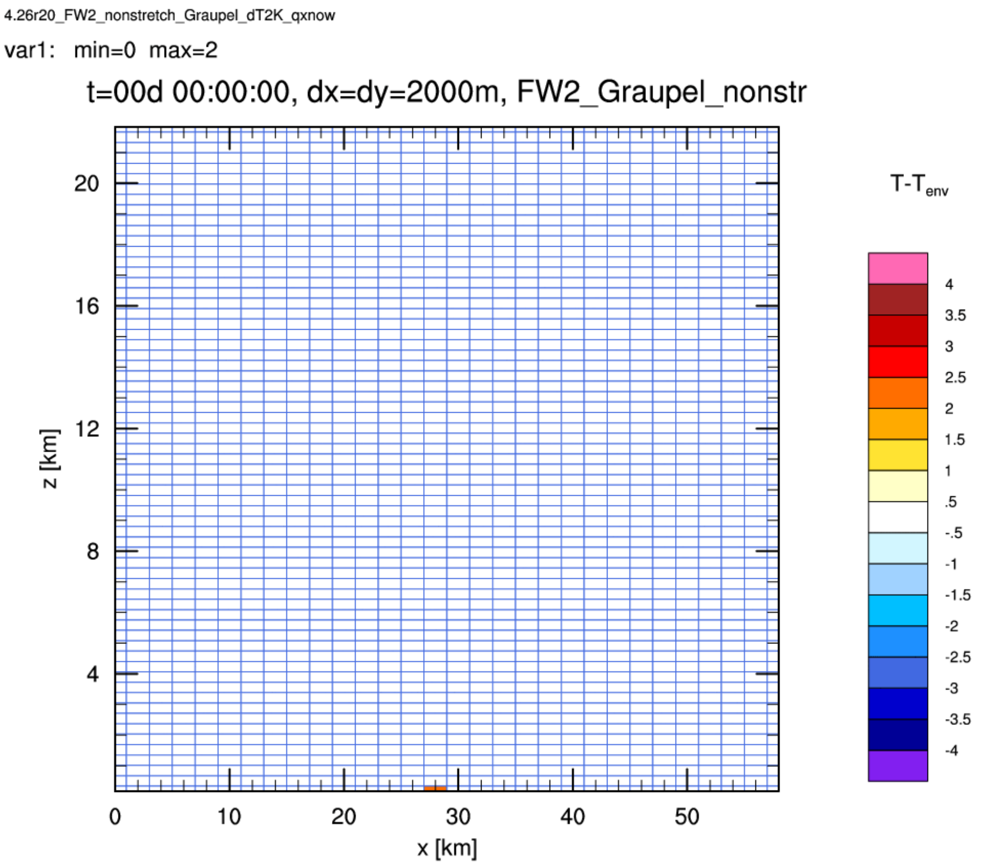


‚nnow'



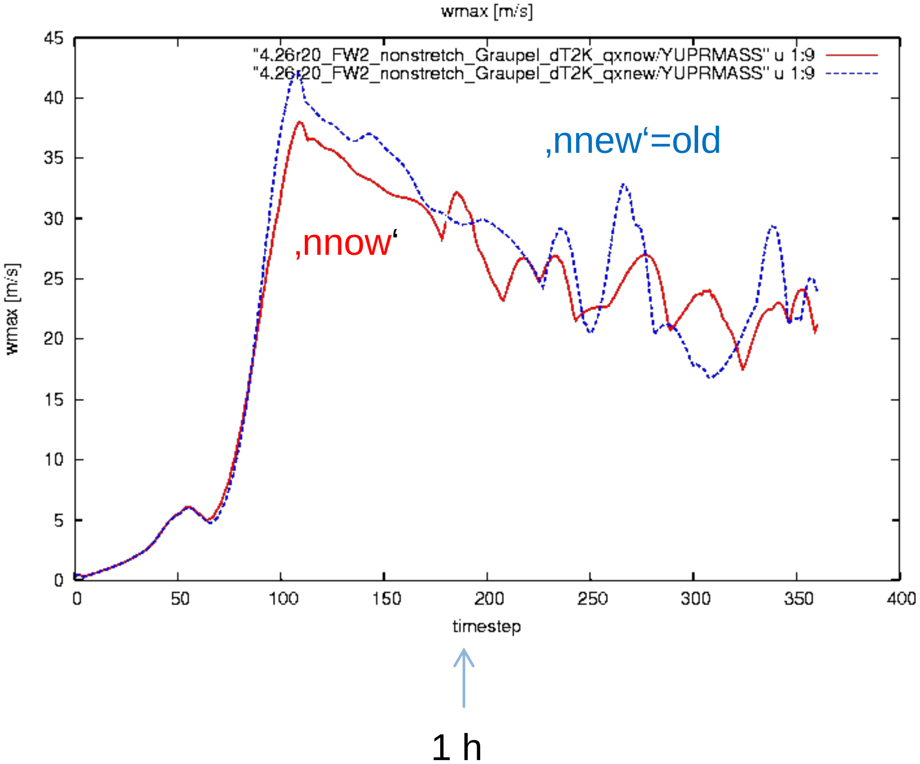
Idealised convection test at the resolution limit of the model

- T-perturbation $\Delta T=+2K$ in only **one** grid box (in z~500m)
- Stratification analogous to Weismann, Klemp (1982) MWR
- Atmosphere at rest
- No turbulence, only cloud physics
- Non-stretched grid

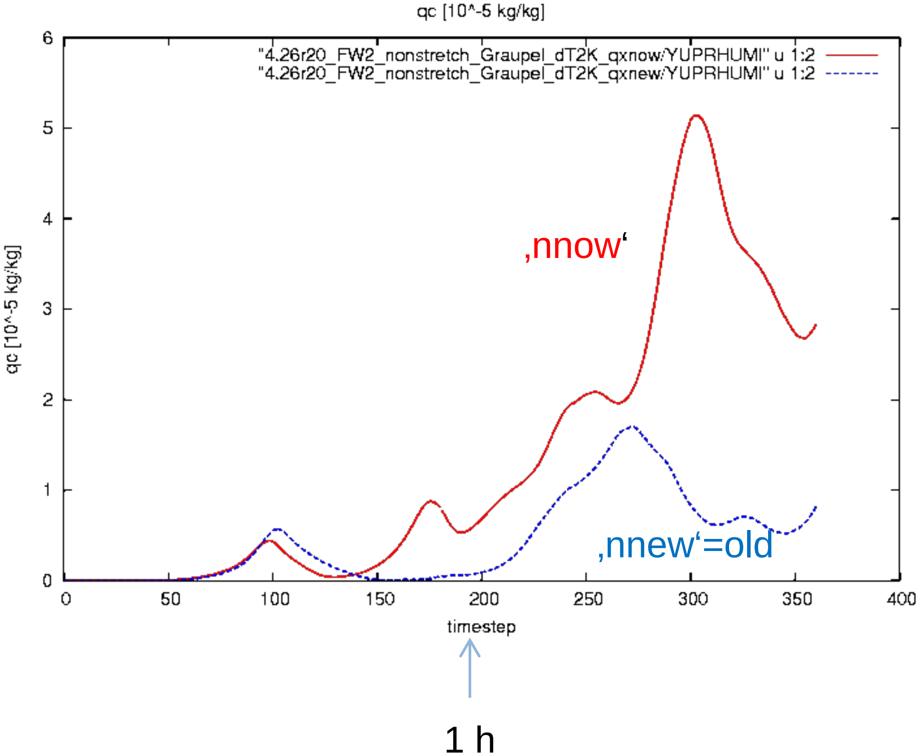


Idealised convection test at the resolution limit of the model

w_{max} [m/s]



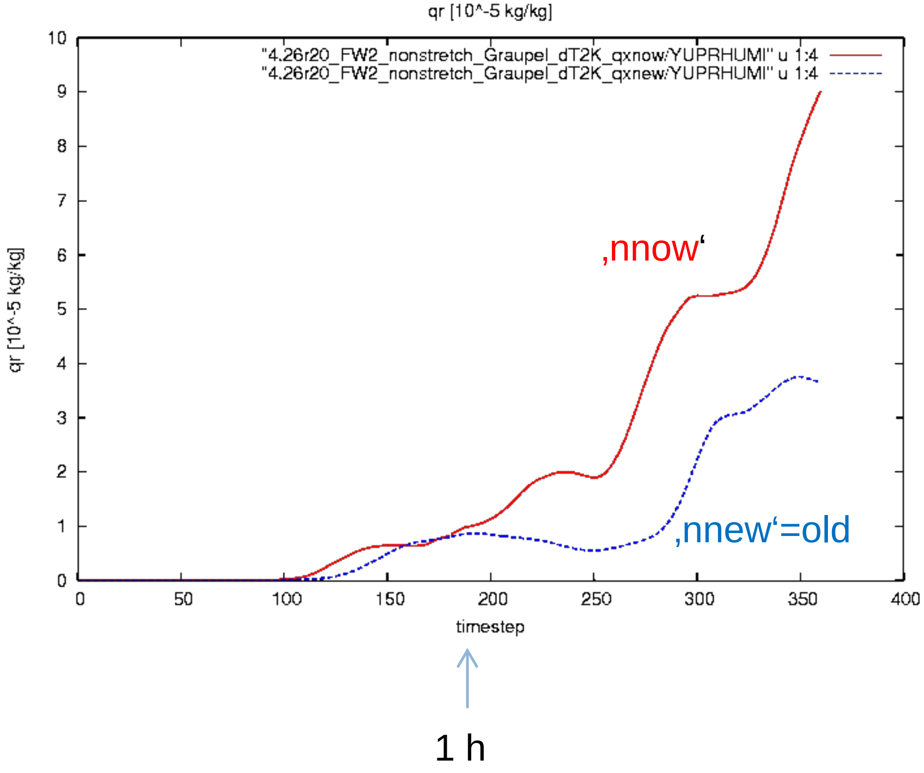
cloud water q_c [0.01 g/kg]



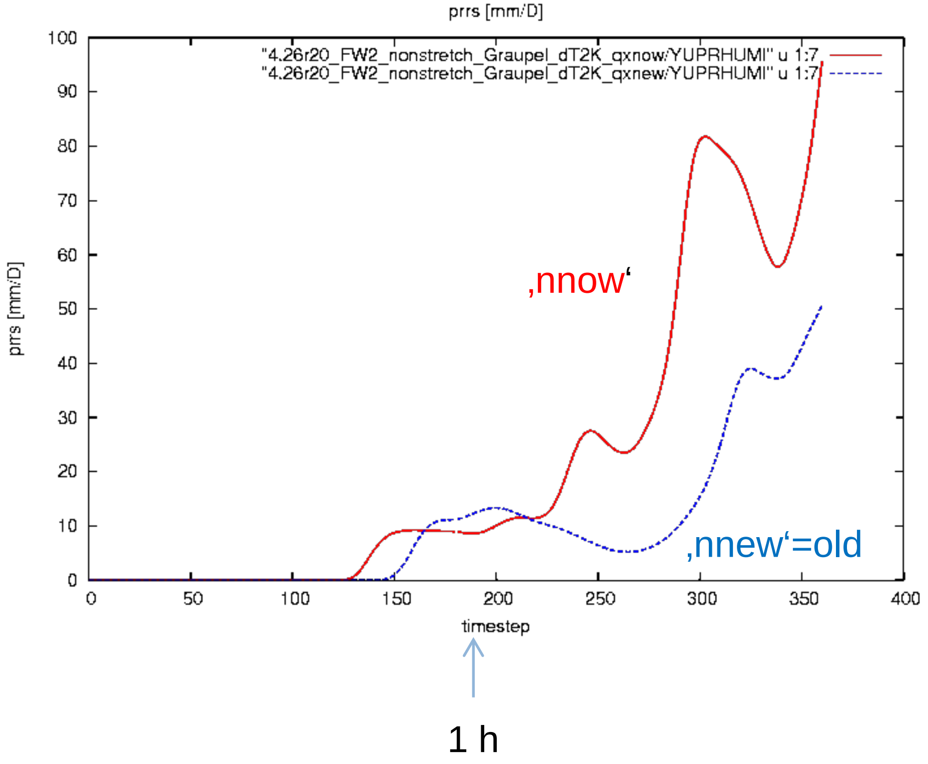
graupel-scheme used (q_v, q_c, q_i, q_r, q_s, q_g)

Idealised convection test at the resolution limit of the model

rain q_r [0.01 g/kg]



precipitation rate [mm/day]



graupel-scheme used (q_v , q_c , q_i, q_r , q_s , q_g)

Summary

- Convection-permitting models are quite sensitive to (among others) the treatment of the buoyancy term (not a new insight, of course)
- The water loading contribution to the buoyancy is relatively large and even the ‚small' error of using moisture variables one time level too late has a strong influence in the evolution of convection
- Experience: the largest improvements in weather forecasting stem from bug removals …

Staggered vs unstaggered grids

… and what does this mean for discontinuous Galerkin methods?

Dynamical core (Euler solver) developments in COSMO

- Current Runge-Kutta dynamical core
  - further maintenance (DWD) (~0.5 FTE)
  - higher order discretizations (Univ. Cottbus) (~1 FTE)

COSMO priority project ‚Conservative dynamical core (2008-2012):

- EULAG as a candidate for the future COSMO dyn. Core

Ziemiański et al. (2011) Acta Geophysica
Rosa et al. (2011) Acta Geophysica
Kurowski et al. (2011) Acta Geophysica

→ follow up PP ‚COSMO-EULAG operationalization (2012-2015) (IMGW, Poland) (~3 FTE)

- fully implicit FV solver ‚CONSOL' (CIRA, Italy) (~0.5 FTE)
  Jameson (1991) AIAA

Project in the framework of the German research foundation

- Dynamical core based on Discontinuous Galerkin methods (DWD, Univ. Freiburg) (~1.08 FTE)

the last three dynamical core developments use an unstaggered (!) grid

1 FTE (full time equivalent) = 1 person/year

Linear 1D wave equation as a prototype for hyperbolic equations

$$\frac{\partial u}{\partial t} = -g\,\delta_x h$$
$$\frac{\partial h}{\partial t} = -H_0\,\delta_x u$$

$$c = \sqrt{gH_0}$$

wave ansatz: $\phi(x,t) = \tilde{\phi}\, e^{i(kx-\omega t)}$

continuous: $\delta_x \phi = \dfrac{\partial \phi}{\partial x} \quad \multimap \quad ik_x\,\tilde{\phi}$

unstaggered $\delta_x \phi_j = \dfrac{\phi_{j+1} - \phi_{j-1}}{2\Delta x} \quad \multimap \quad i\,\dfrac{\sin k_x \Delta x}{\Delta x}\,\tilde{\phi}$

staggered: $\delta_x \phi_j = \dfrac{\phi_{j+\frac{1}{2}} - \phi_{j-\frac{1}{2}}}{\Delta x} \quad \multimap \quad i\,\dfrac{2\sin\frac{k_x \Delta x}{2}}{\Delta x}\,\tilde{\phi}$

‚modified' wavenumber

e.g. *D. R. Durran: Numerical methods …*

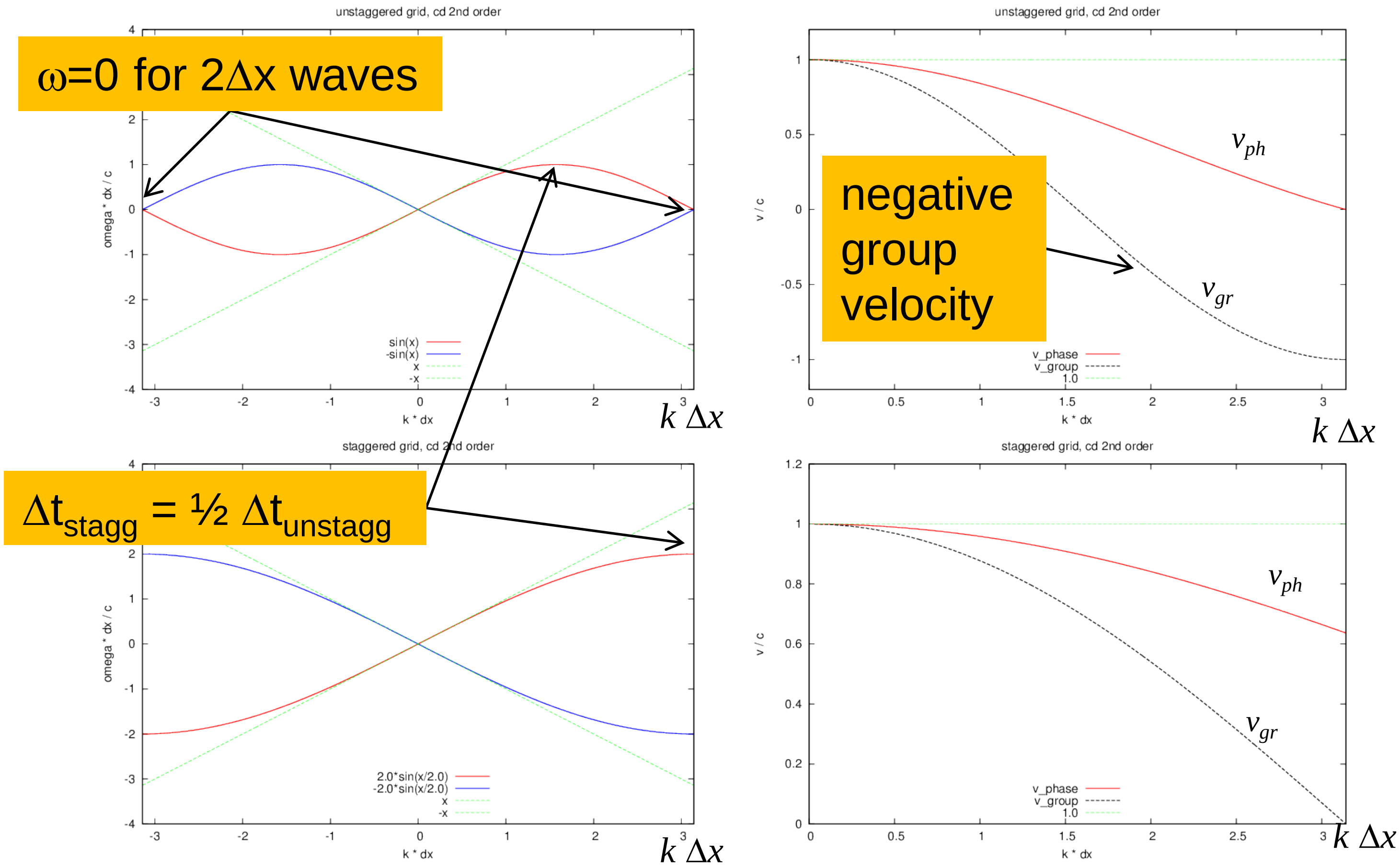
Dispersion relation of the 1D wave equation

Frequency ω | Phase-, group-velocity

unstaggered

grid

staggered



1D wave expansion with a Discontinuous Galerkin (DG) discretization

$$\frac{\partial u}{\partial t} = -\frac{\partial g\, h}{\partial x}$$
$$\frac{\partial h}{\partial t} = -\frac{\partial H_0\, u}{\partial x}$$

$$c = \sqrt{gH_0}$$

Literature*:*

Hu, Hussaini, Rasetarinera (1999) JCP: 1D advection-, 2D wave-equation

Hu, Atkins (2002) JCP: non-uniform grids → $k=k(\omega)$

Ainsworth (2004) JCP: multi-dim. advection equation

→ talk by F. Giraldo

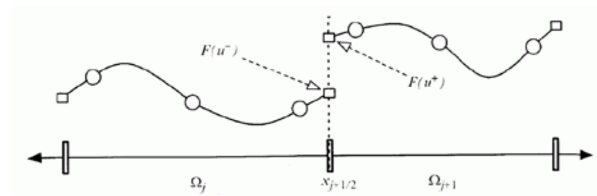
Discontinuous Galerkin (DG) methods in a nutshell

→ talk by F. Giraldo

$$\frac{\partial q^{(k)}}{\partial t} + \nabla \cdot \mathbf{f}^{(k)}(q) = S^{(k)}(q), \qquad k = 1, ..., K$$

$$\int_{\Omega_j} dx \; v(\mathbf{x}) \cdot \ldots$$

→ weak formulation (increases solution space)



From Nair et al. (2011) in ‚Numerical techniques …

Finite-element ingredient

$$q^{(k)}(x,t) = \sum_{l=0}^{p} q_{j,l}^{(k)}(t) \; p_l(x - x_j)$$

e.g. Legendre-Polynomials

e.g.
Cockburn, Shu (1989) Math. Comput.
Cockburn et al. (1989) JCP

Finite-volume ingredient

$$\mathbf{f}(q) \to \mathbf{f}^{num}(q^+, q^-) = \frac{1}{2}\left(\mathbf{f}(q^+) + \mathbf{f}(q^-) - \alpha(q^+ - q^-)\right)$$

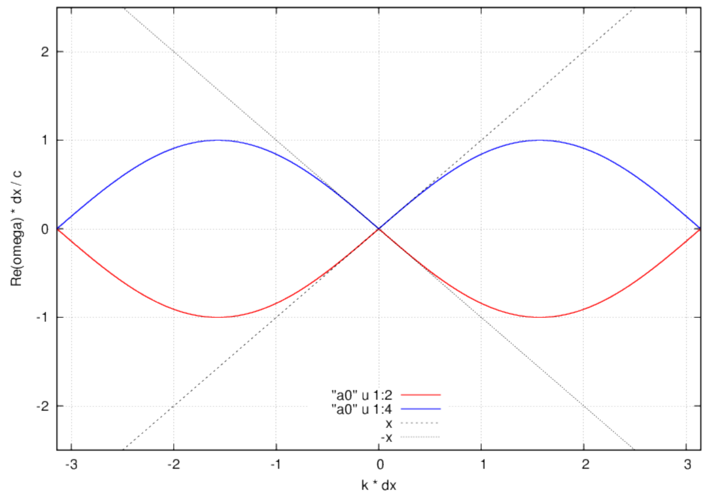
Lax-Friedrichs flux

→ ODE-system for $q^{(k)}{}_{jl}$

DG with p=0
(=classical FV-method)

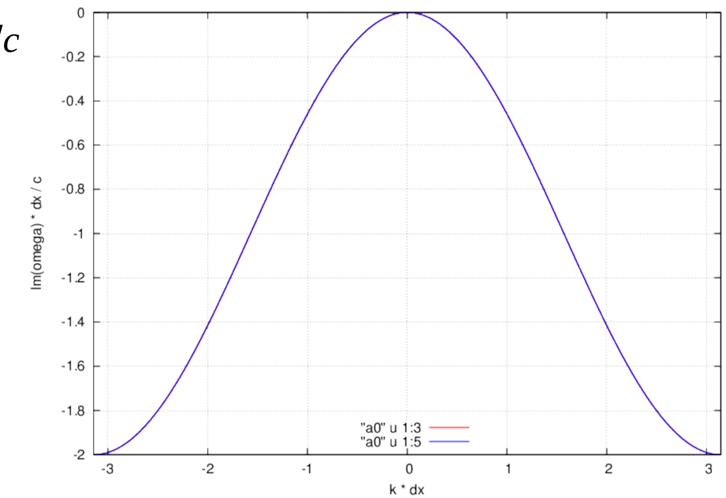
dispersion relation is the same as for the
2^{nd} order cent. diff. scheme
on an *unstaggered* grid
+ 2^{nd} order *(hyper-)diffusion*

Re $\omega \, \Delta x / c$



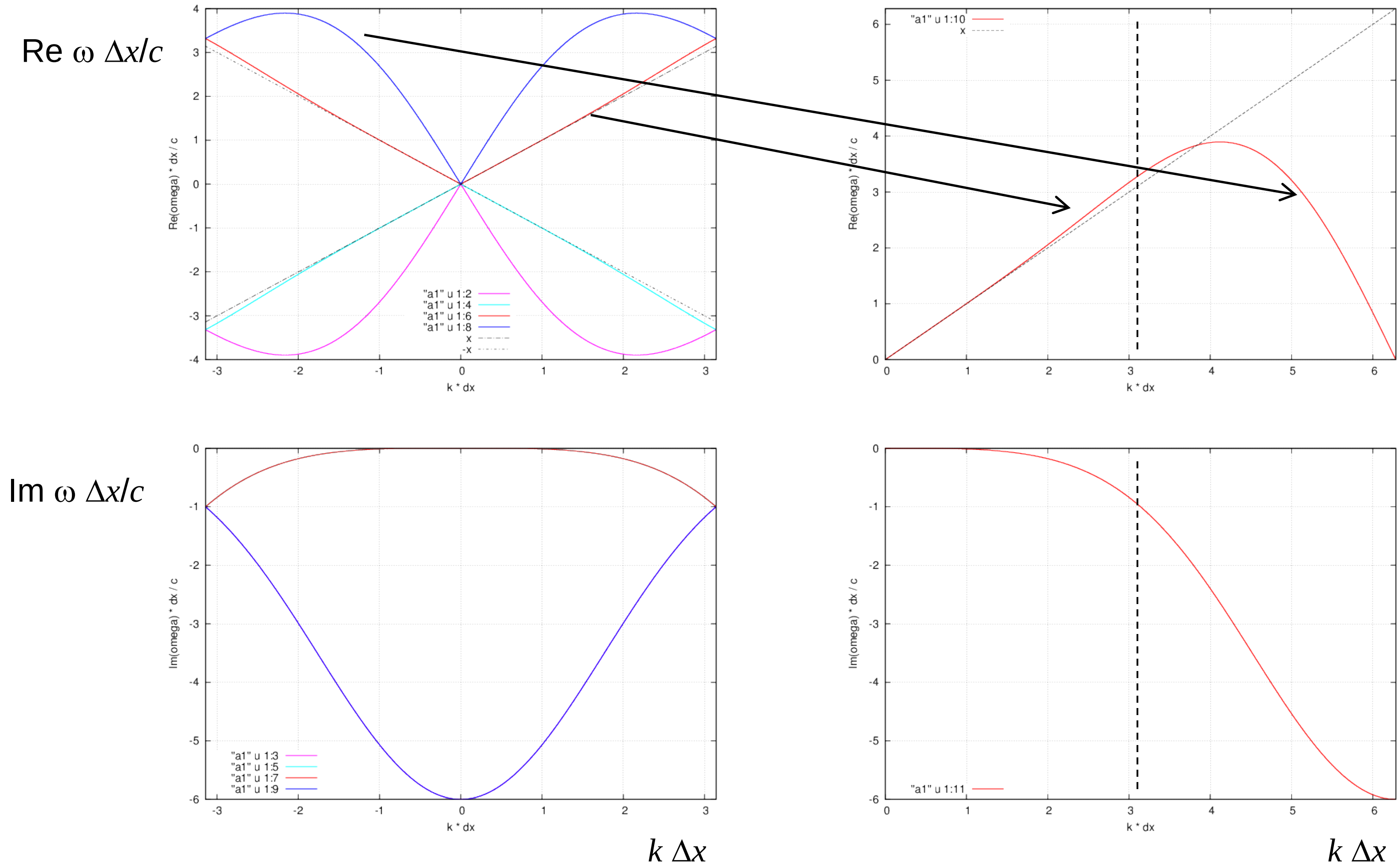
$k \, \Delta x$

Im $\omega \, \Delta x / c$



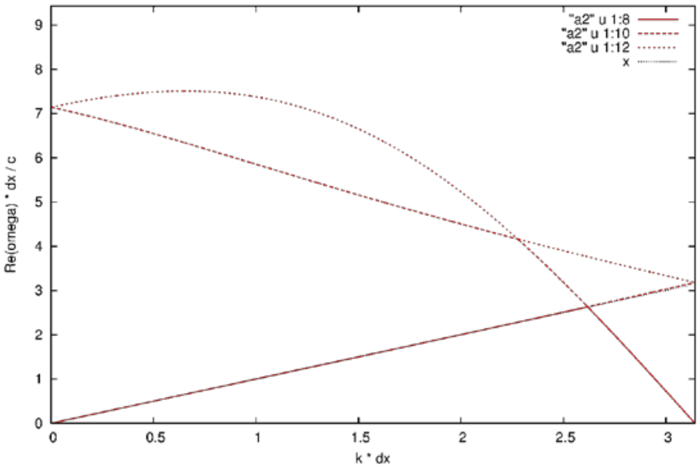
$k \, \Delta x$

DG with p=1 → 2 physically relevant (!) modes (*not* spurious/parasitic mode)



DG with p=2

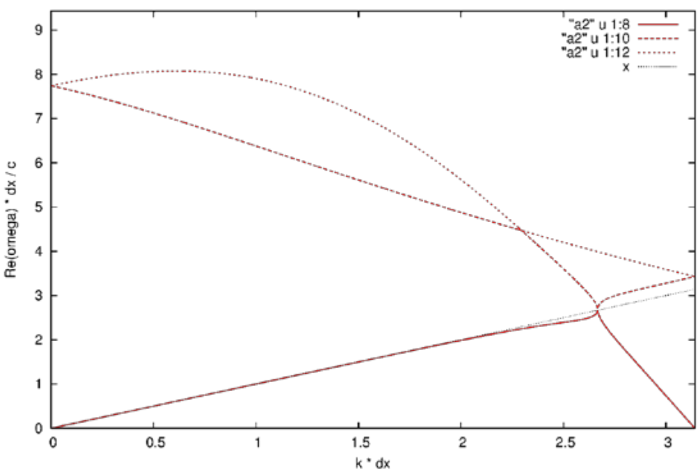
α=c



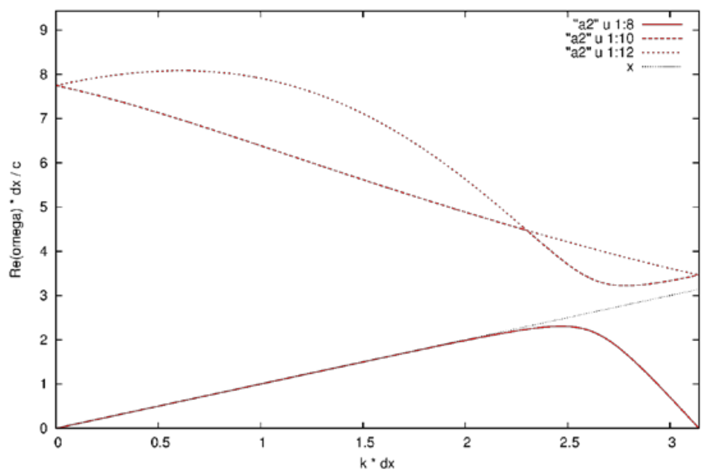
lowest mode has completely wrong behaviour near k $\Delta x = \pm\pi$

→ $\alpha > 0.15$ c necessary!

α=0.11 c



α=0



} frequency gap

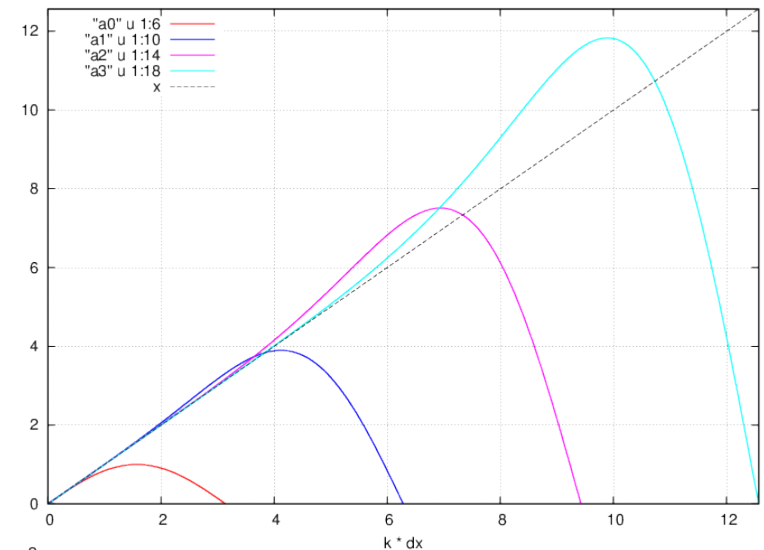
DG with p=0,1,2,3

(α=c used)

p	max $|\omega|\cdot\Delta x/c$
0	1
1	3.9
2	7.51
3	11.83
4	16.86
5	22.58
6	28.96
10	60.75
15	113.68

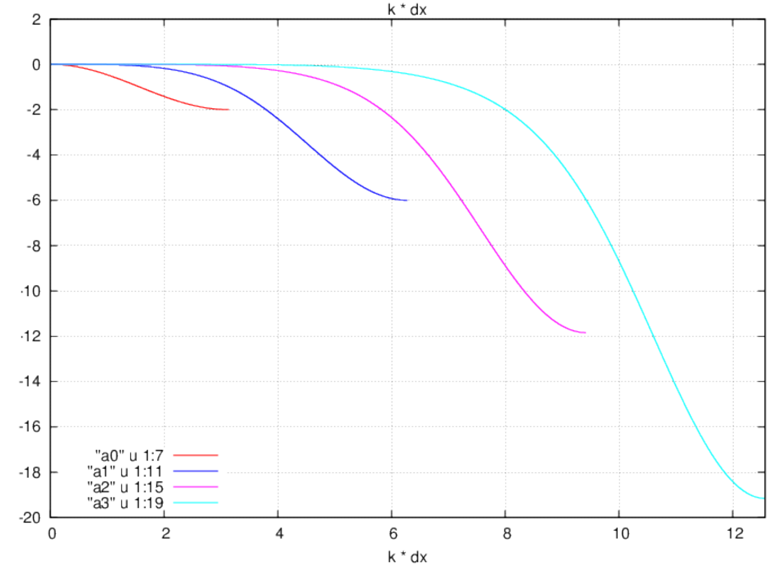
→ max $|\omega|\ \Delta x/c \approx 1 + 2.6\ p + 0.33\ p^2$
increases slightly stronger
than linear with p.
Choose not too large p!

Re $\omega\ \Delta x/c$



$k\ \Delta x$

Im $\omega\ \Delta x/c$



$k\ \Delta x$

Conclusions from 1D wave expansion with DG method:

- Wave expansion with DG methods behaves as on an unstaggered grid, but with strong damping of short waves
- There is no spurious (or ‚parasitic') mode in wave expansion: the dispersion relation is continuous and smooth until wavelength 2 dx / (p+1) if the numerical diffusive flux is not too small (but this is automatically fulfilled if α = max EV of $\boldsymbol{f}'(\boldsymbol{q})$)
- Maximum of frequency increases slightly stronger than linear with p → Choose not too large polynomial degree p

Thank you very much for your attention