

# The ECMWF model, progress and challenges

**Nils Wedi\***, Mats Hamrud, George Mozdzyński

\*European Centre for Medium-range Weather Forecasts, [wedi@ecmwf.int](mailto:wedi@ecmwf.int)

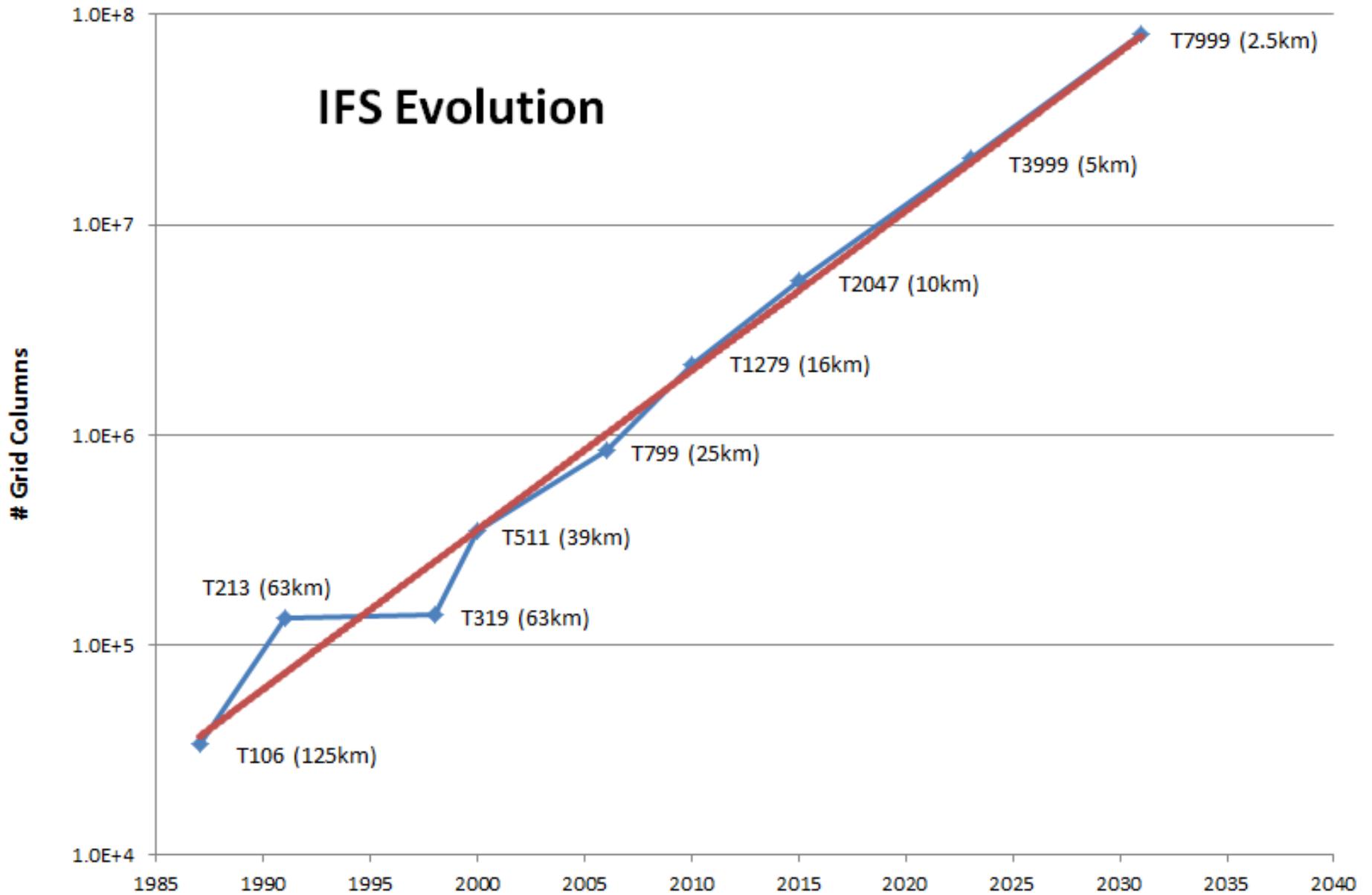
Many thanks to Linus Magnusson and Jean Bidlot

# Technology

## ◆ 4 foci:

- ◆ **Enabling technology for computations on massively parallel computers for science today**
- ◆ **A cost-effective, low-energy consuming but highly accurate model and data assimilation system**
- ◆ **Code resilience and understanding, what does each part of the model contribute, where is accuracy required, where not ?**
- ◆ **Quantitative measures on how good the forecast is**

# IFS Evolution

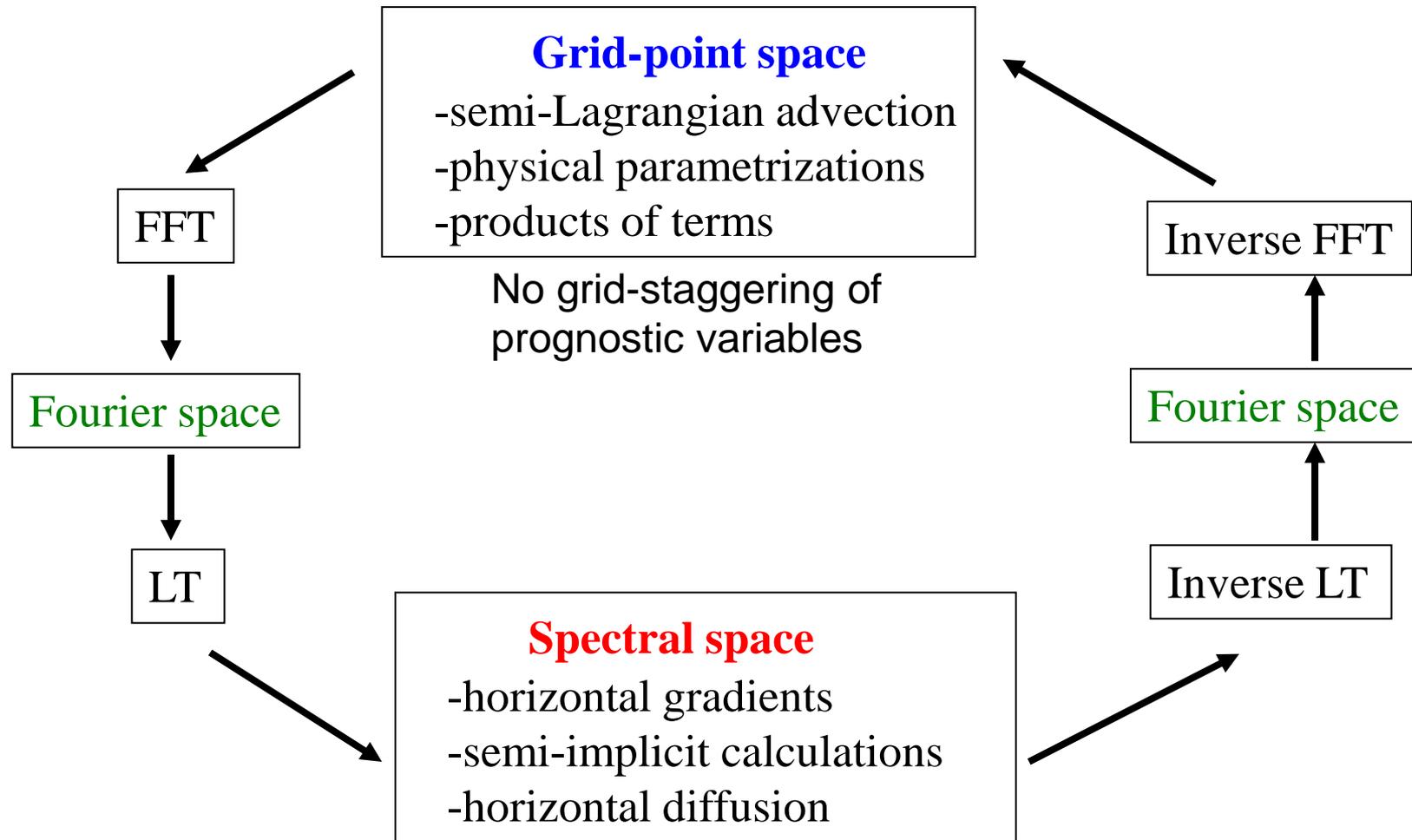


# The Integrated Forecasting System (IFS)

technology applied at ECMWF for the last 30 years ...

A spectral transform, semi-Lagrangian, semi-implicit  
(compressible) (non-)hydrostatic model

# Schematic description of the spectral transform method in the ECMWF IFS model



FFT: Fast Fourier Transform, LT: Legendre Transform

# Direct spectral transform (Forward)

Fourier transform:

$$\zeta_m(\theta) = \frac{1}{2\pi} \int_0^{2\pi} \zeta(\lambda, \theta) e^{-im\lambda} d\lambda$$

**FFT (fast Fourier transform)**

using

$$N_F \geq 2N+1$$

points (linear grid)

(3N+1 if quadratic grid)

Legendre transform:

$$\zeta_n^m = \frac{1}{2} \int_{-1}^1 \zeta_m \overline{P_n^m(\cos(\theta))} d\cos(\theta).$$

**Direct Legendre transform**

by **Gaussian quadrature**

using  $N_L \geq (2N+1)/2$

“Gaussian” latitudes (linear grid)

((3N+1)/2 if quadratic grid)

$$w_k = \frac{2N+1}{[P_N^{m=1}(x_k)]^2}$$



$$\zeta_n^m = \sum_{k=1}^K w_k \zeta_m(x_k) \overline{P_n^m(x_k)}$$

(normalized) associated Legendre polynomials

# Inverse spectral transform (Backward)

Inverse Legendre transform

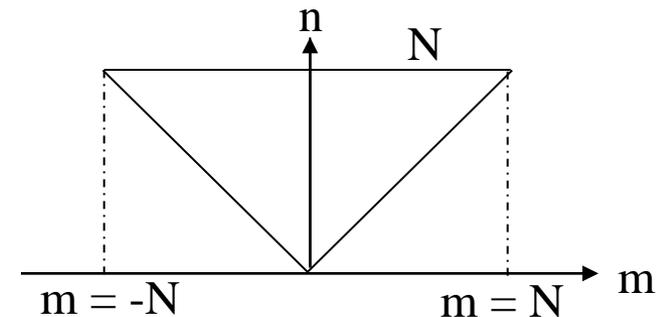
$$\zeta(\theta, \lambda) = \sum_{m=-N}^N e^{im\lambda} \sum_{n=|m|}^N \zeta_n^m \overline{P_n^m}(\cos(\theta)),$$

$$\zeta(\lambda, \mu, \eta, t) = \sum_{m=-N}^N \sum_{n=|m|}^N \zeta_n^m(\eta, t) Y_n^m(\lambda, \mu)$$

Triangular truncation  
(isotropic)

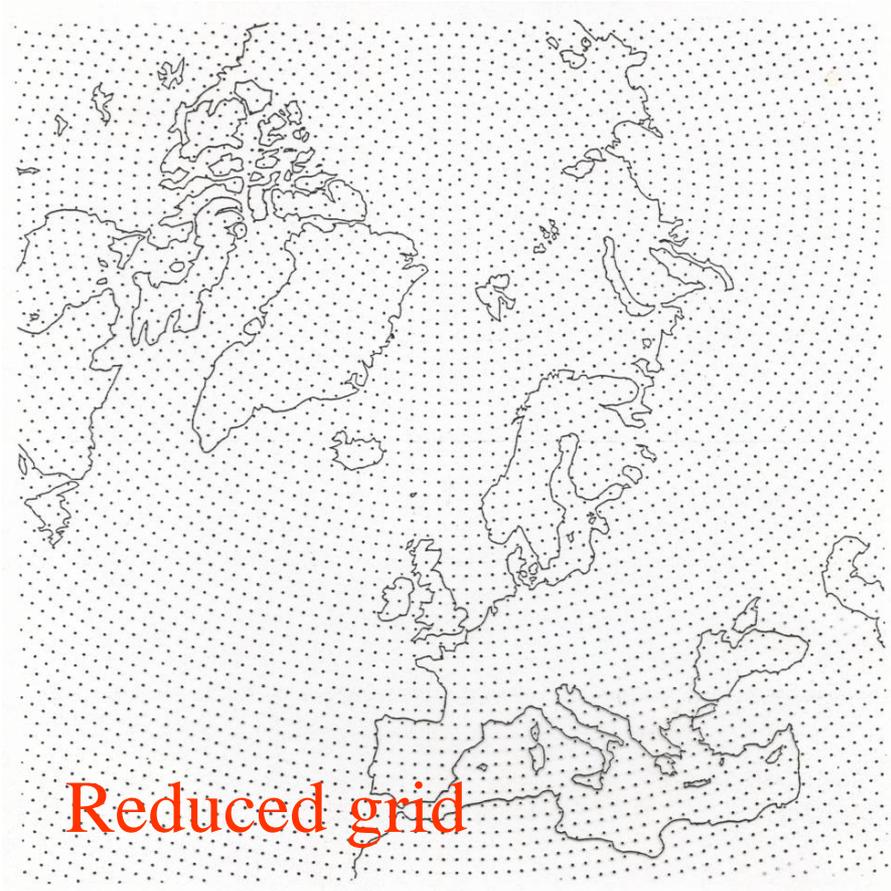
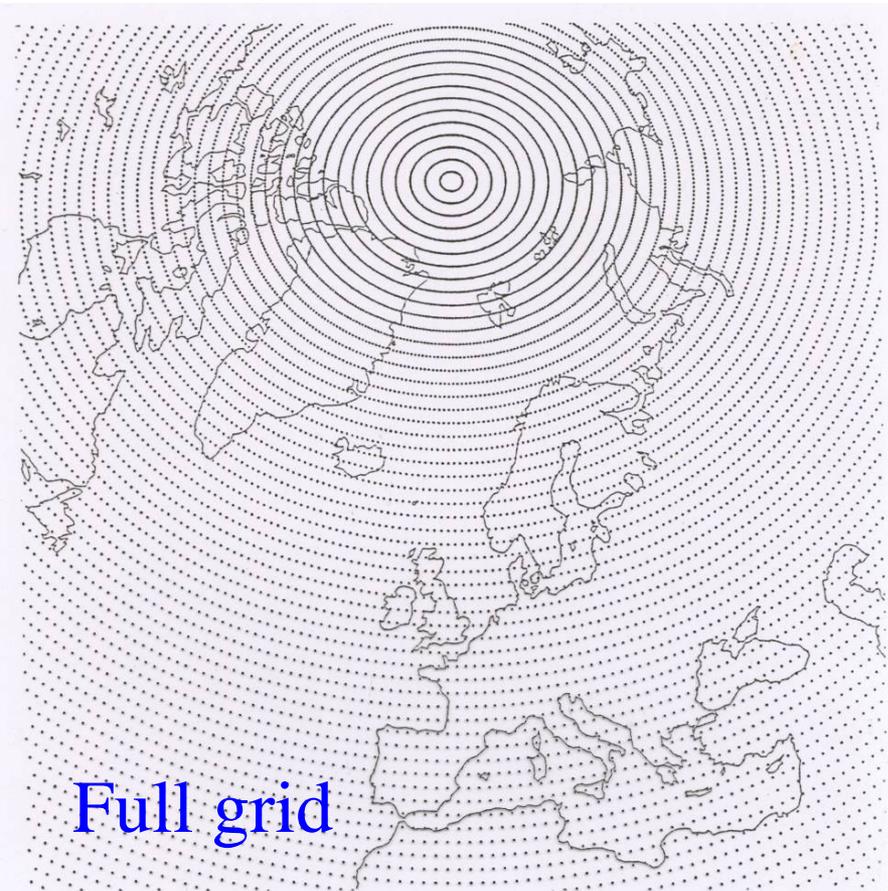
Spherical harmonics

Triangular truncation:



# The Gaussian grid

About 30% reduction in number of points



Reduction in the number of Fourier points at high latitudes is possible because the associated Legendre polynomials are very small near the poles for large  $m$ .

Note: number of points nearly equivalent to quasi-uniform icosahedral grid cells of the ICON model.

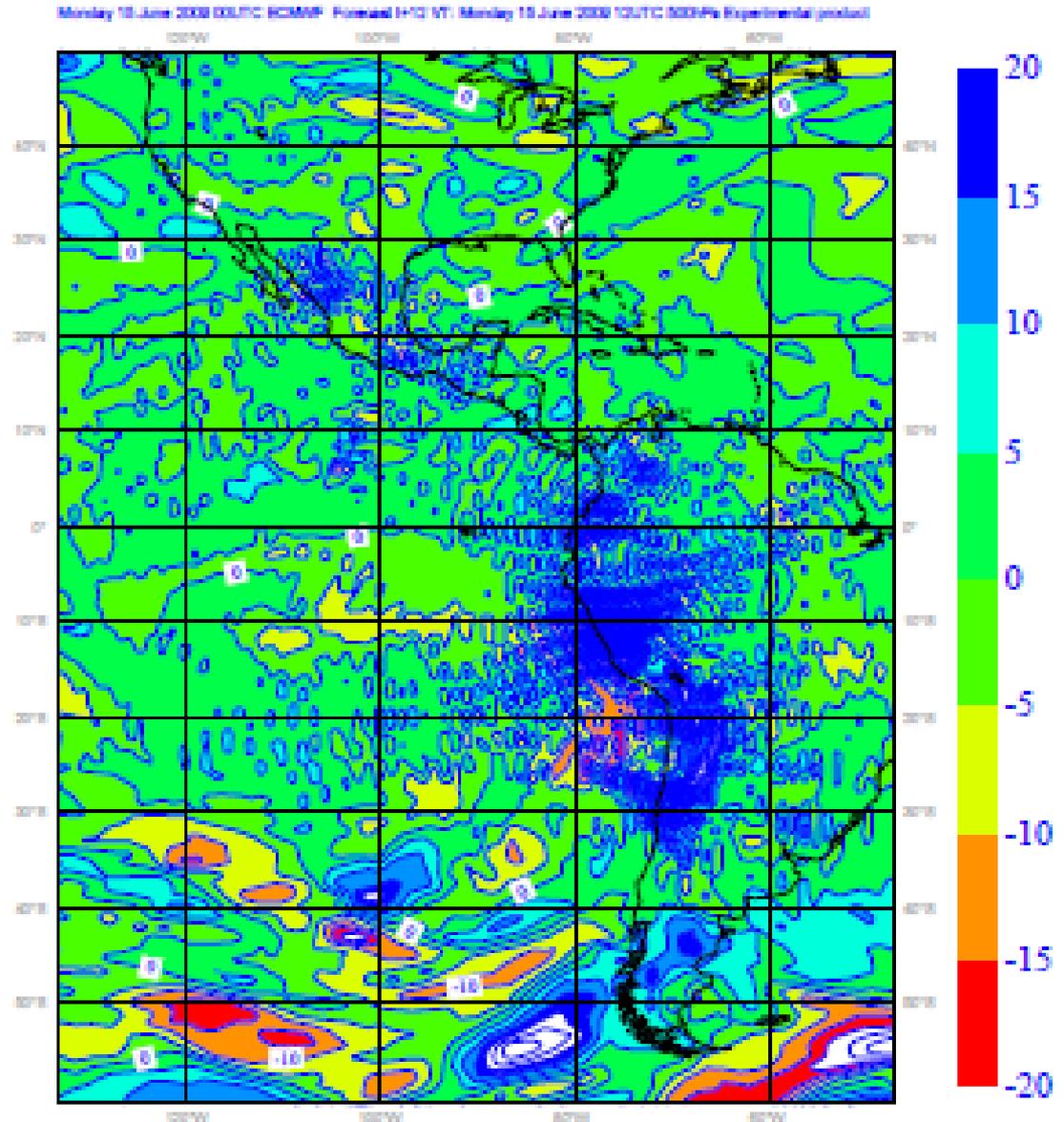
# Aliasing

- ◆ **Aliasing of quadratic terms on the linear grid ( $2N+1$  gridpoints per  $N$  waves), where the product of two variables transformed to spectral space cannot be accurately represented with the available number of waves (as quadratic terms would need a  $3N+1$  ratio).**
- ◆ **Absent outside the tropics in E-W direction due to the design of the reduced grid (obeying a  $3N+1$  ratio) but present throughout (and all resolutions) in N-S direction.**
- ◆ ***De-aliasing in IFS:* By subtracting the difference between a specially filtered and the unfiltered pressure gradient term at every time-step the stationary noise patterns can be removed at a **cost of approx. 5% at T1279** (2 extra transforms).**

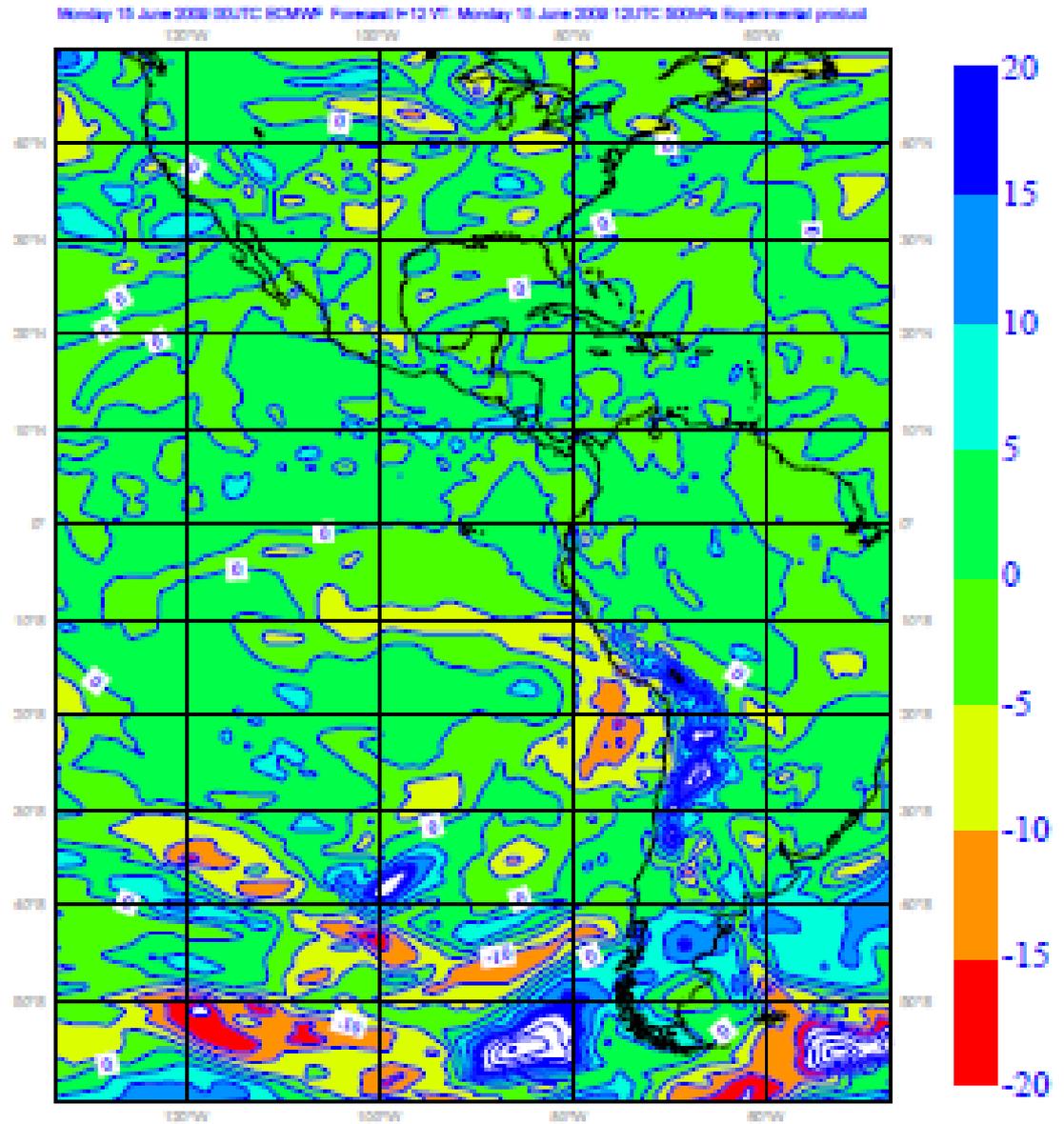
# De-aliasing

E-W

500hPa adiabatic  
zonal wind  
tendencies (T159)



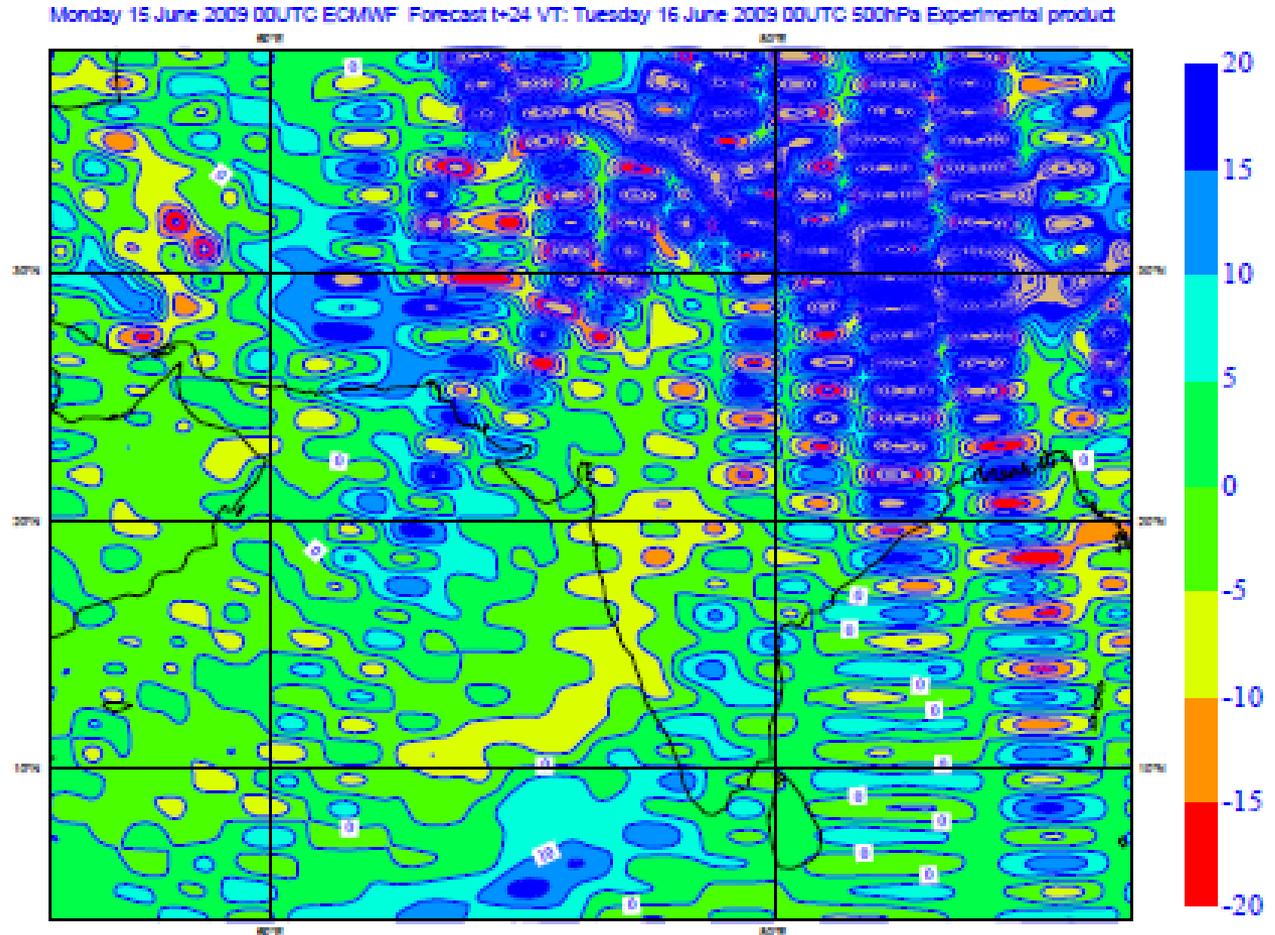
# De-aliasing



# De-aliasing

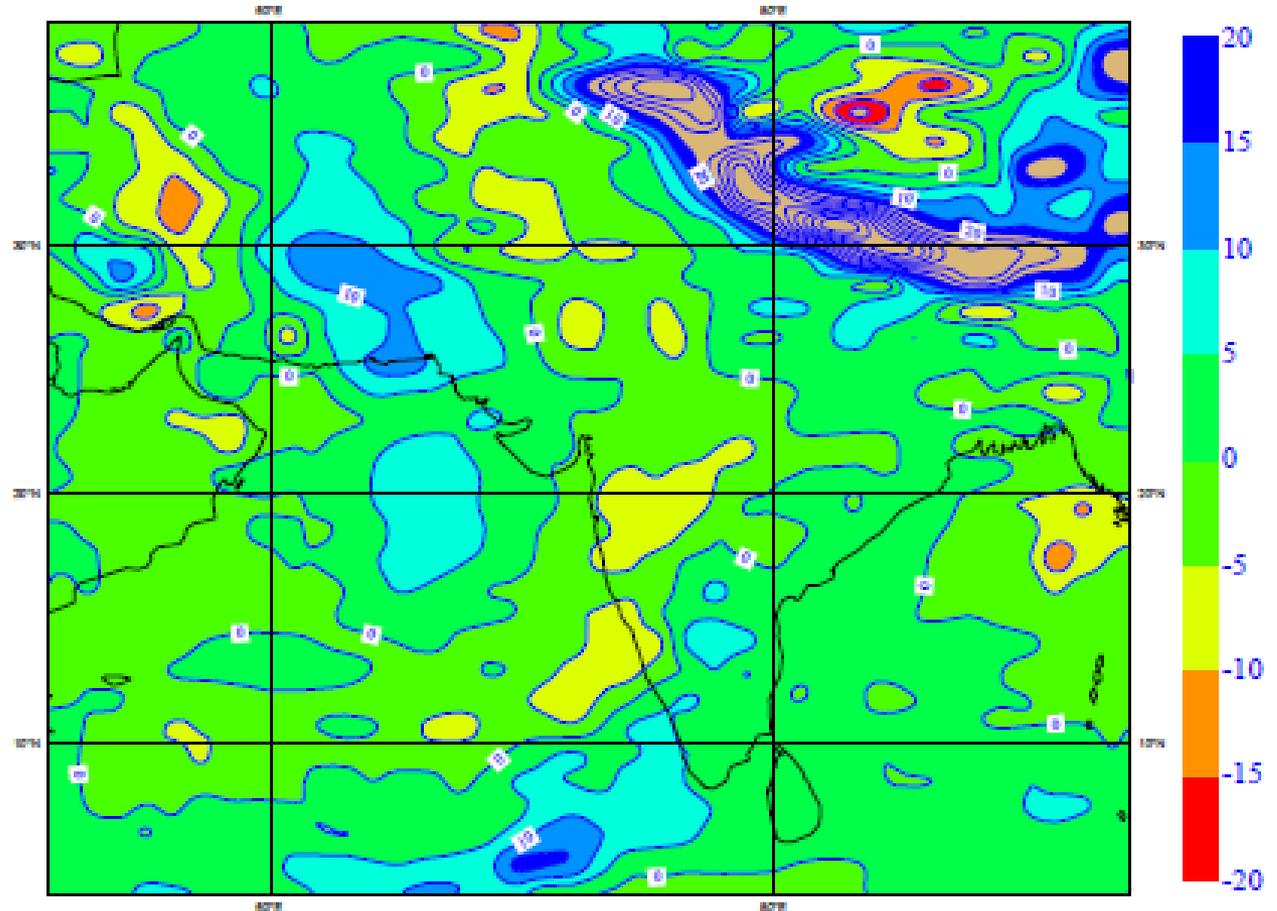
N-S

500hPa adiabatic  
meridional wind  
tendencies (T159)

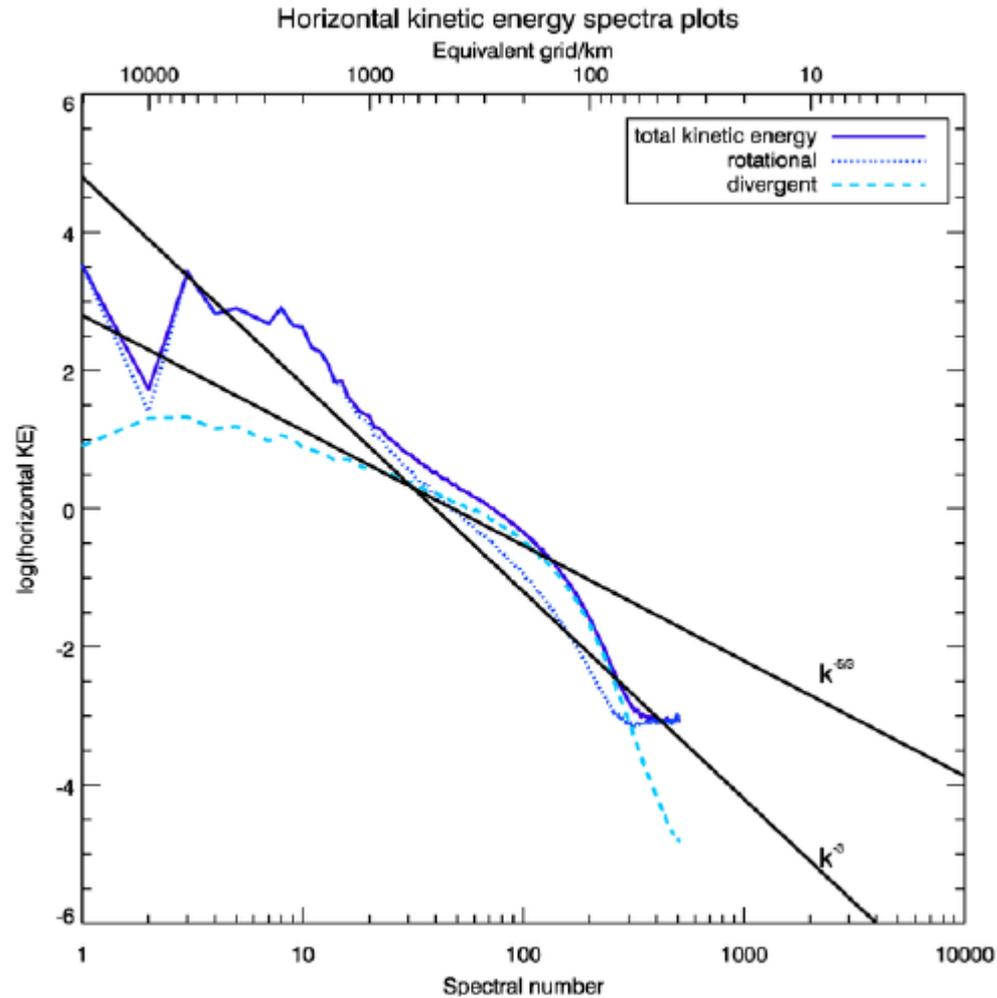


# De-aliasing

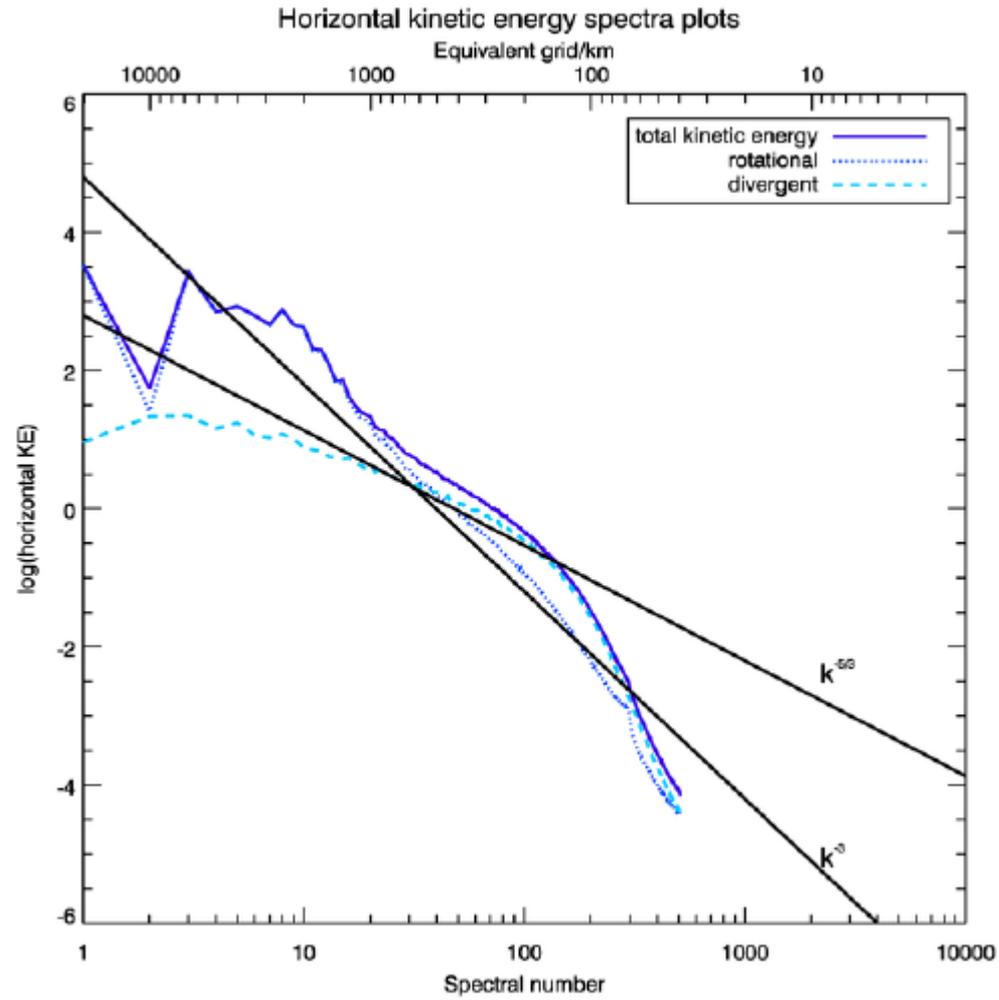
Monday 15 June 2009 00UTC ECMWF Forecast t+24 VT: Tuesday 16 June 2009 00UTC 500hPa Experimental product



# Kinetic Energy Spectra – 100 hPa



# Kinetic Energy Spectra – 100 hPa



# Fast Multipole Method (FMM) and spectral filtering

(*Jakob-Chien and Alpert, 1997; Tygert 2008*)

$$f_j = \sum_{k=1}^N \frac{\beta_j P_k}{\tilde{\mu}_j - \mu_k} \quad \text{For all } j=1, \dots, J$$

FMM: We can do above sum for all points  $j$  in  $O(J+N)$  operations instead of  $O(J*N)$  !

Example: From Christoffel-Darboux formula for associated Legendre polynomials  
We can do a direct and inverse Legendre transform for a single Fourier mode as:

$$\begin{aligned} \tilde{\zeta}^m(\tilde{\theta}_j) = & \epsilon_{N+1}^m \overline{P}_{N+1}^m(\mu_j) \sum_{i=1}^J \frac{\zeta^m(\theta_i) w_i \overline{P}_N^m(\mu_i)}{\tilde{\mu}_j - \mu_i} \\ & - \epsilon_{N+1}^m \overline{P}_N^m(\tilde{\mu}_j) \sum_{i=1}^J \frac{\zeta^m(\theta_i) w_i \overline{P}_{N+1}^m(\mu_i)}{\tilde{\mu}_j - \mu_i} \end{aligned}$$

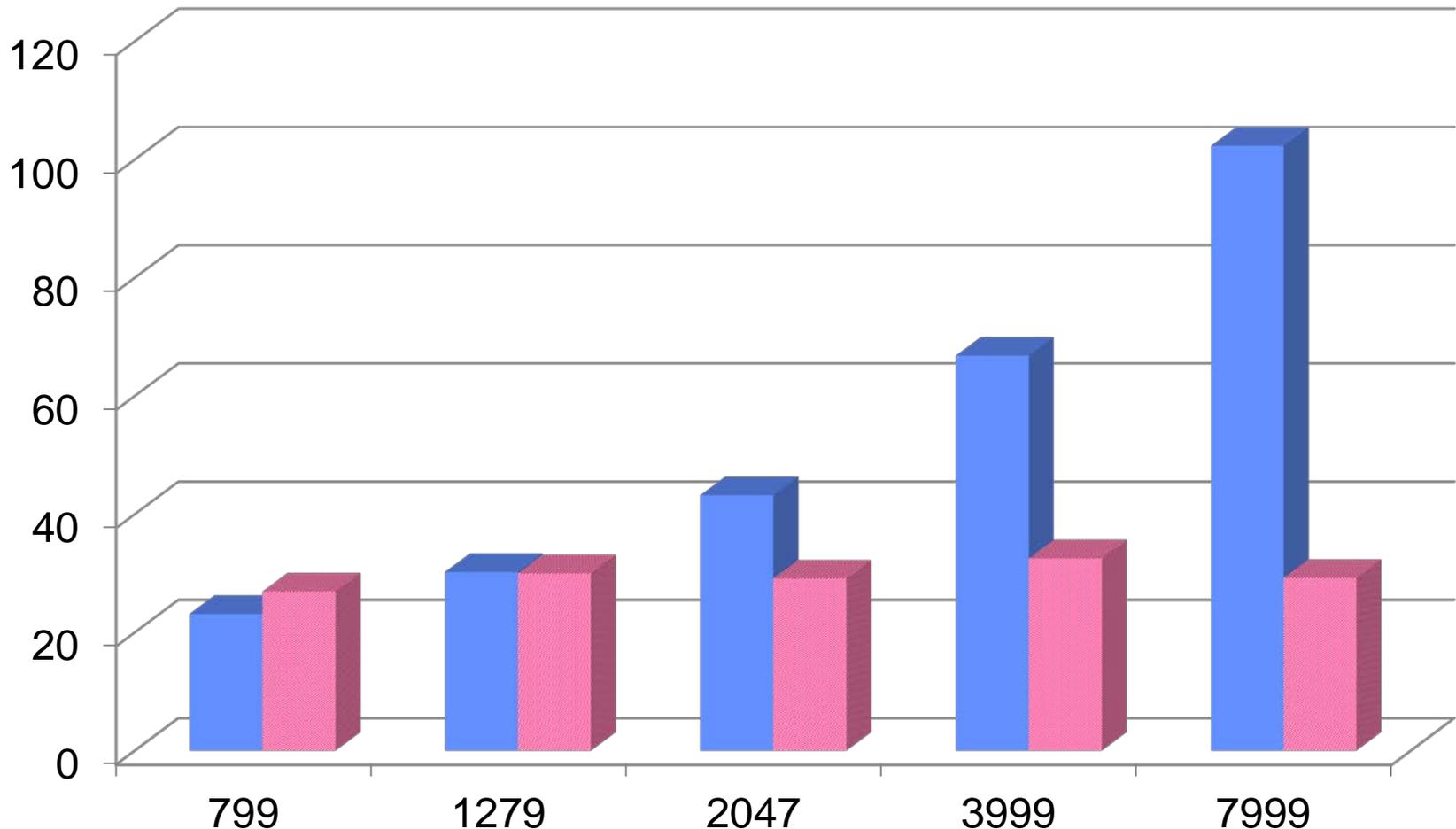
# A fast Legendre transform (FLT)

*(O'Neil, Woolfe, Rokhlin, 2009; Tygert 2008, 2010)*

- ◆ **The computational complexity of the ordinary spectral transform is  $O(N^3)$  (where  $N$  is the truncation number of the series expansion in spherical harmonics) and it was therefore believed to be *not computationally competitive with other methods at very high resolution***
- ◆ **The FLT is found to be  $O(N^2 \log N^3)$  for horizontal resolutions up to T7999 (*Wedi et al, 2013*)**

Number of floating point operations for direct or inverse spectral transforms of a single field, scaled by  $N^2 \log^3 N$

■ dgemm ■ FLT



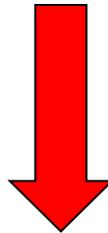
# Matrix-matrix multiply for each zonal wavenumber $m$

$$\begin{pmatrix} \zeta_{m,1}(x_1) & \cdots & \zeta_{m,l_{tot}}(x_1) \\ \vdots & \ddots & \vdots \\ \zeta_{m,1}(x_K) & \cdots & \zeta_{m,l_{tot}}(x_K) \end{pmatrix} = \begin{pmatrix} \overline{P}_1^m(x_1) & \cdots & \overline{P}_N^m(x_1) \\ \vdots & \ddots & \vdots \\ \overline{P}_1^m(x_K) & \cdots & \overline{P}_N^m(x_K) \end{pmatrix} \begin{pmatrix} \zeta_{1,1}^m & \cdots & \zeta_{1,l_{tot}}^m \\ \vdots & \ddots & \vdots \\ \zeta_{N,1}^m & \cdots & \zeta_{N,l_{tot}}^m \end{pmatrix}$$

$$\begin{bmatrix} \zeta^m(x_k)_l \end{bmatrix} = \begin{bmatrix} \overline{P}_n^m(x_k) \end{bmatrix} \begin{bmatrix} \zeta_{n,l}^m \end{bmatrix}$$

Gaussian latitude

Field,  
vertical level



apply **butterfly compression**,  
this step is *precomputed* only  
once!

total wavenumber

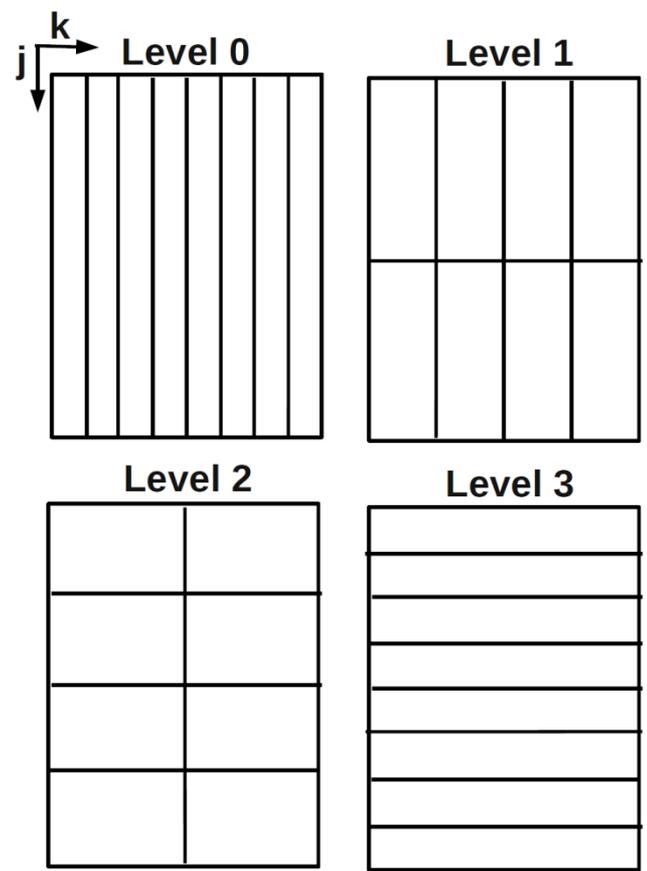
zonal  
wavenumber

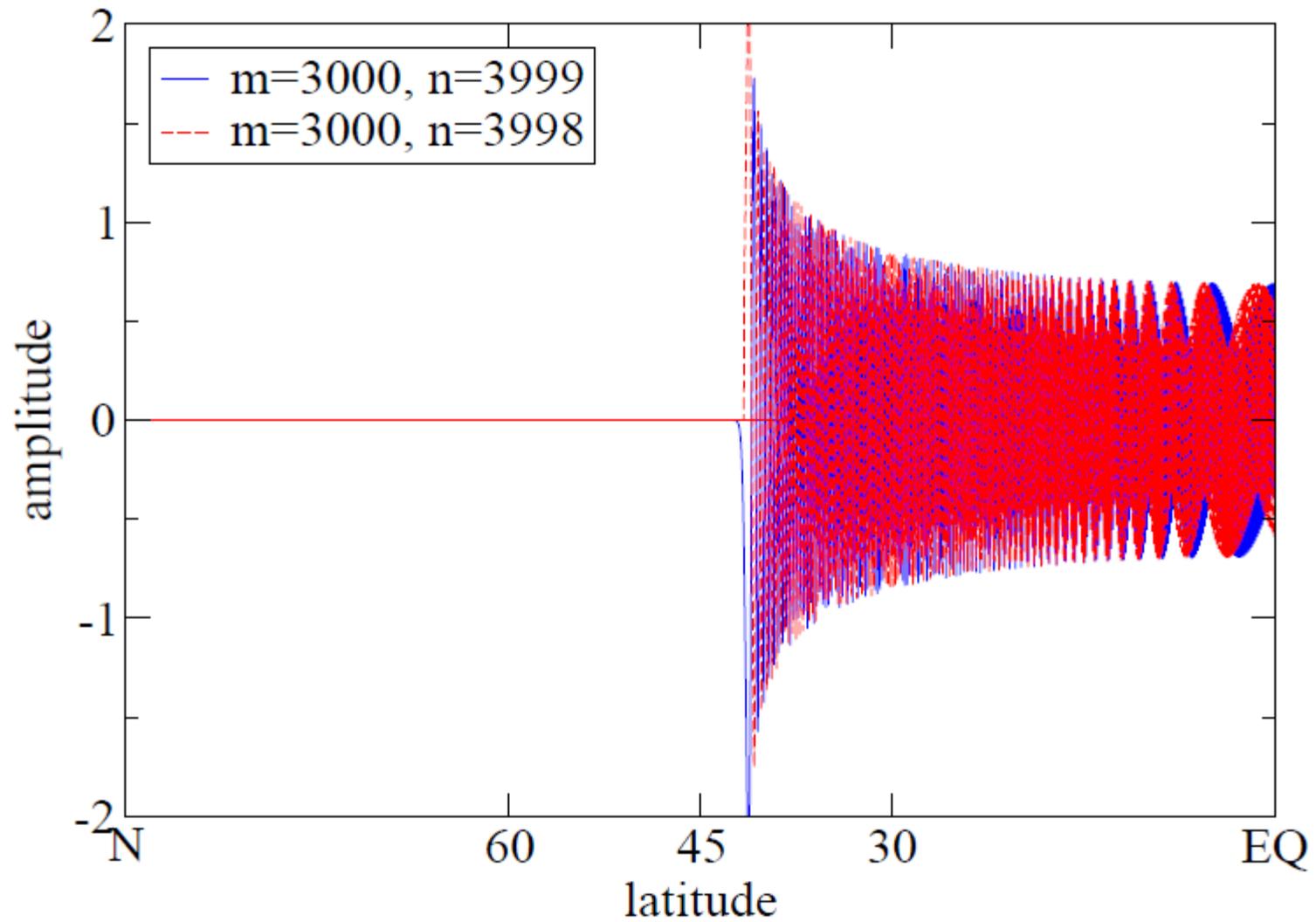
# Butterfly algorithm:

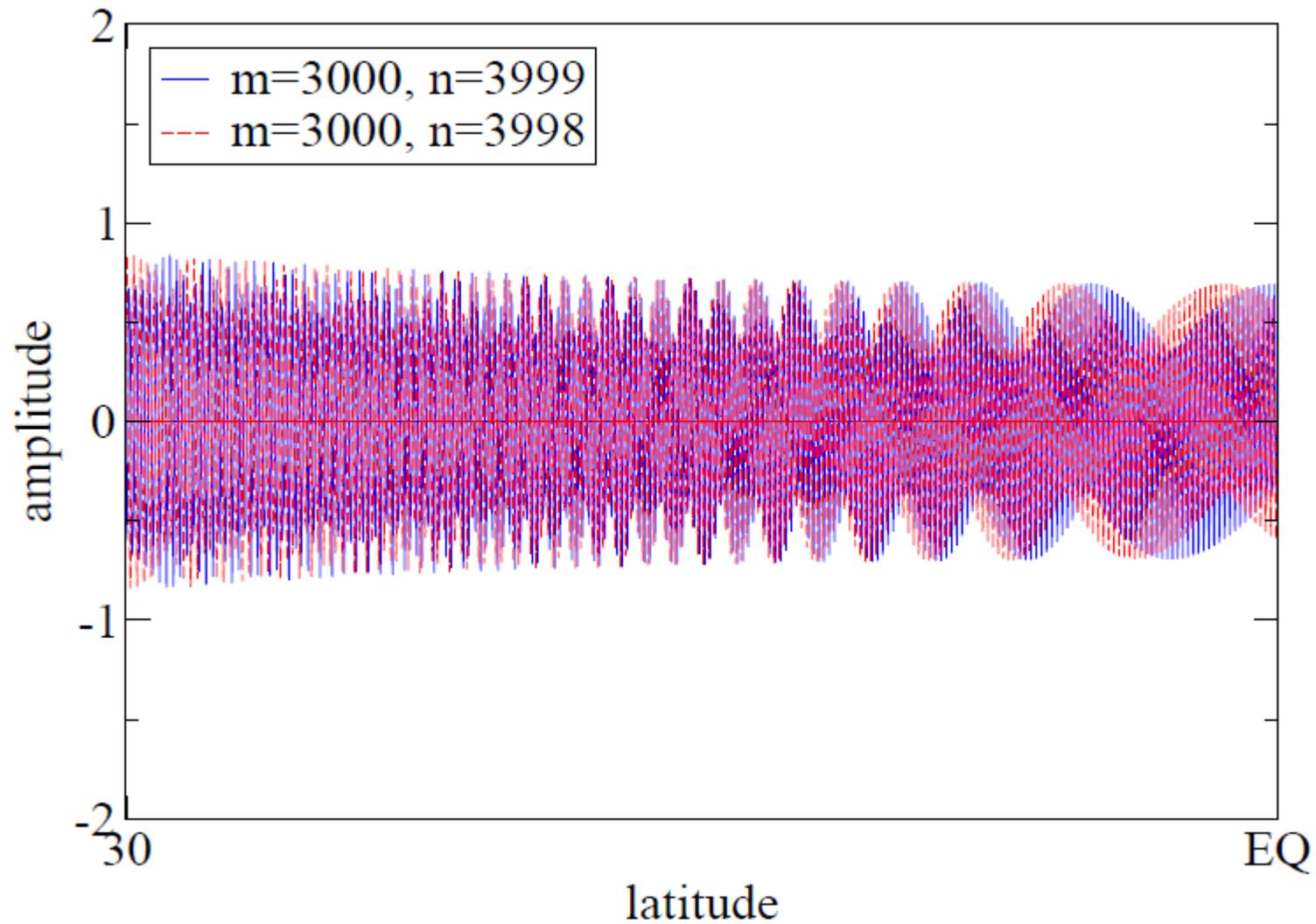
pre-compute  $S_{rxs} \cong C_{rxk} A_{kxs}$

```
for l = 0 → L do
  for all j, k boxes do
    if l = 0 then
      S0,k = extract_sub_matrix()
      compr_sub_matrix(S0,k, A0,k)
      store A0,k
    else
      Sl,j,k = comb_compr_l_and_r_neighb(C, l - 1)
      compr_sub_matrix(Sl,j,k, Al,j,k)
      store Al,j,k
    end if
  if l = L then
    store CL,j
  end if
end for
end for
```

With each level 1,  
double the columns  
and half the rows







# Butterfly algorithm: apply $f = S\alpha$

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for  $l = 0 \rightarrow L$  do

  for all  $j, k$  boxes do

    if  $l = 0$  then

      store  $\beta_{0,k} = A_{0,k}\alpha_k$

    else

      store  $\beta_{l,j,k}$

$= A_{l,j,k} \times \text{comb\_l\_and\_r\_neighb}(\beta, l - 1)$

    end if

  if  $l = L$  then

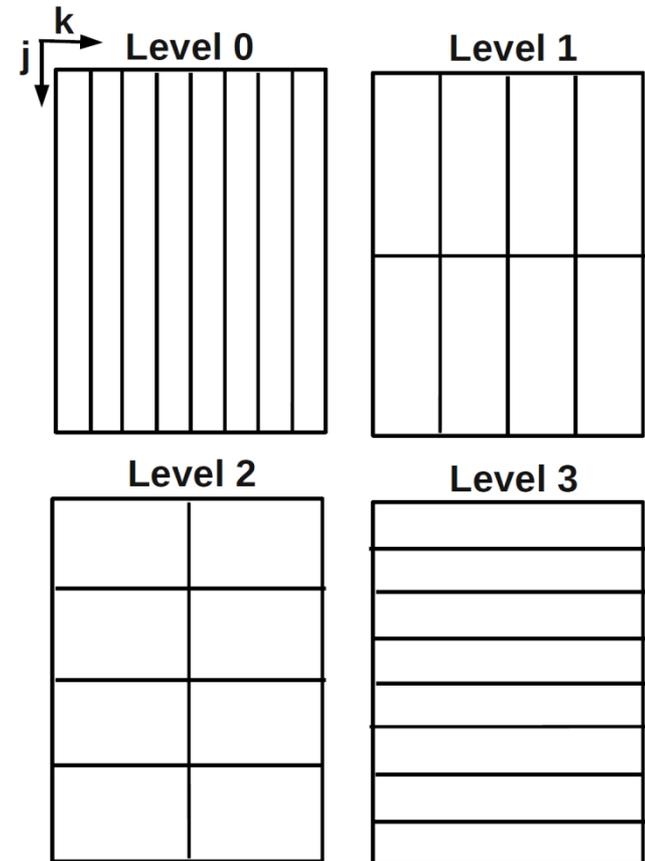
    store  $f_{L,j} = C_{L,j}\beta_{L,j}$

  end if

end for

end for

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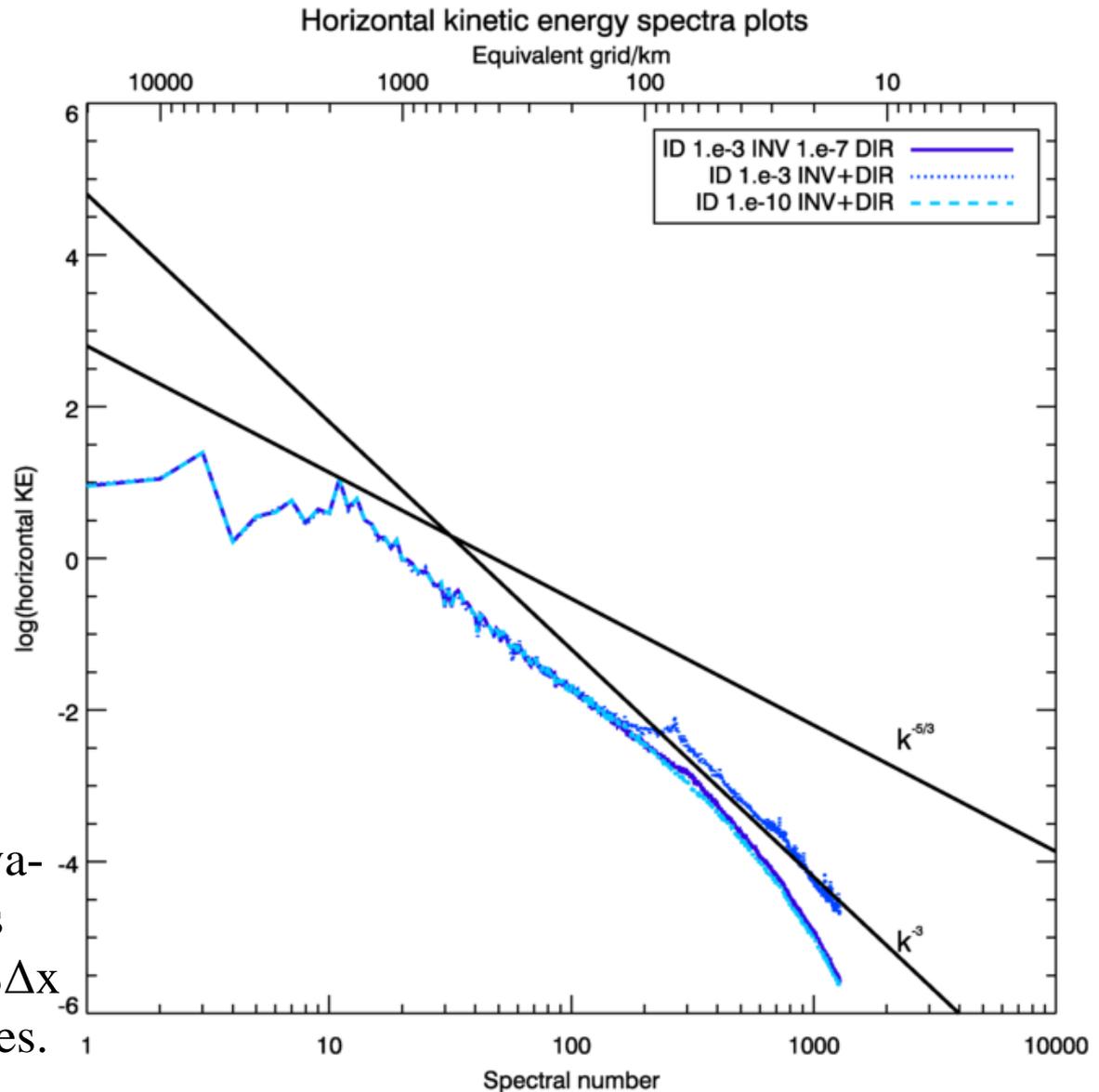
# Interpolative Decomposition (ID)

- ◆ The *compression* uses the interpolative decomposition (ID) described in [Cheng et al \(2005\)](#).
- ◆ The  $r \times s$  matrix  $S$  may be *compressed* such that

$$\left\| S_{r \times s} - C_{r \times k} A_{k \times s} \right\| \leq \varepsilon$$

With an  $r \times k$  matrix  $C$  constituting a subset of the columns of  $S$  and the  $k \times s$  matrix  $A$  containing a  $k \times k$  identity as a submatrix.  $k$  is the  $\varepsilon$ -rank of the matrix  $S$  ([see also e.g. Martinsson and Rokhlin, 2007](#)).

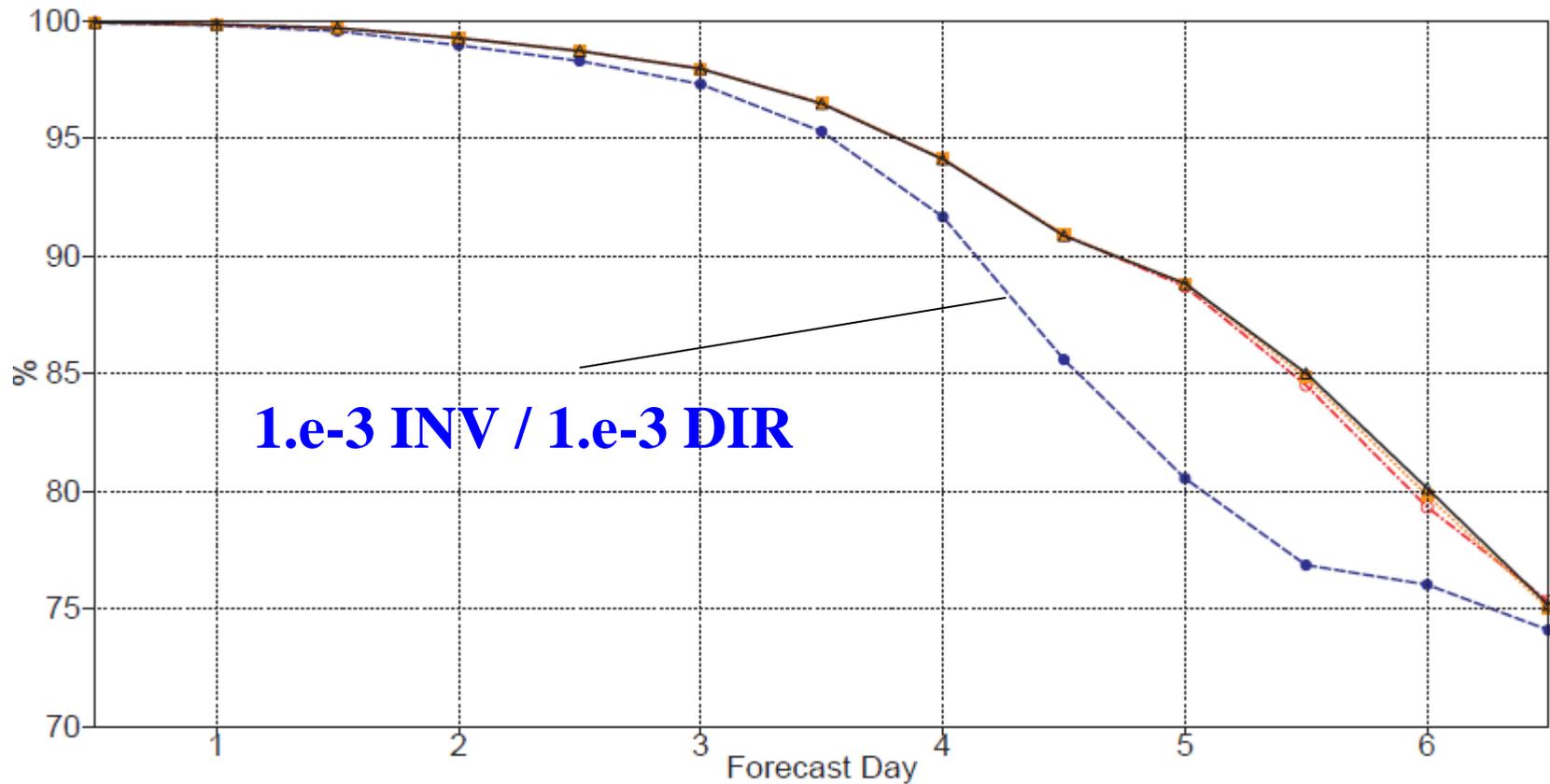
# T1279 FLT using different ID epsilons for INV (1.e-3) + DIR (1.e-7)



A recent comparison with observational surface wind data suggests that IFS fully resolves scales at  $8\Delta x$  and partially resolves  $4\Delta x$  features. (*Abdalla et al, 2013*)

500hPa geopotential  
Anomaly correlation  
lat 20.0 to 90.0, lon -180.0 to 180.0

- △— fsi5 T2047 ctrl no FLT
- fsk4 T2047 lossy 1.e-10
- fsi0 T2047 lossy 1.e-7
- fsjb T2047 lossy 1.e-3



**1.e-3 INV / 1.e-3 DIR**

# Comparison T1279 1.e-4 INV / 1.e-7 DIR

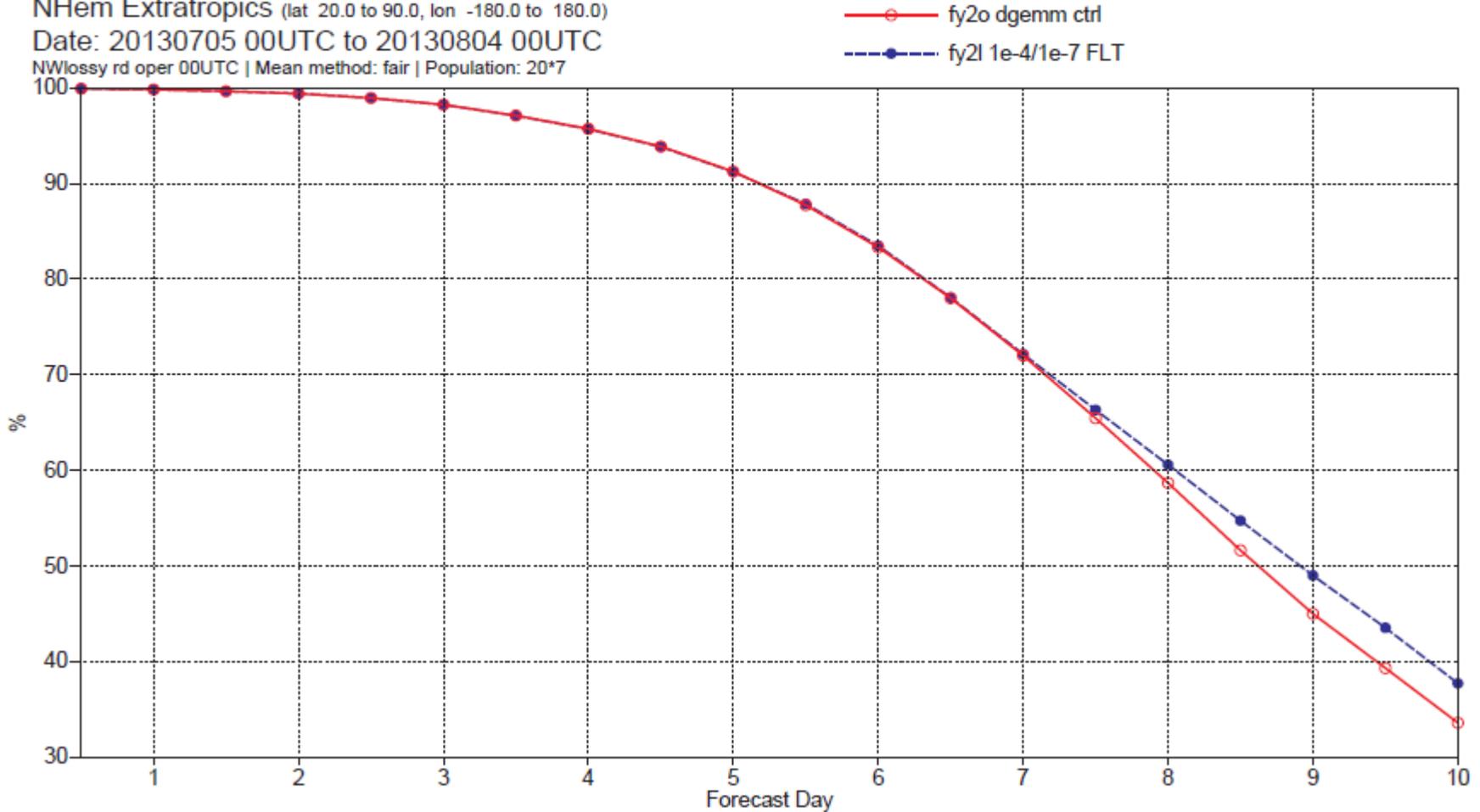
500hPa geopotential

Anomaly correlation

NHem Extratropics (lat 20.0 to 90.0, lon -180.0 to 180.0)

Date: 20130705 00UTC to 20130804 00UTC

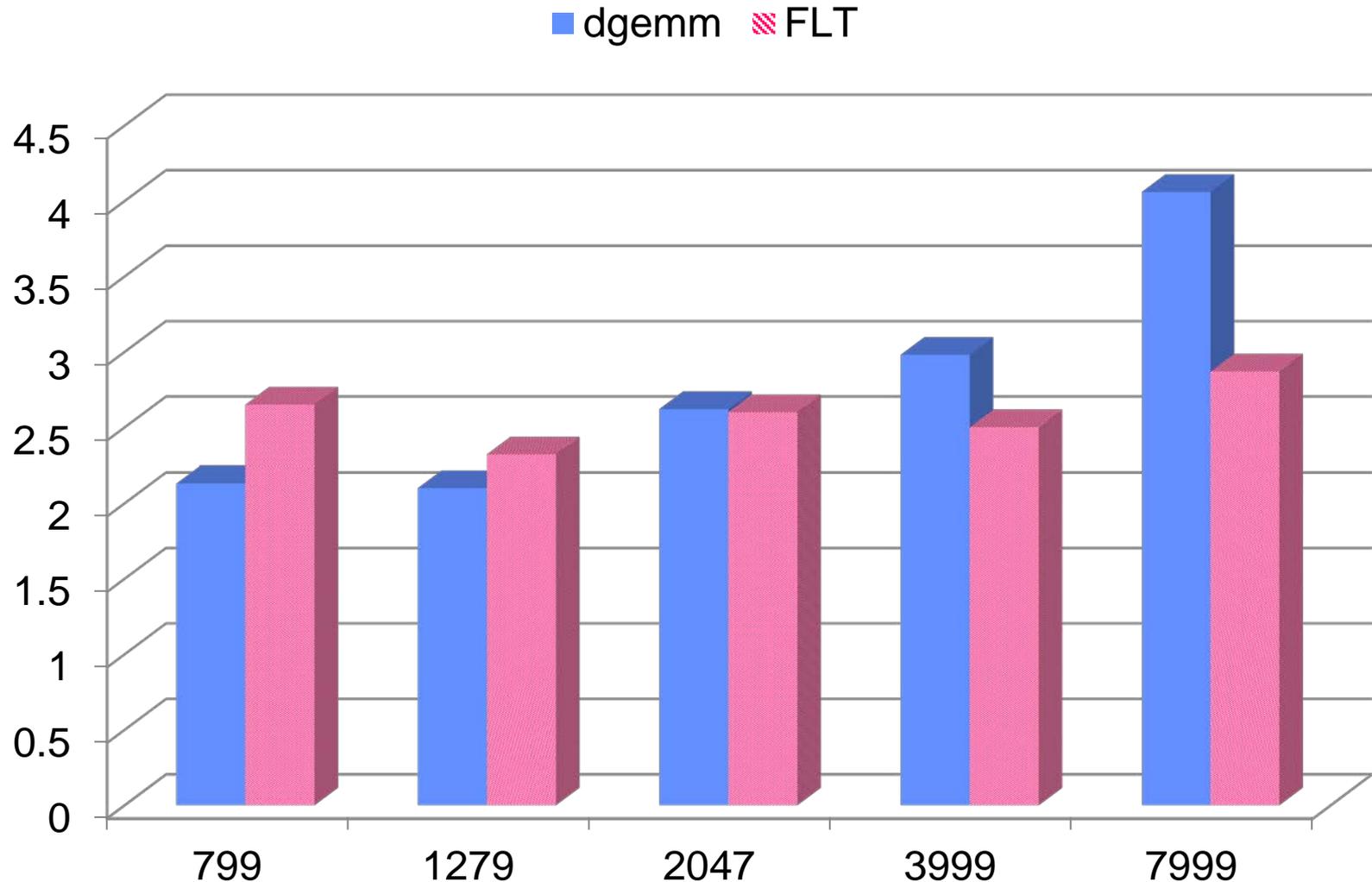
NWlossy rd oper 00UTC | Mean method: fair | Population: 20\*7



## The FLT in a nutshell – $O(N^2 \log^3 N)$

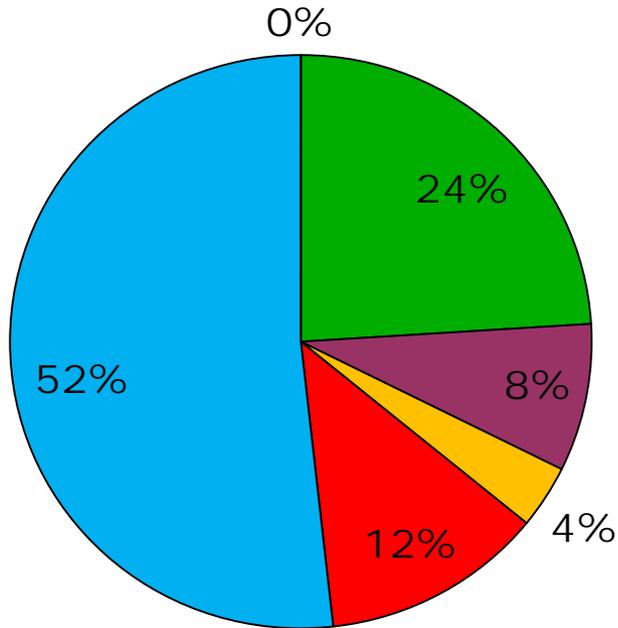
- ◆ **Speed-up the sums of products between associated Legendre polynomials at all Gaussian latitudes and the corresponding spectral coefficients of a field (e.g. temperature on given level)**
- ◆ **The essence of the FLT:**
  - ◆ **Exploit similarities of associated Legendre polynomials at all (Gaussian) latitudes but different total wave-number**
  - ◆ **Pre-compute** (once, 0.1% of the total cost of a 10 day forecast) a **compressed** (approximate) representation of the matrices (for each  $m$ ) involved
  - ◆ **Apply** the compressed (reduced) representation at every time-step of the simulation.

# Average wall-clock time compute cost of $10^7$ spectral transforms scaled by $N^2 \log^3 N$



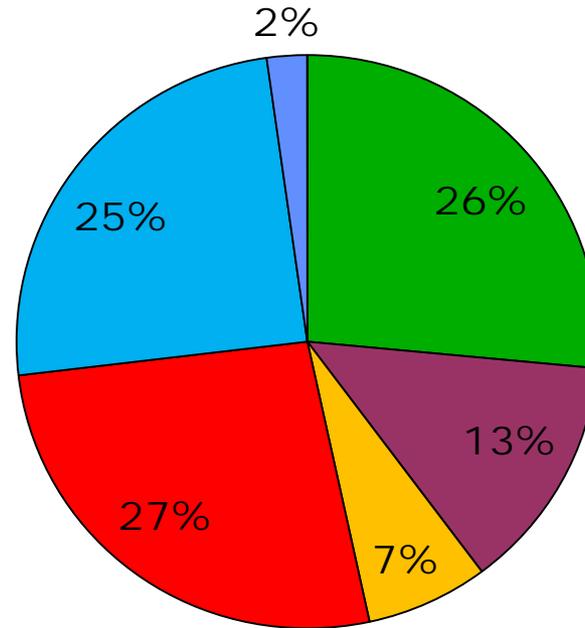
# Computational Cost: NH at T3999 vs. H T2047

SP\_DYN was 23 percent for this model configuration, and is now 7 percent. Improvement due to exposing 'greater OpenMP parallelism' from 4K threads to a maximum of 4K \* 91 threads ; in this case 16K threads.



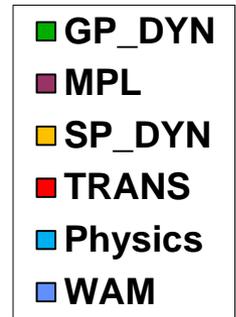
## H T<sub>2047</sub> L91

Tstep=450s, 0.8s/iteration  
With 896x16 ibm\_power7

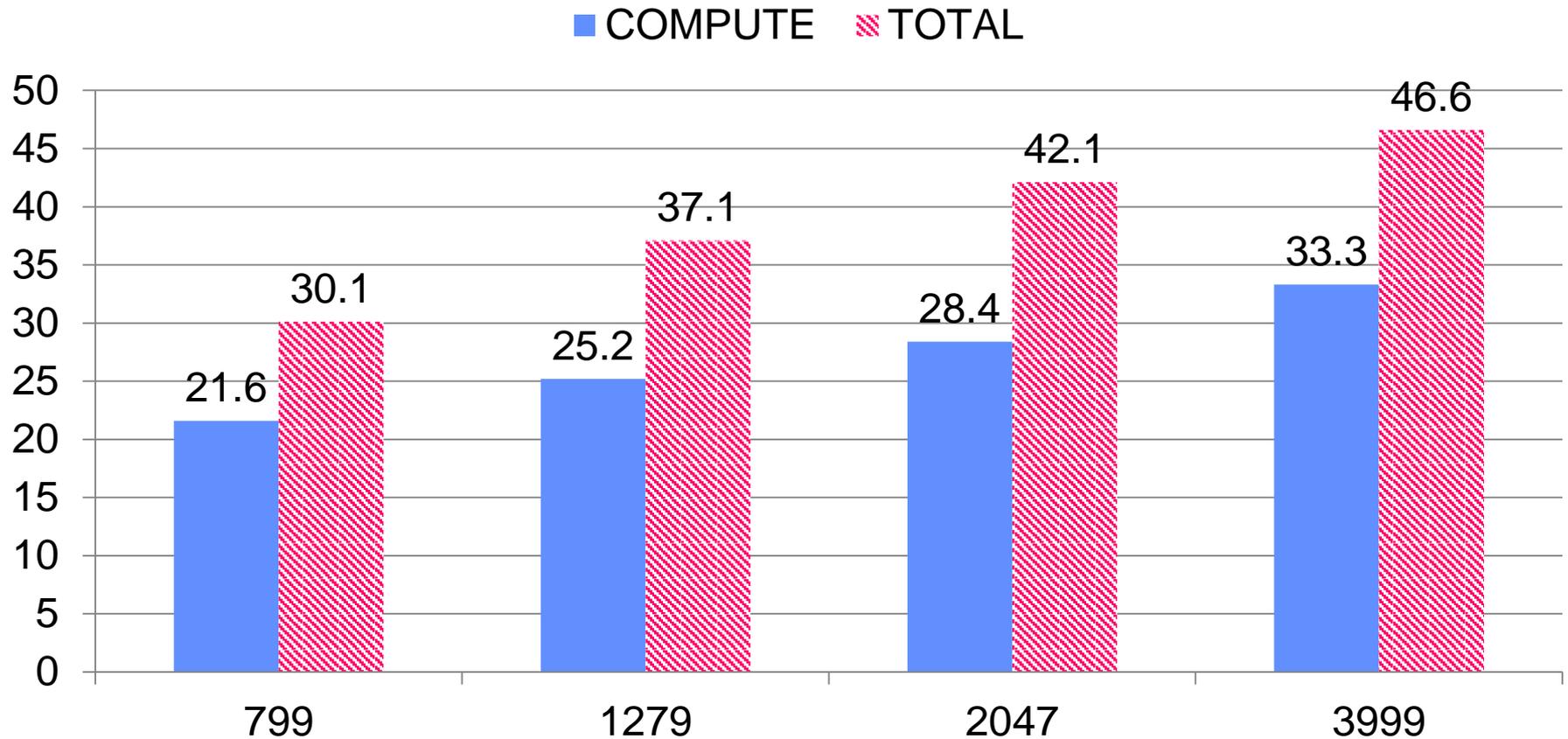


## NH T<sub>3999</sub> L91

Tstep=180s, 3.1s/iteration  
With 1024x16 ibm\_power7

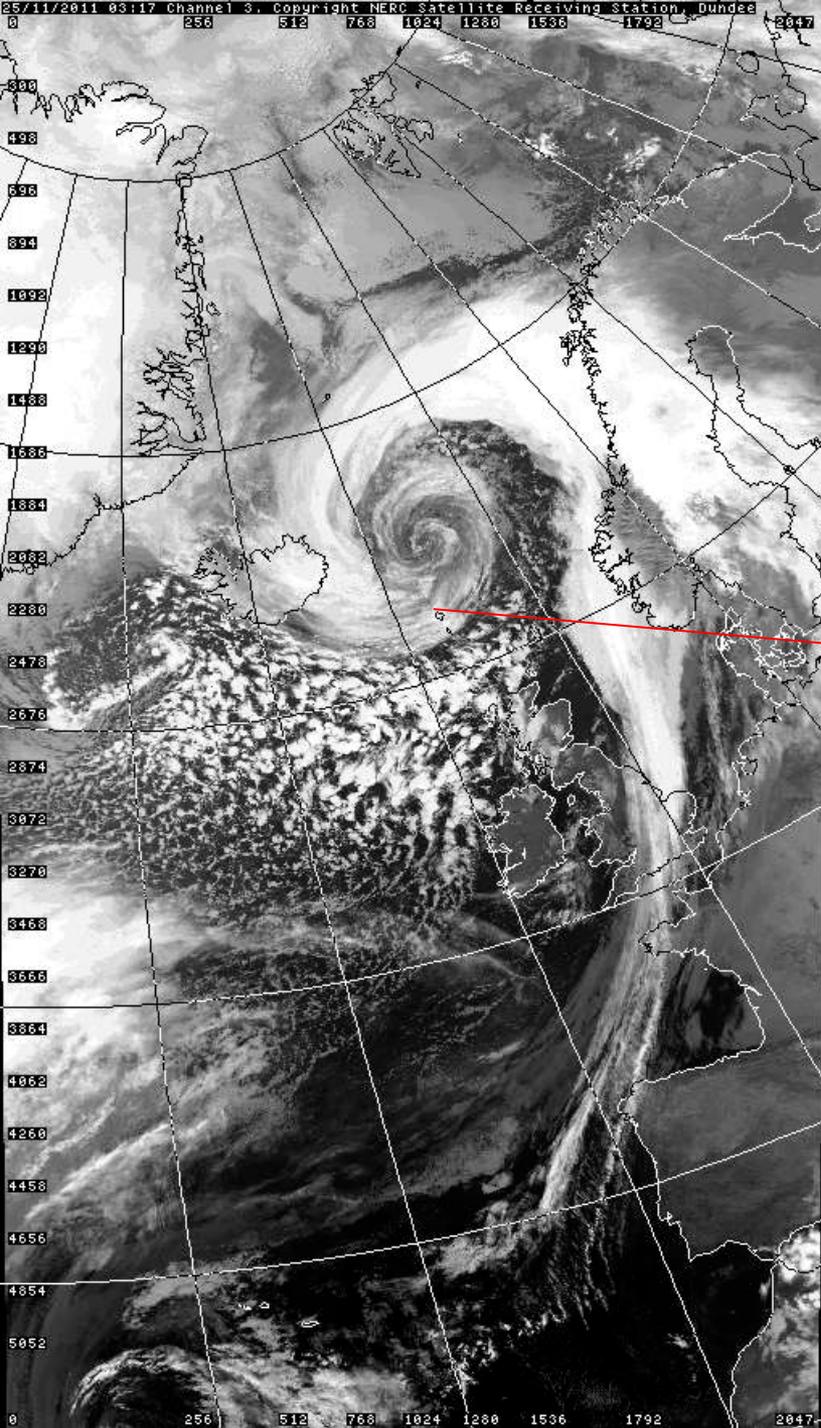


% (of total execution time) cost of spectral part of the model on IBM Power7 (all L91, all NH for comparison); Total includes communications



We expect significant reductions in future cores -> vector instr. / GPU

All these can be run with hydrostatic code == 1/2 of above numbers !



# Norwegian storm Berit(Xaver) 22-27<sup>th</sup> November 2011

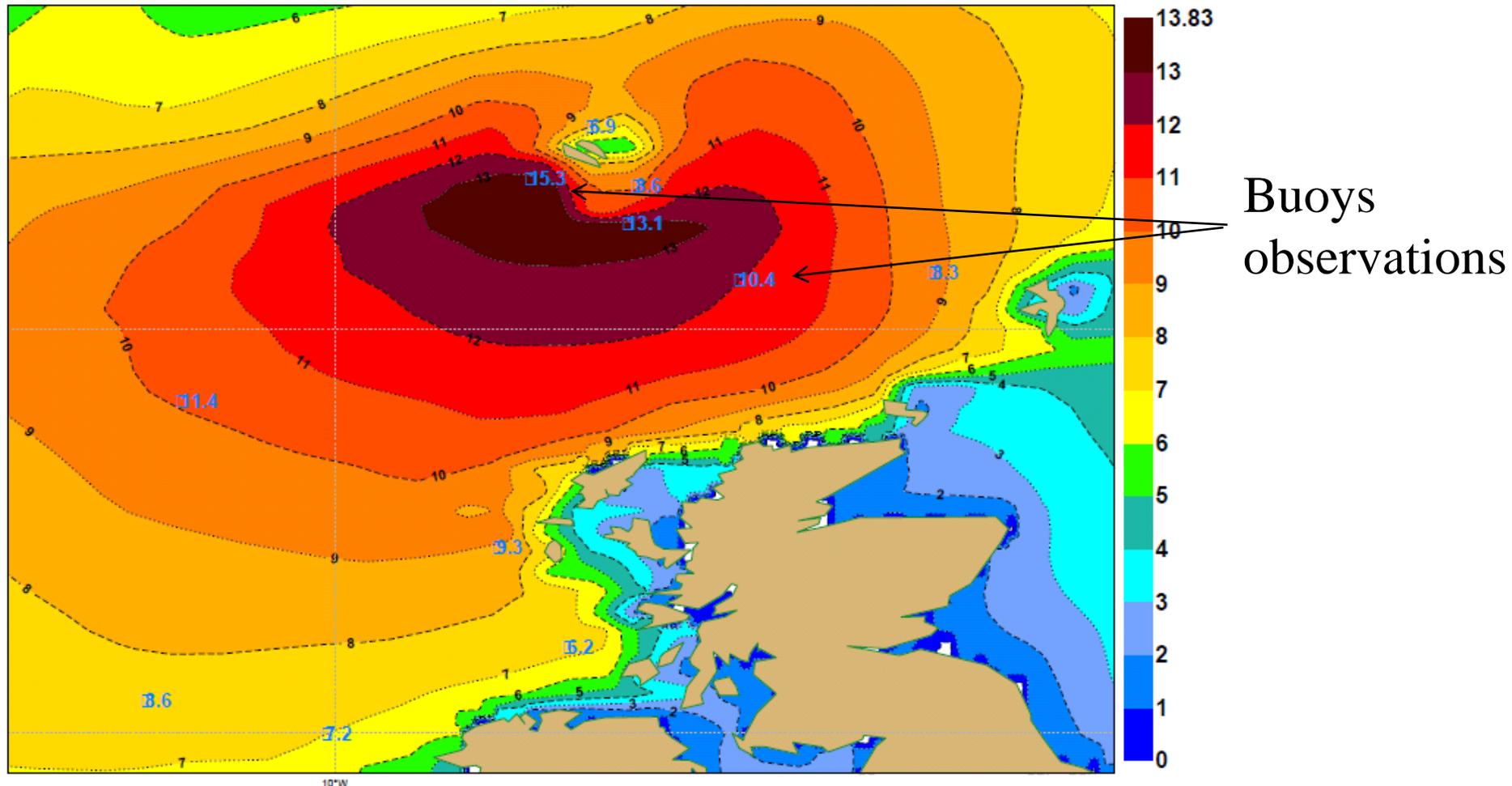
Faroe Islands

Monster waves battered the shores of the Faroe Islands, and Norway later, both reporting severe structural damage from the storm(s) during this period.

# T1279 coupled to 0.25 degree wave-model

## Faroe Islands hit by storm Xaver (Berit) on 25<sup>th</sup> November 2011

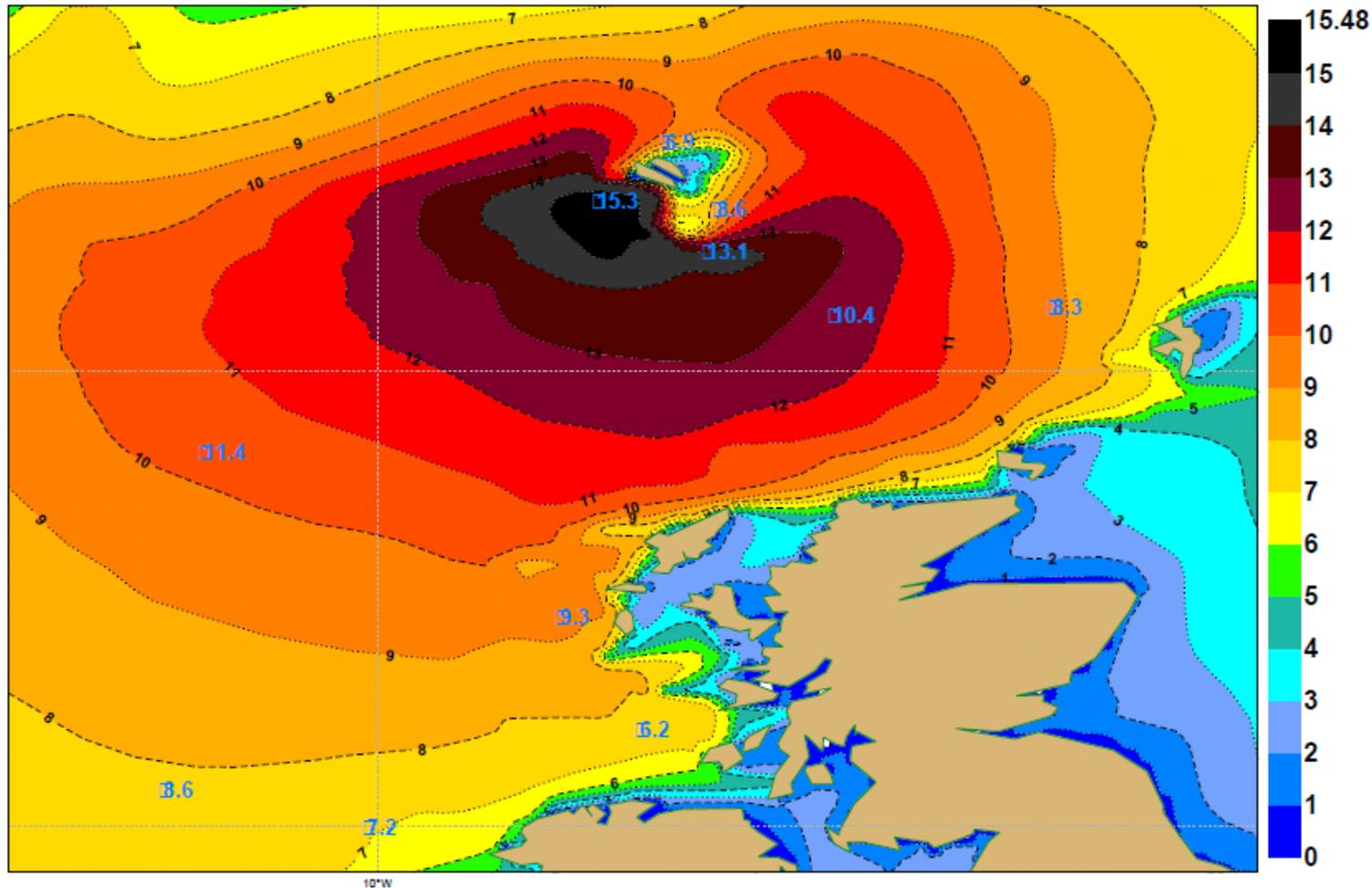
Thursday 24 November 2011 00UTC ECMWF Forecast t+24 VT: Friday 25 November 2011 00UTC Surface: Significant wave height (Exp: 0001 )  
T1279 with 0.25 global WAM (operations)  
2 hourly averaged wave height observations



# T3999 coupled to 0.1 degree wave model

## Faroe Islands hit by storm Xaver (Berit) on 25<sup>th</sup> November 2011

Thursday 24 November 2011 00UTC ECMWF Forecast t+24 VT: Friday 25 November 2011 00UTC Surface: Significant wave height (Exp: flmw )  
T3999 with 0.1 global WAM  
2 hourly averaged wave height observations

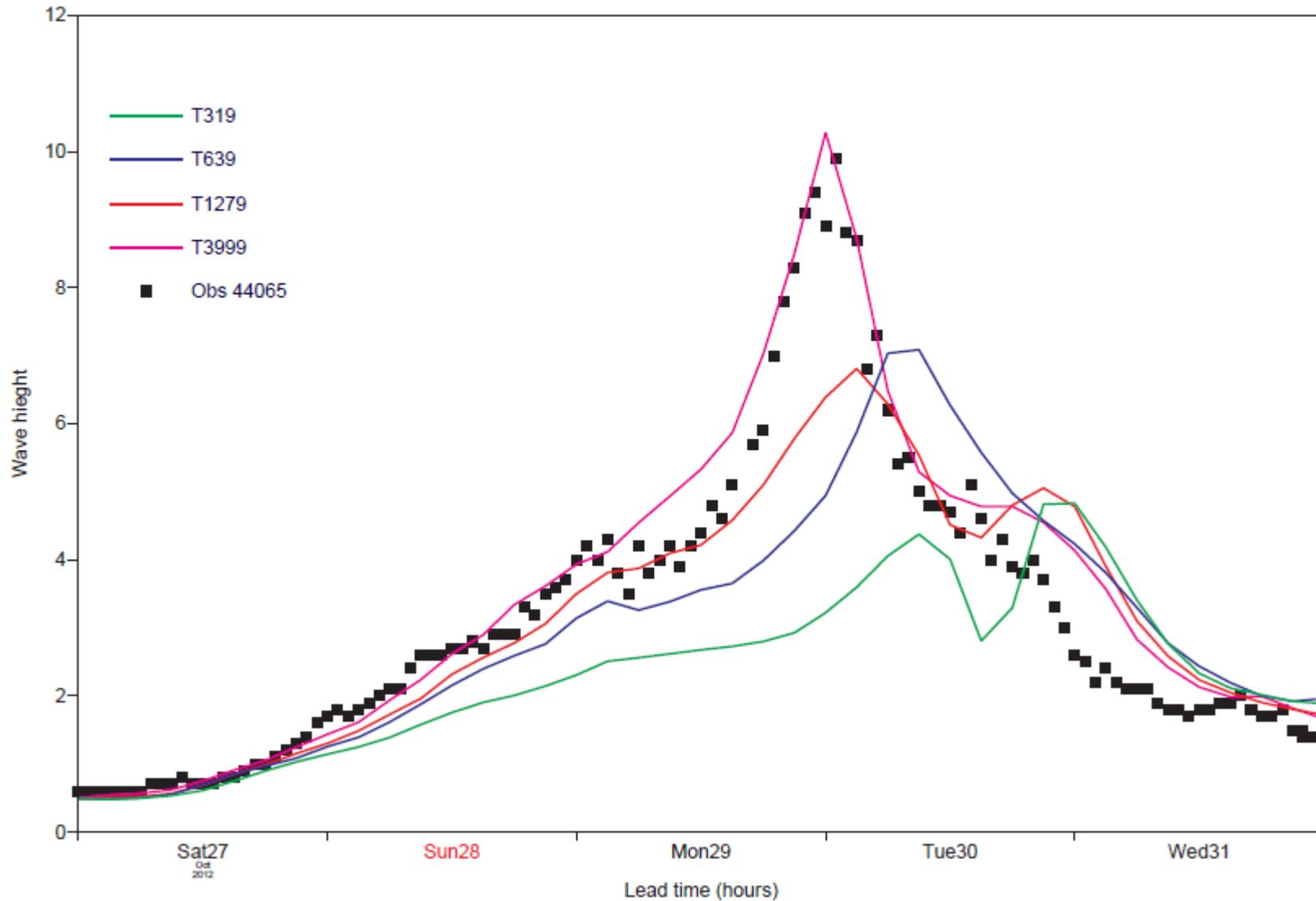


# Extreme events: Hurricane Sandy

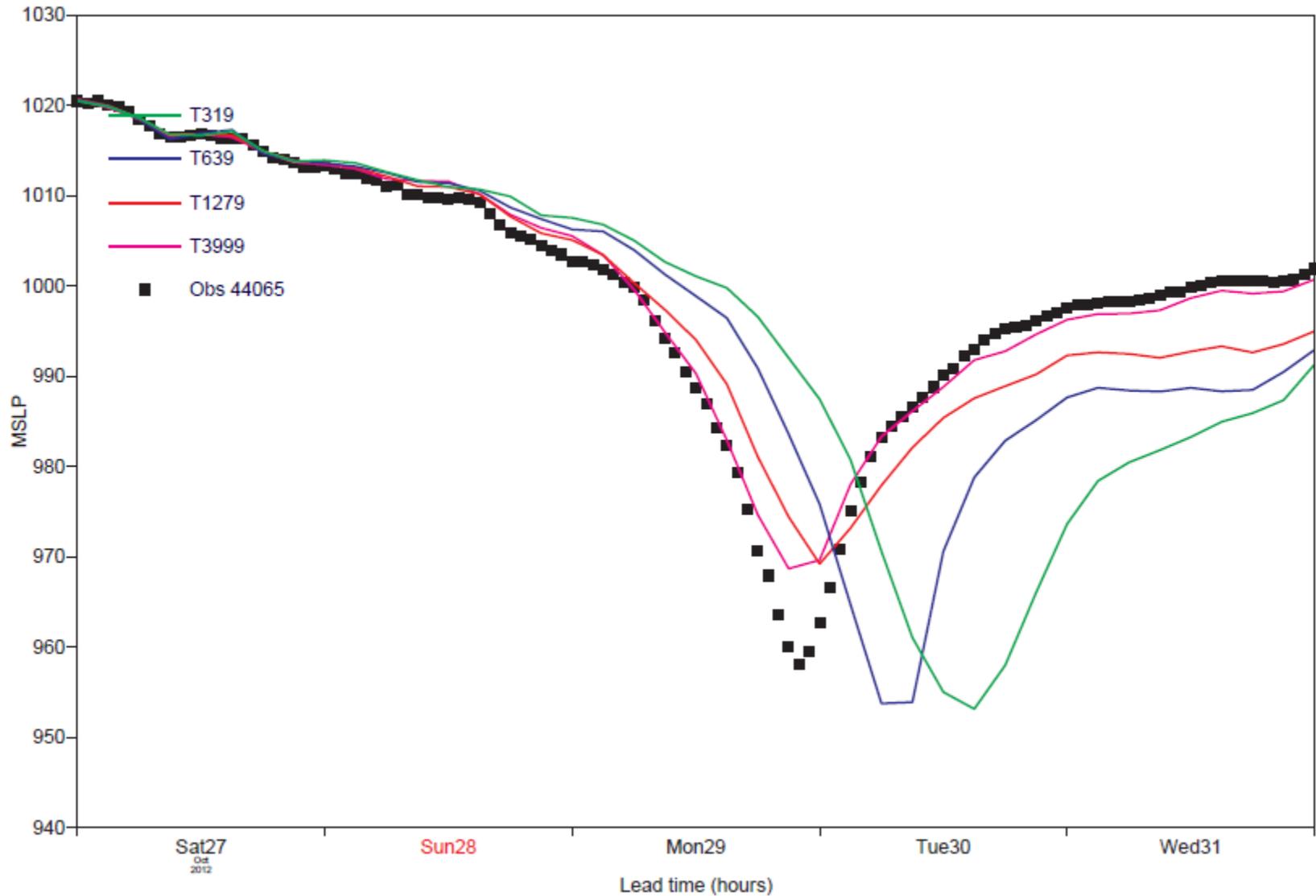


NASA Earth Observatory, 28<sup>th</sup> October 2012

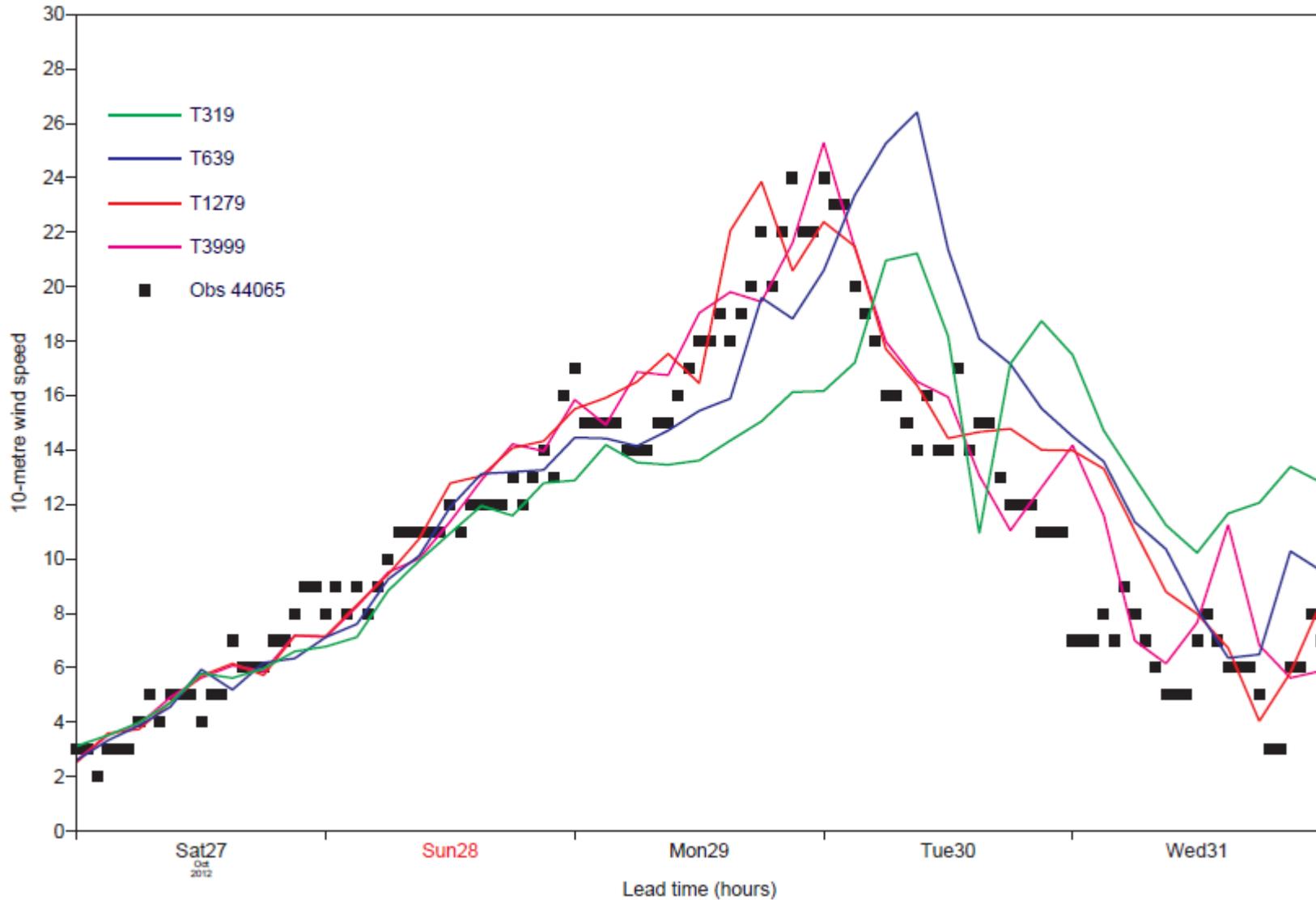
# Wave height 72h forecast



# MSL pressure 72h forecast



# 10m - wind speed 72h forecast





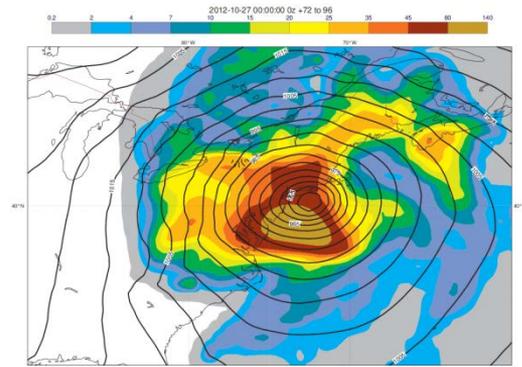
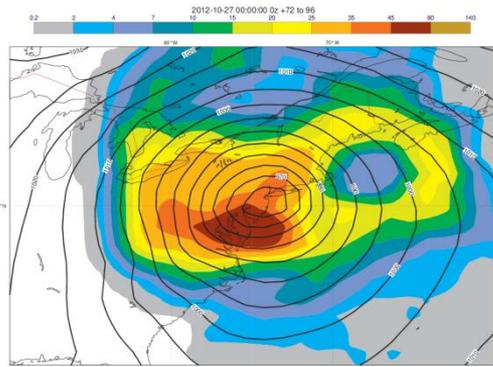
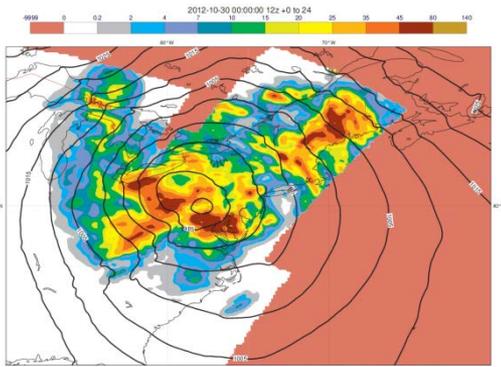
# 24-hour precipitation and MSLP

2012-10-27 00z, precip 72 to 96h, 84h for MSLP

NEXRAD 24h precip  
Oper analysis of MSLP

T159

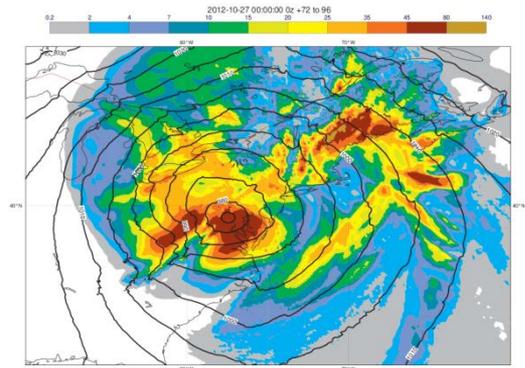
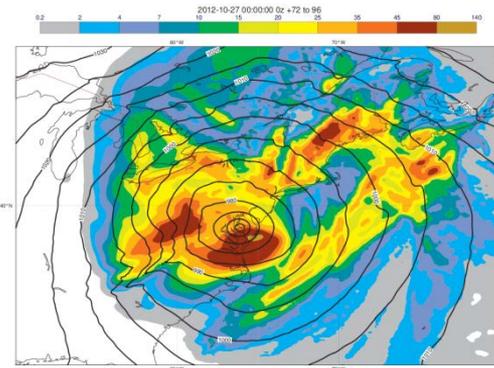
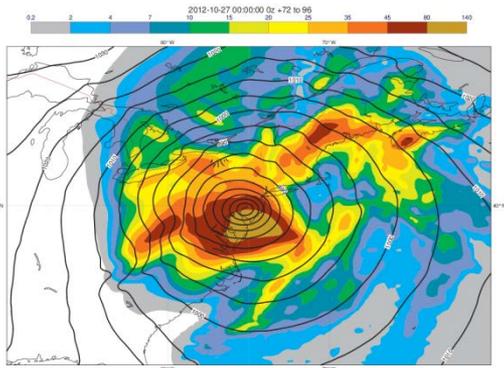
T319



T639

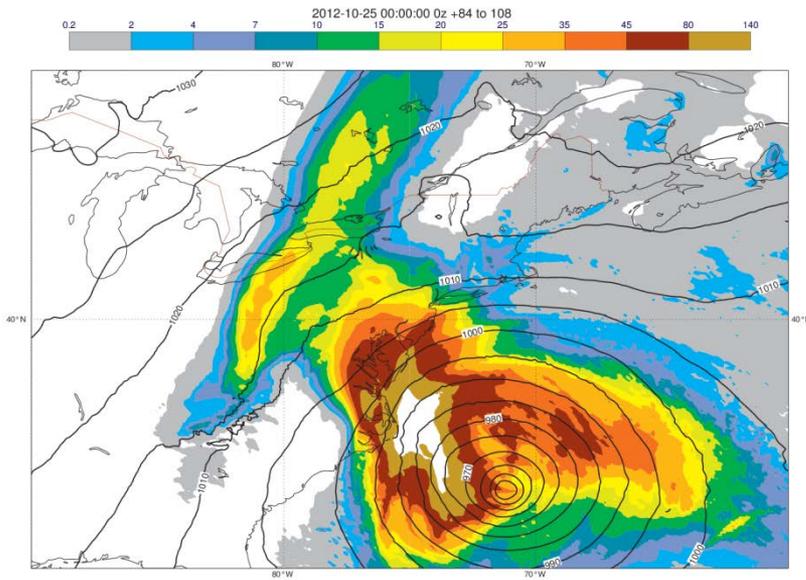
T1279

T3999

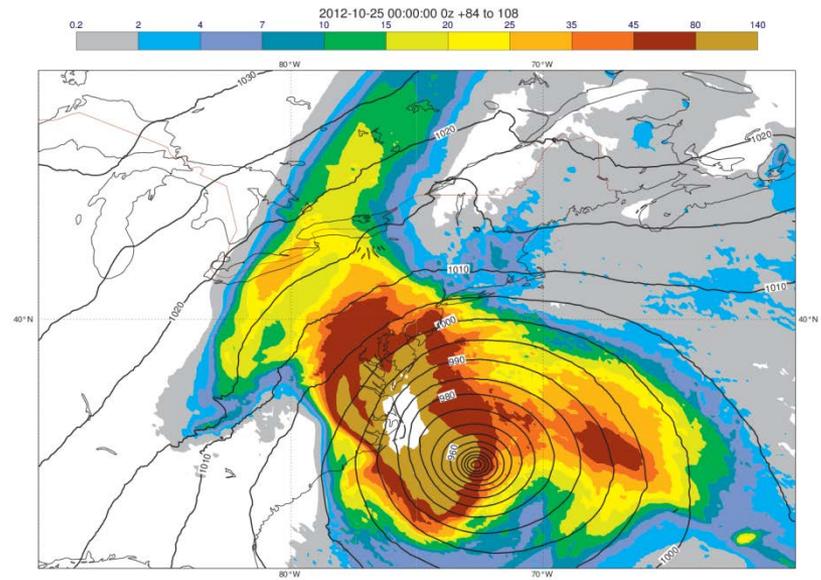


# Hydrostatic vs Nonhydrostatic

Hydrostatic  
Precip 84h to 108h



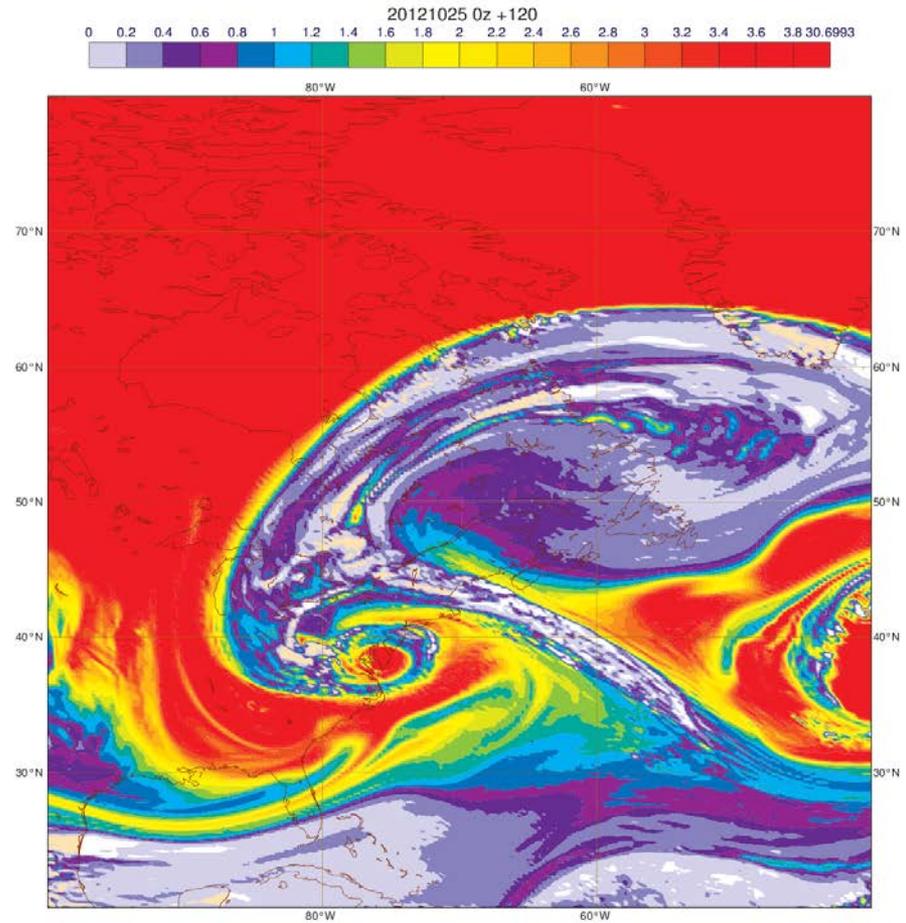
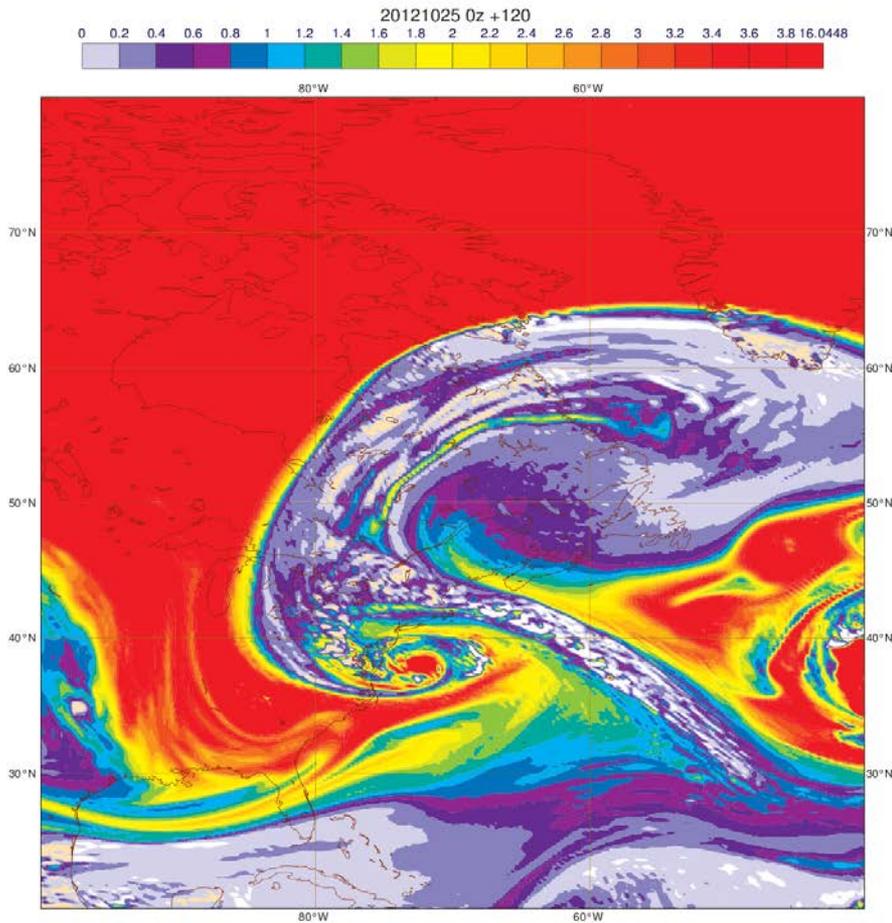
Non-Hydrostatic  
Precip 84h to 108h



# Hydrostatic vs Nonhydrostatic

Hydrostatic  
PV330 +120

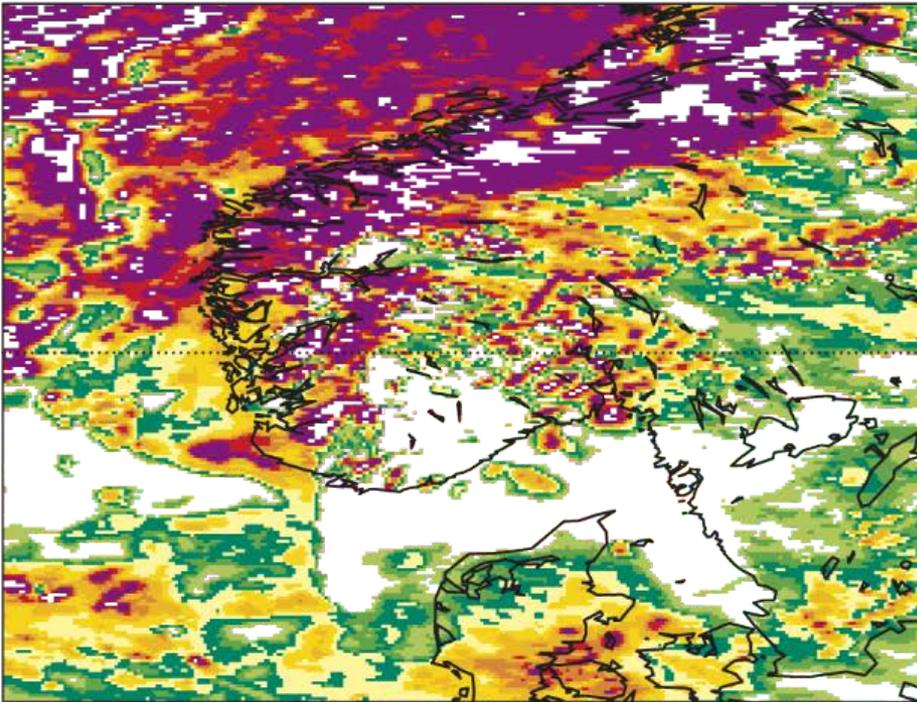
Non-Hydrostatic  
PV330 +120



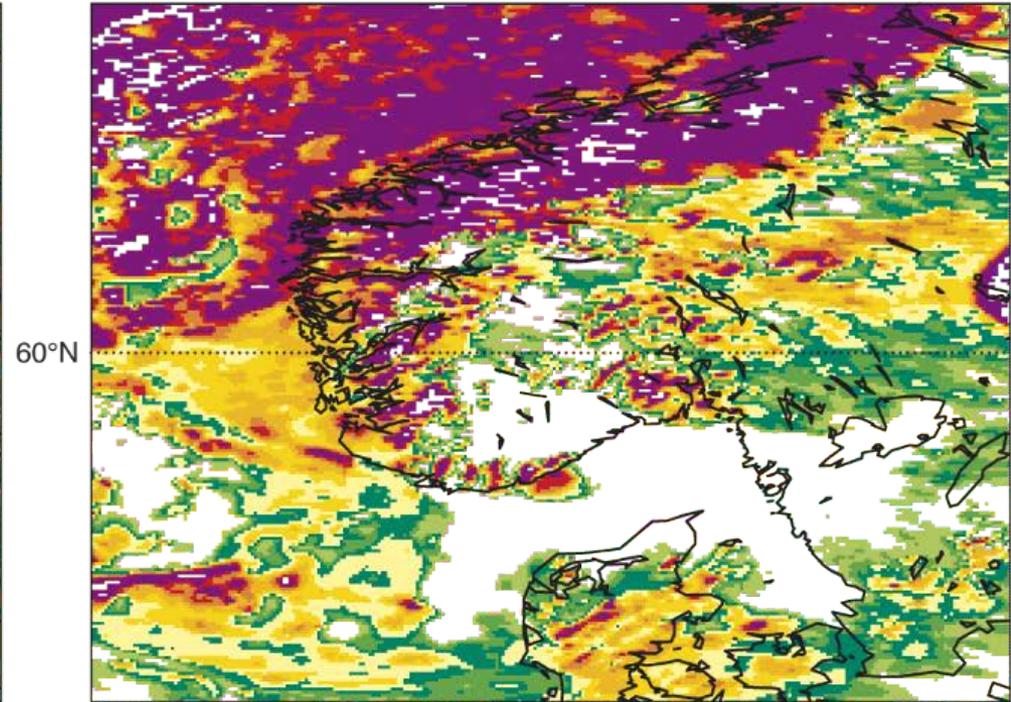
T3999 L91 (~5km)

# Hydrostatic vs Nonhydrostatic

a Non-hydrostatic simulation



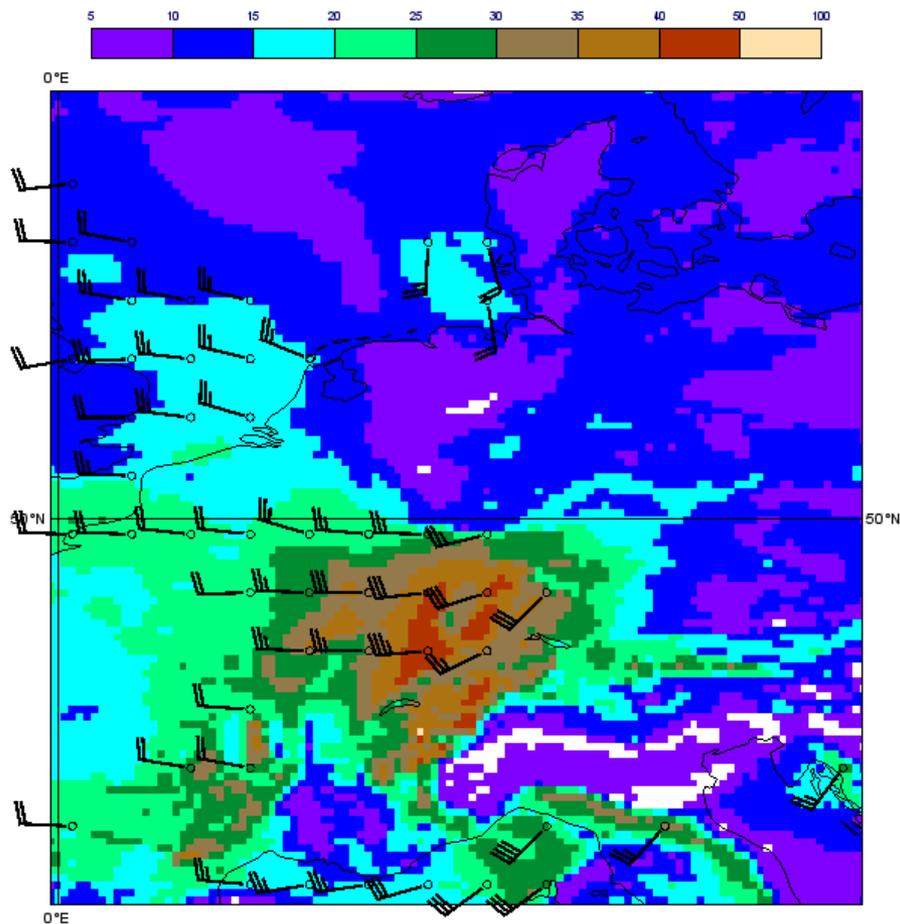
b Hydrostatic simulation



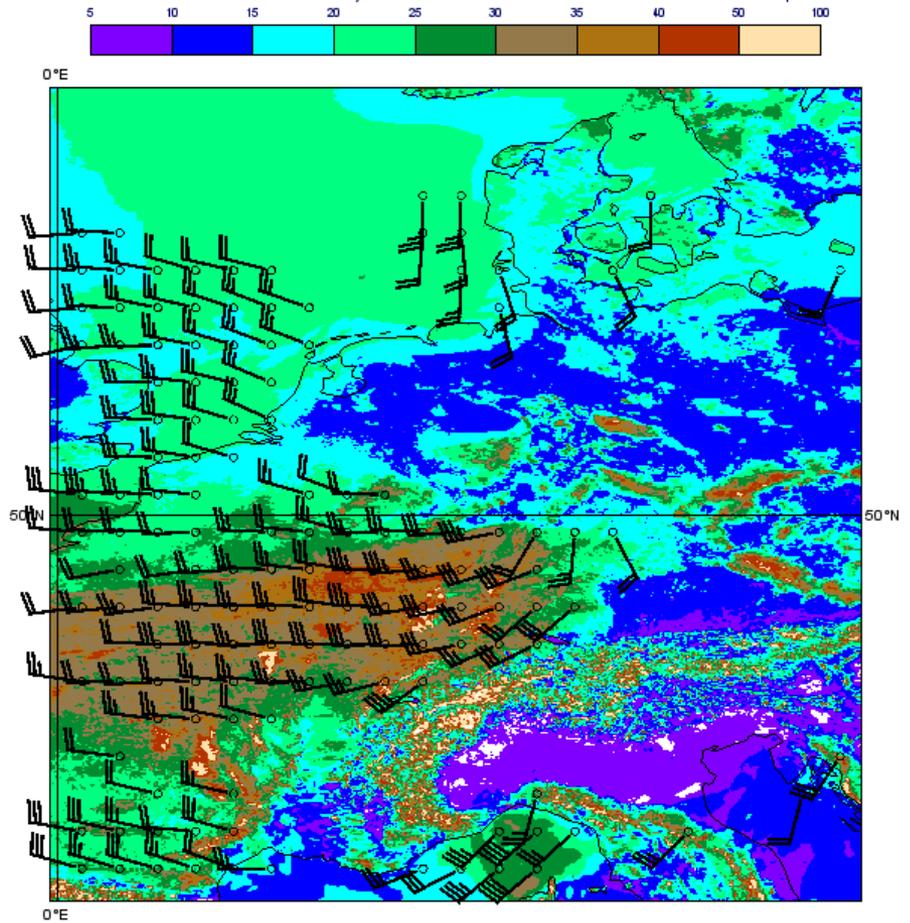
Cloud cover 24h forecast T3999 (~5km)

Era-Interim shows a wind shear with height in the troposphere over the region!

# Lothar "Christmas storm"

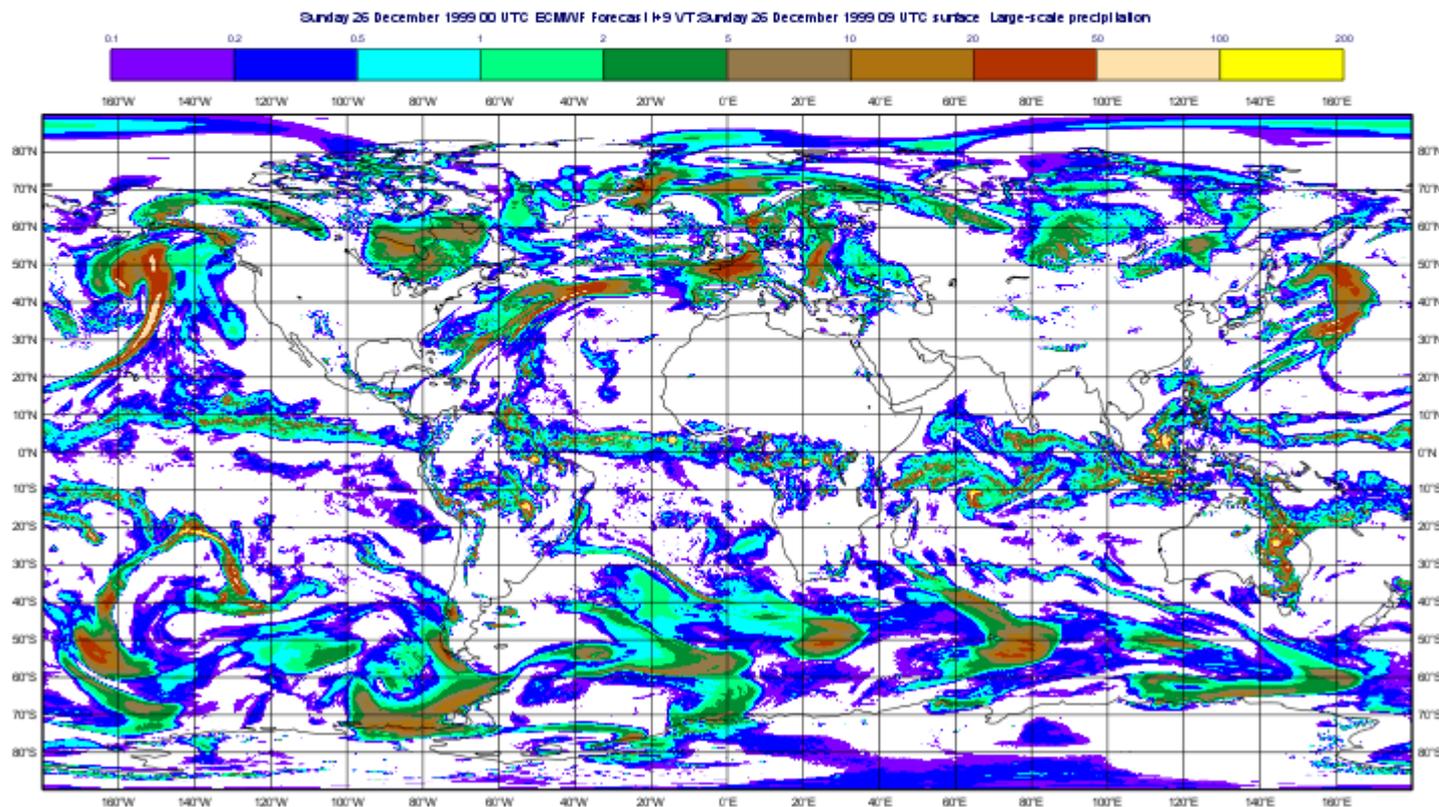


T1279 (~16km)

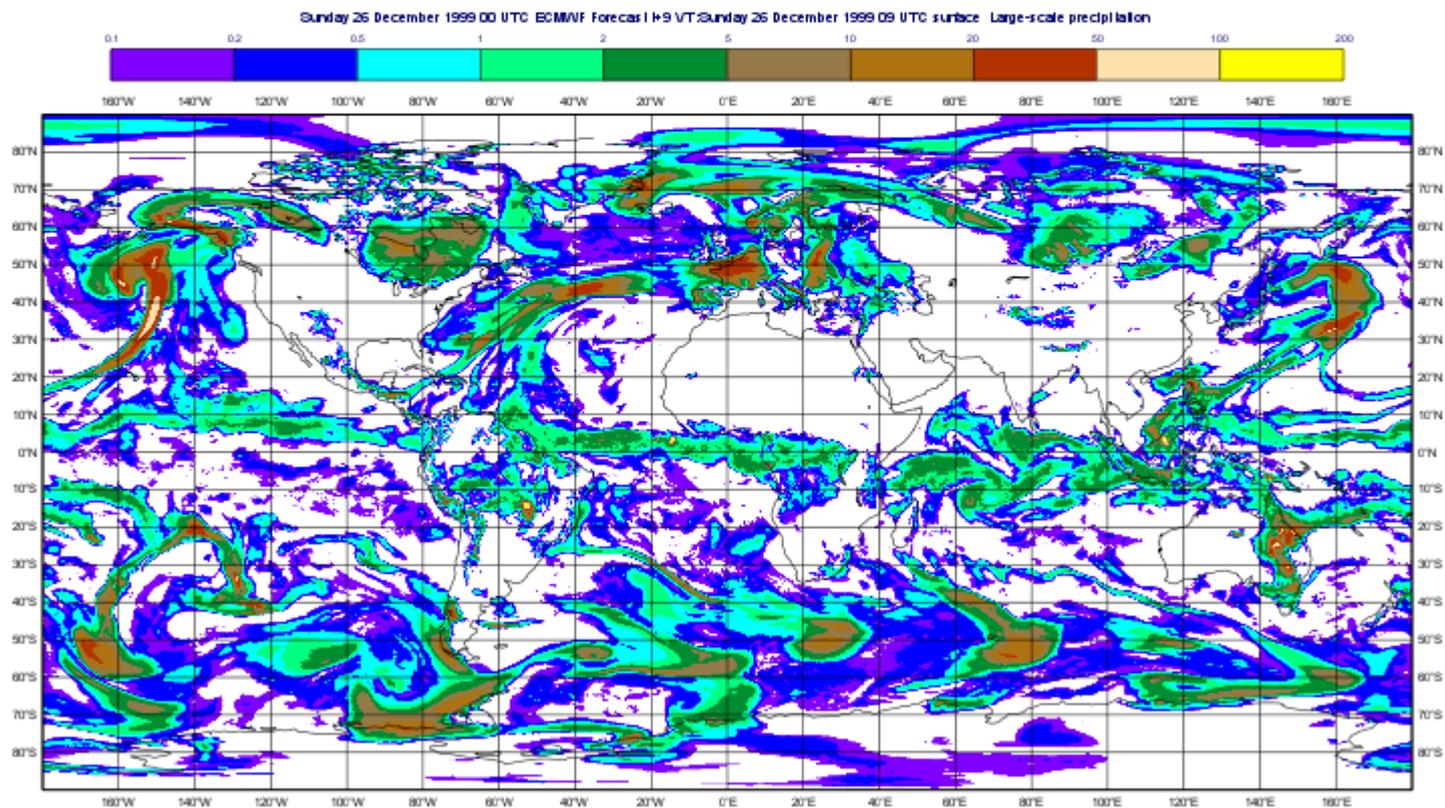


T7999 (~2.5km)

# T7999 (~2.5km global) large-scale precipitation (ran **without** deep convection parametrization)



# T1279 large-scale precipitation



# Identified Issues

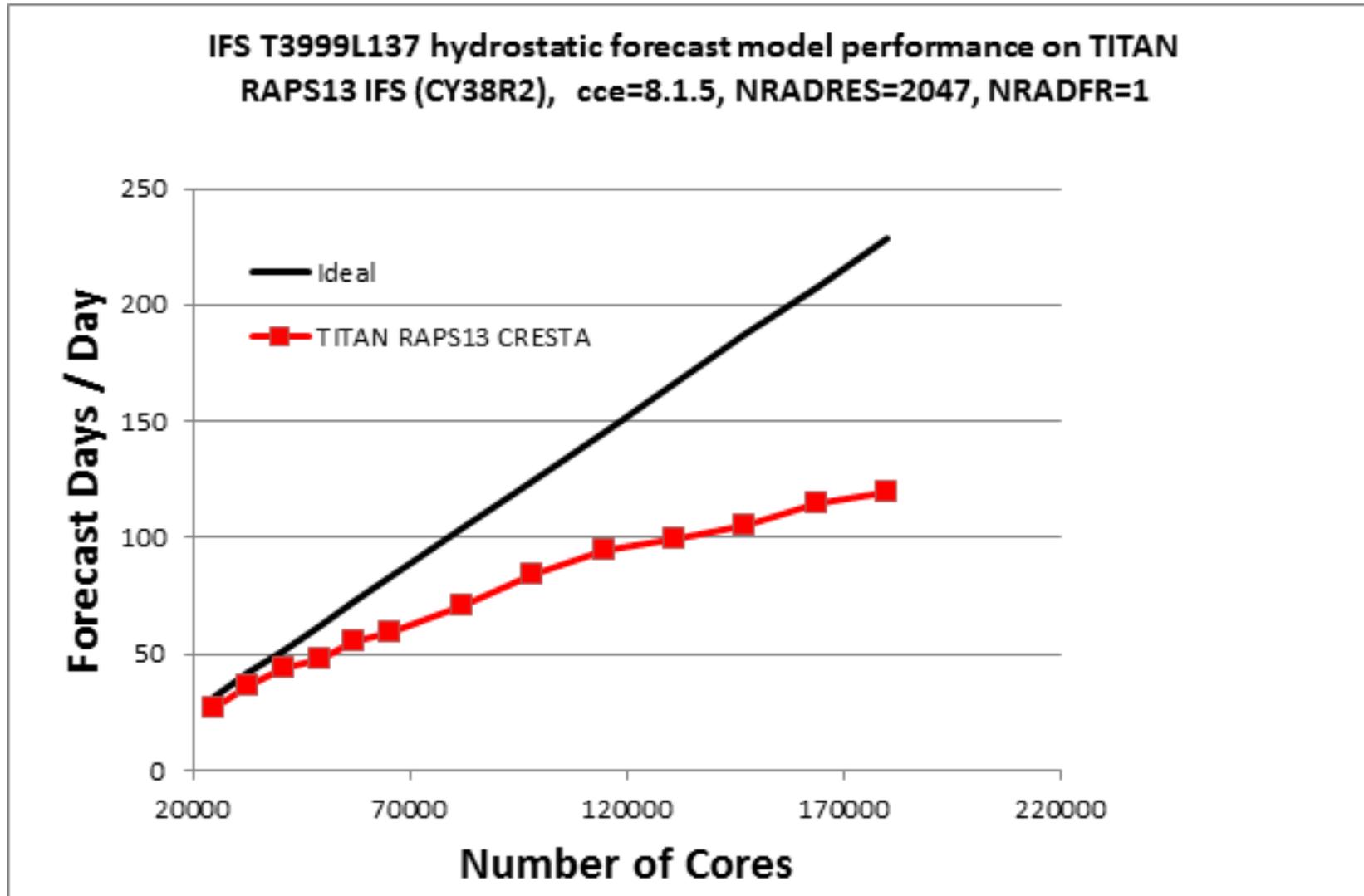
- ◆ **The overall scalability of the IFS and in particular the communication cost of (global) transpositions (gridpoint to spectral to gridpoint).**
- ◆ **Efficient exploitation of and lacking sufficient local parallelism (outside the spectral transforms) suitable for future computing architectures.**
- ◆ **The constant-coefficient, semi-implicit scheme is explicit in the boundary forcing (potentially a steep orography issue).**
- ◆ **Lacking the ability to compute local derivatives and fluxes and a satisfactory treatment of vertical derivatives.**
- ◆ **Lacking the ability to solve an elliptic equation that (nonlinearly) couples horizontal and vertical discretization.**
- ◆ **Coupling of moist physics and dynamics at non-hydrostatic scales.**

# CRESTA project

- ◆ **Parallelism too limited, need to explore new ways to unlock more parallelism**
- ◆ **Overlapping of communication and computations**
- ◆ **Reducing communications and data movement**
- ◆ **New data structures allowing alternative grids**
- ◆ **New approaches to data parallelism and I/O given that ultra-high resolution spectral transform models may be viable in ensemble mode**

**>> See George's presentation**

# IFS scalability



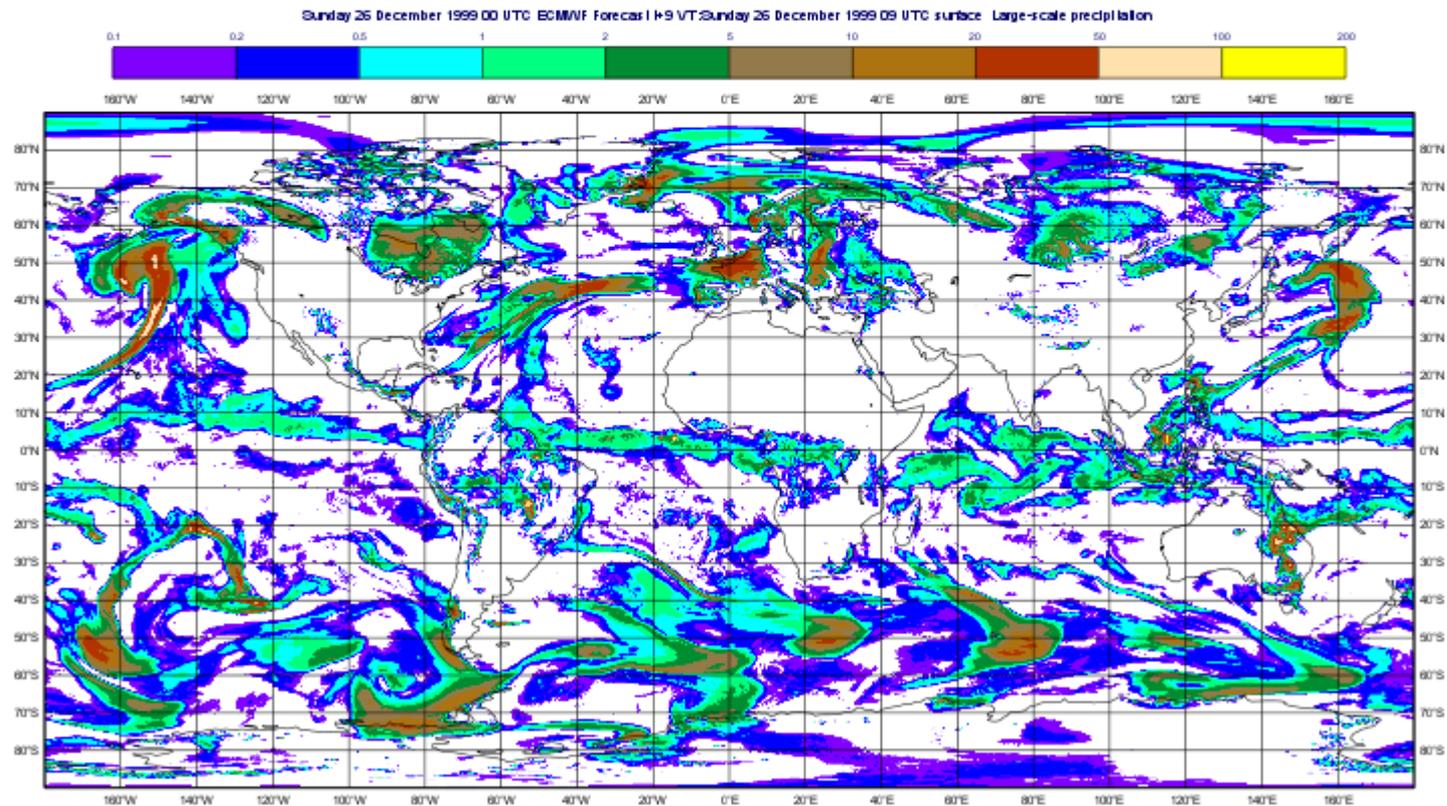
# Panta Rhei Project - ERC Advanced Grant

- ◆ **Focus on alternative pathways for accurate modelling of multi-scale fluid flows**
- ◆ **Combining several disciplines of Earth System sciences via a unifying modelling framework with the initial focus on advancing a global nonhydrostatic forecast model, and finding an “optimal” equation set/time-stepping algorithm**
- ◆ **Step change in forecast skill, model accuracy and cost-effectiveness**
- ◆ **A pioneering numerical approach, where a non-hydrostatic global model is conditioned by global hydrostatic solutions within a single code framework via numerical procedures expressed in time-dependent generalized curvilinear coordinates**

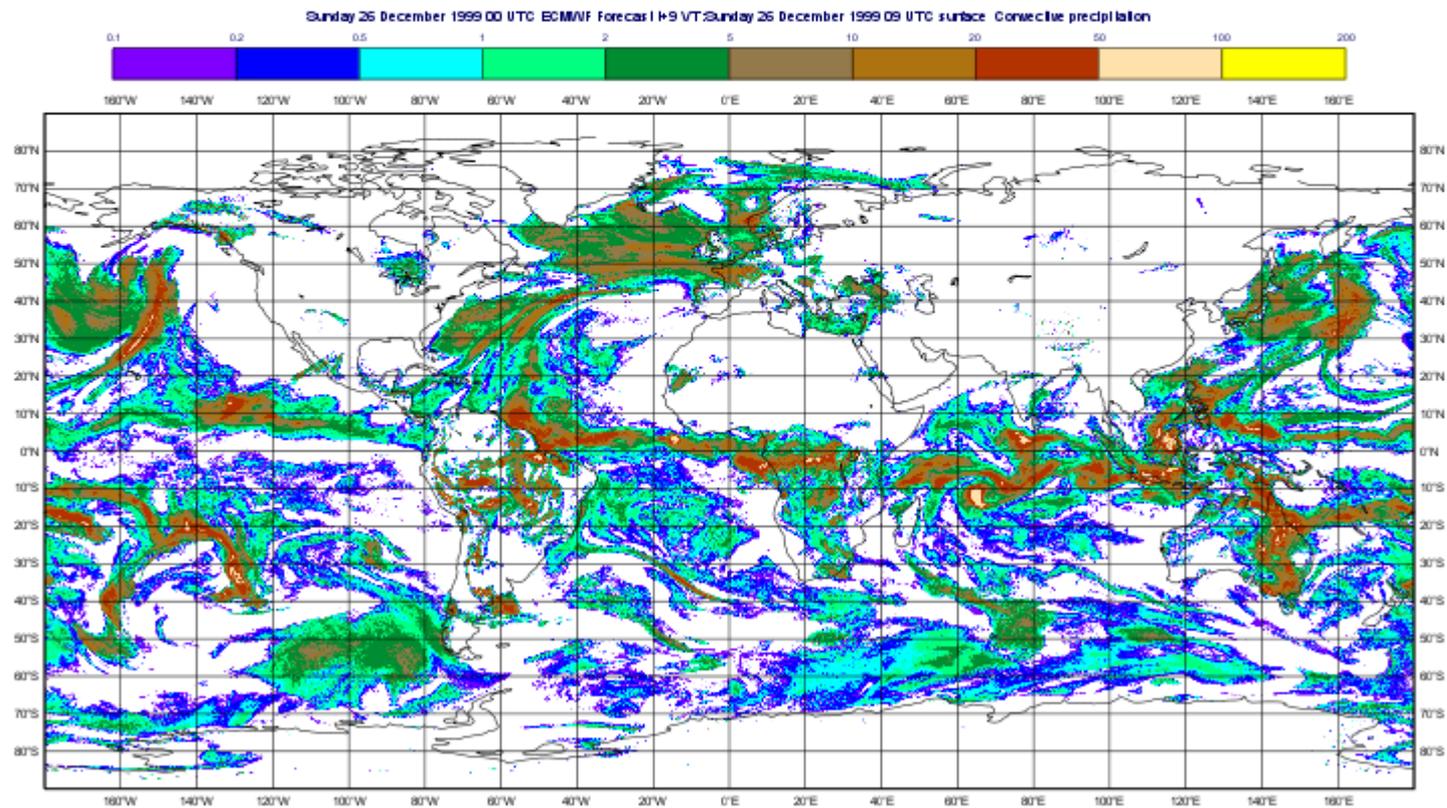
**>> see Piotr's presentation**

# Additional slides

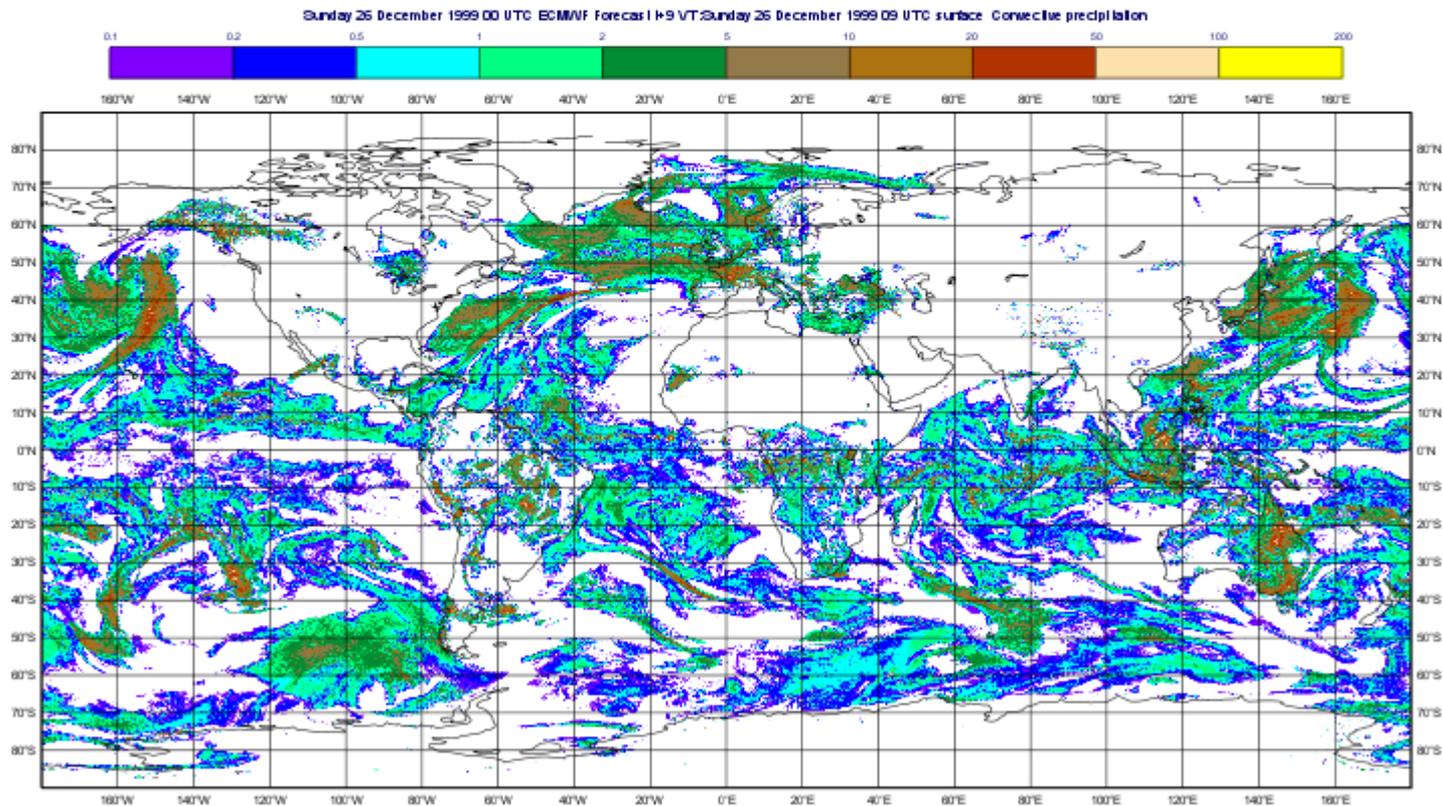
# T7999 large-scale precipitation (ran with deep convection parametrization)



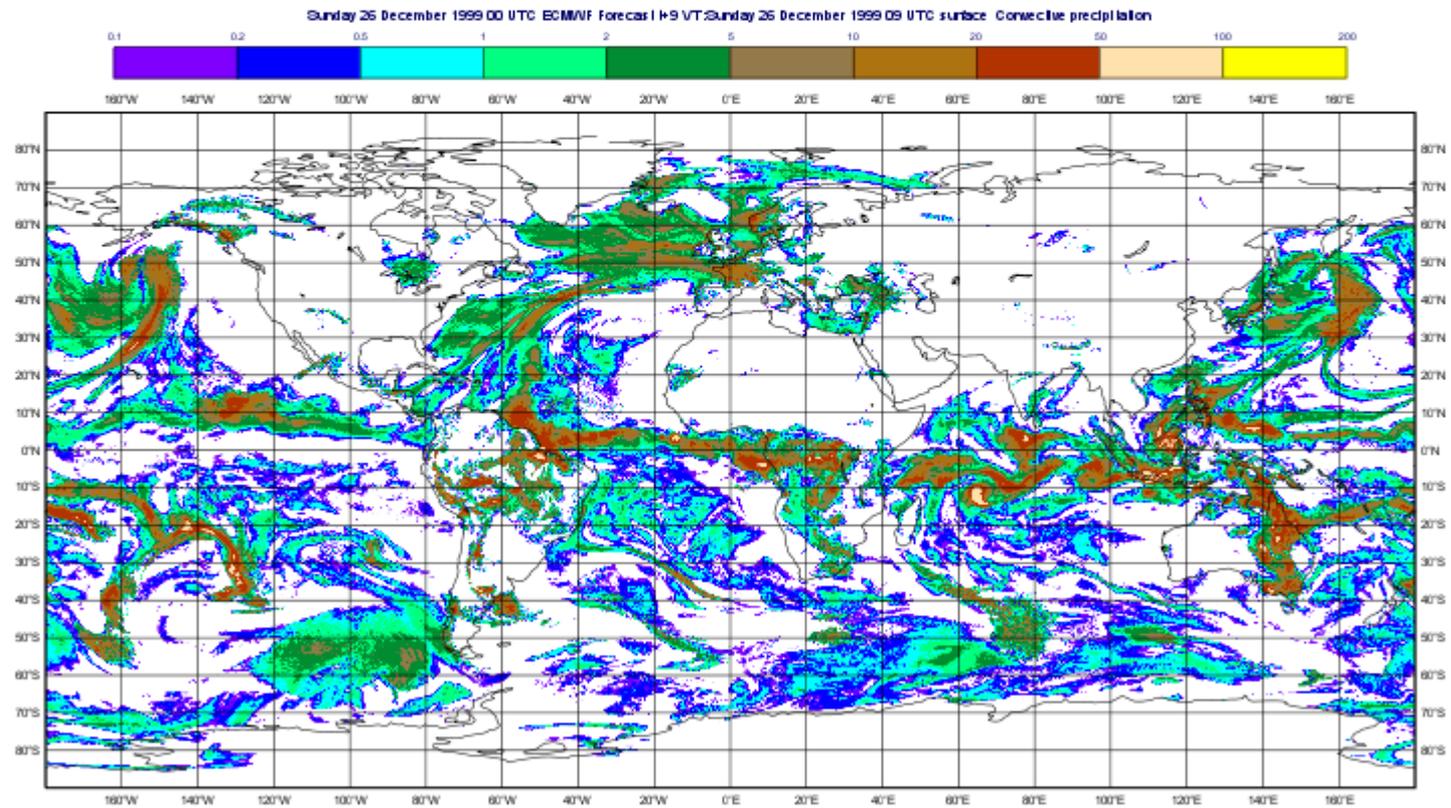
# T7999 convective precipitation (ran with deep convection parametrization)



# T7999 convective precipitation (ran **without** deep convection parametrization)



# T1279 convective precipitation





# Nonhydrostatic IFS (NH-IFS)

*Bubnová et al. (1995); Bénard et al. (2004), Bénard et al. (2005), Bénard et al. (2010), Wedi et al. (2009), Yessad and Wedi (2011)*

- ◆ **Arpégé/ALADIN/Arome/HIRLAM/ECMWF nonhydrostatic dynamical core, which was developed by Météo-France and their ALADIN partners and later incorporated into the ECMWF model and also adopted by HIRLAM.**

# Hydrostatic Primitive Equations (HPE)

$$\frac{d\mathbf{V}}{dt} = -2(\boldsymbol{\Omega} \times \mathbf{V}) - \nabla\Phi - RT\nabla(\log \Pi) + \mathbf{P}_V$$

$$\frac{dT}{dt} = \frac{RT}{c_p} \frac{\omega}{\Pi} + P_T$$

$$\frac{\partial\Phi}{\partial\Pi} = -\frac{RT}{\Pi}$$

# Non-hydrostatic shallow atmosphere (NHS)

Distinction between **hydrostatic pressure** and **total pressure**

$$\frac{\partial \Pi}{\partial z} = -\rho g$$

$$p = \rho RT$$

Introduce:  $\hat{Q} \equiv \log(p/\Pi)$  ‘Nonhydrostatic pressure departure’

Here hydrostatic pressure follows from the prognostic surface pressure equation as before !

Note that the geopotential is derived from

$$\frac{\partial \Phi}{\partial \Pi} = -\frac{RT}{p}$$

# NHS – continued ...

$$\frac{d\mathbf{V}}{dt} = -2\boldsymbol{\Omega} \times \mathbf{V} - \frac{\partial p}{\partial \Pi} \nabla \Phi - RT \frac{\nabla p}{p} + \mathbf{P}_V$$

$$\frac{dw}{dt} = g_0 \frac{\partial(p - \Pi)}{\partial \Pi} + P_w$$

$$\frac{dT}{dt} = -\frac{RT}{c_v} D_3 + \frac{c_p}{c_v} P_T$$

$$\frac{d\hat{Q}}{dt} = -\frac{c_p}{c_v} D_3 - \frac{\omega}{\Pi} + \frac{c_p}{c_v T} P_T$$

‘Physics’

Projecting on  
temperature and horizontal  
velocities only,  
quasi-anelastic coupling ?!

# NHS – For the solution we need the three-dimensional divergence

$$D_3 = D + \mathcal{X} + \frac{R_d}{R}d$$

horizontal divergence

$$D \equiv \nabla_{\eta} \cdot \mathbf{V}$$

‘vertical divergence’

$$d \equiv -(g_0 p / m R_d T) \partial w / \partial \eta$$

‘X-term residual’

$$\mathcal{X} = \frac{p}{mRT} \nabla \Phi \cdot \left( \frac{\partial \mathbf{V}}{\partial \eta} \right)$$

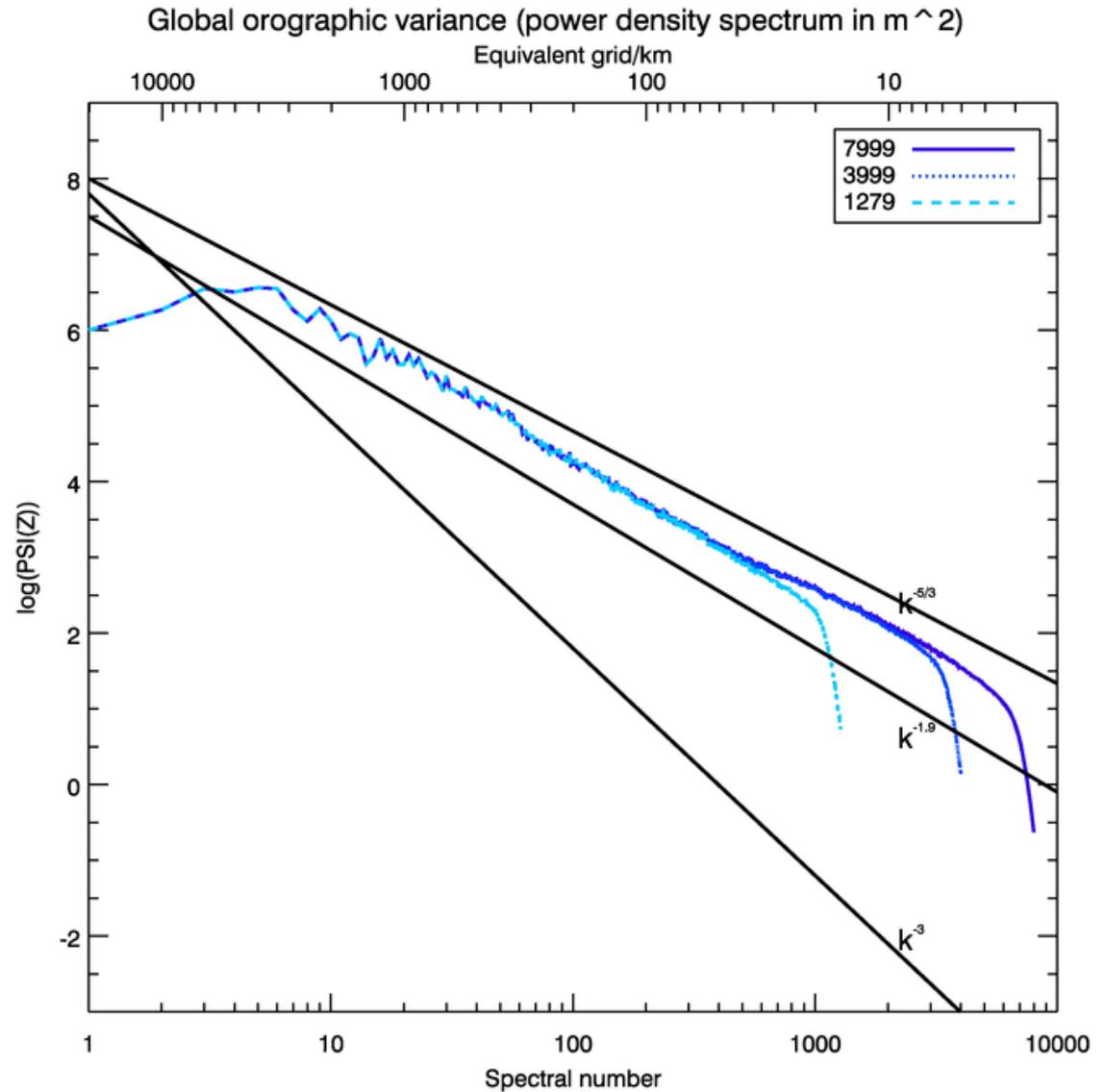
Arises due to the formulation of divergence in time-dependent curvilinear coordinates !

# Numerical solution

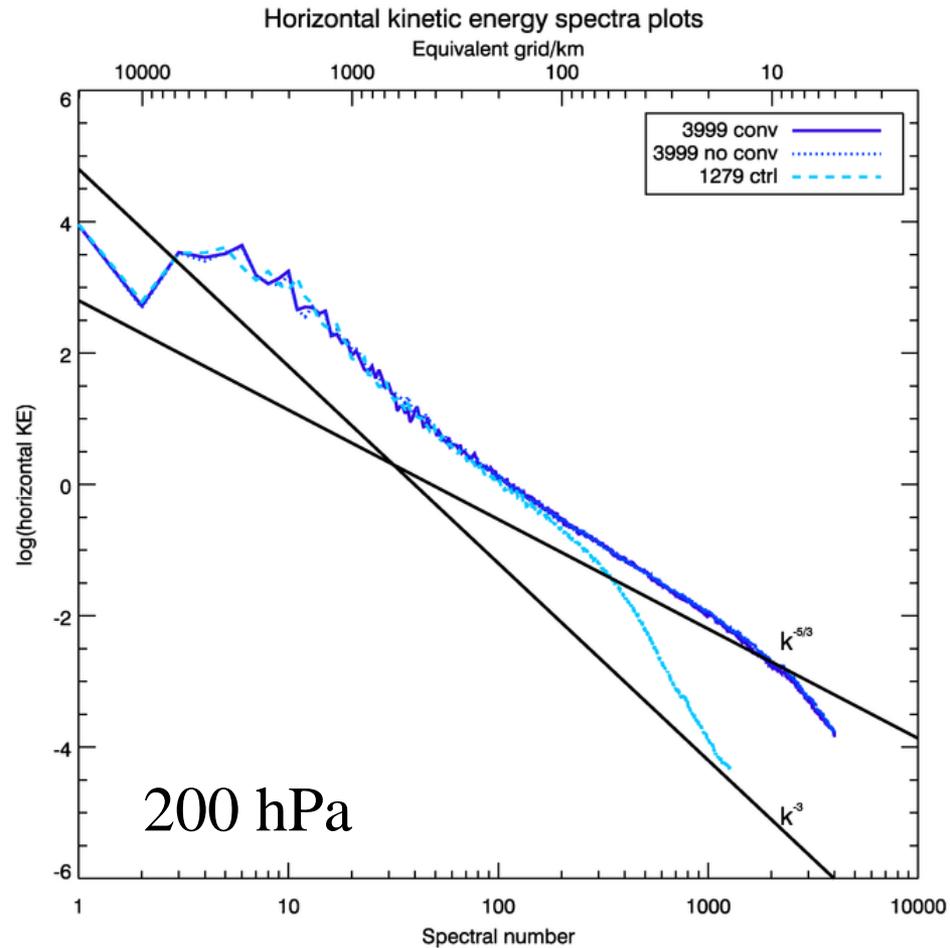
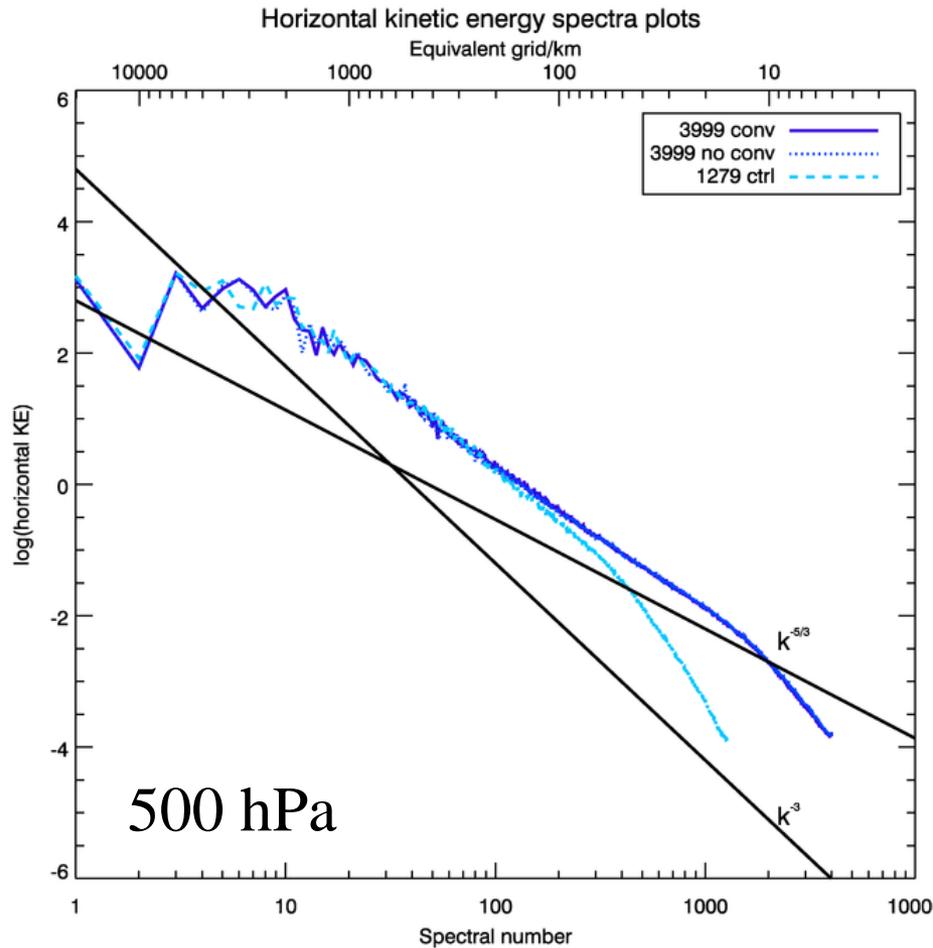
- ◆ **Two-time-level, semi-implicit, semi-Lagrangian.**
- ◆ **Semi-implicit procedure with two reference states, with respect to gravity and acoustic waves, respectively.**
- ◆ **The resulting **Helmholtz equation** can be solved (subject to some constraints on the vertical discretization) with a **direct spectral method**, that is, a mathematical separation of the horizontal and vertical part of the linear problem in spectral space, with the remainder representing at most a pentadiagonal problem of dimension  $NLEV^2$ . Non-linear residuals are treated explicitly (or iteratively implicitly)!**

*(Robert, 1972; Bénard et al 2004,2005,2010)*

# Orographic forcing



# T3999 vs T1279 kinetic energy spectra



After 10 days

# Global horizontal kinetic energy spectra at 500 hPa height for the first 12 hours of the T7999

