Snowpack modelling and data assimilation

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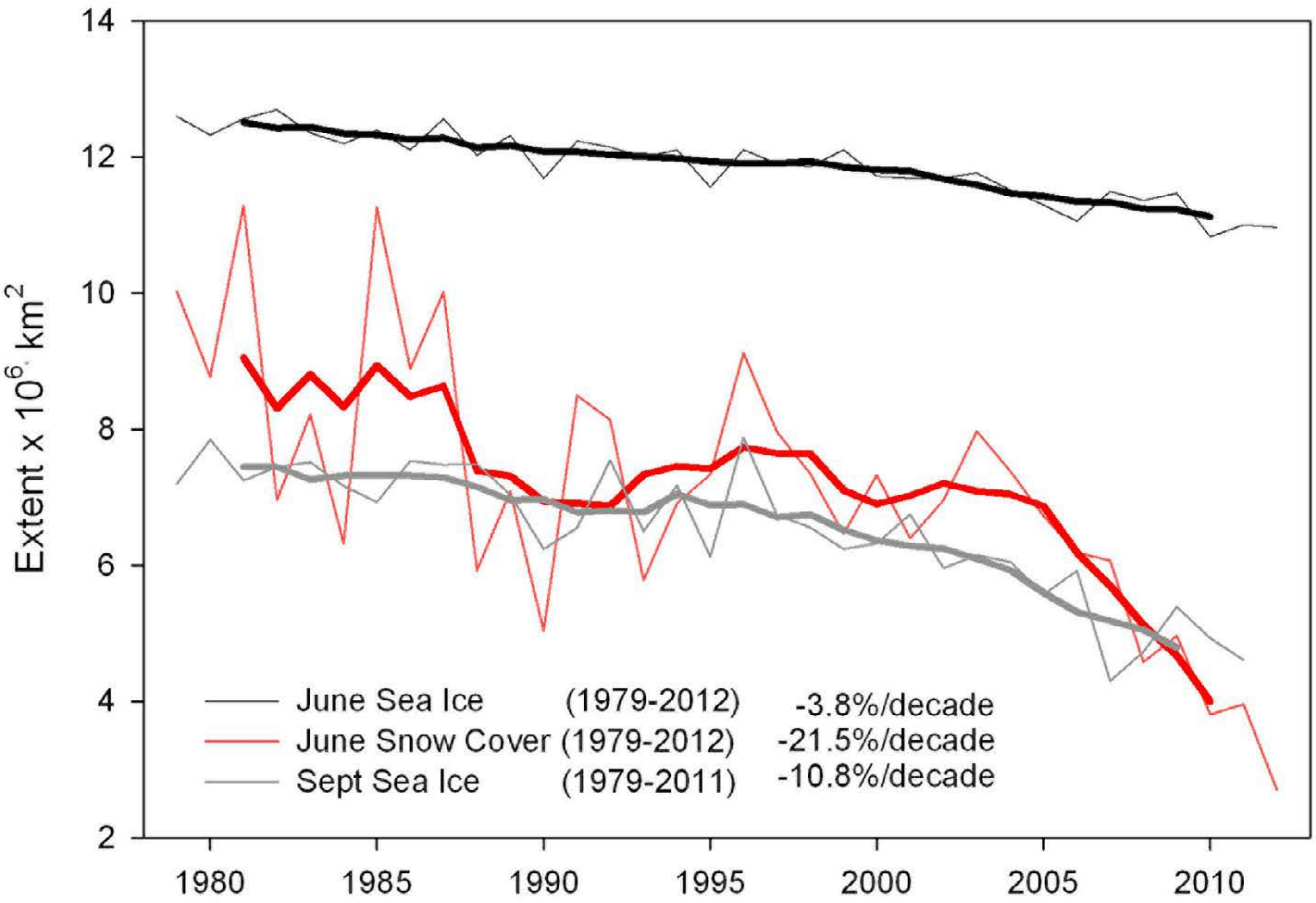
ECMWF-WWRP/THORPEX workshop on polar prediction

Northern Hemisphere ice and snow extents



Derksen and Brown, 2012. *GRL*, **39**, doi:10.1029/2012GL053387

Snowpack model state variables

N_{snow} model layers, each with state variables:

m_j mass (kg m^{-2})

H_j heat content (J m^{-2}) → temperature, liquid water content

ρ_j density (kg m^{-3})

…

Bulk variables:

snow mass $M = \sum_{j=1}^{N_{snow}} m_j$, snow depth $\mathrm{d} = \sum_{j=1}^{N_{snow}} m_j/\rho_j$

	model	layers	variables / layer
	UKMO	1	1
Surfex	ISBA-FR	1	2
Surfex	ISBA-ES	3	3
Surfex	Crocus	≤50	6

Prognostic equations for state variables

Mass balance:

$$\frac{dM}{dt} = S_f - E - R$$

Liquid water drainage:

$$\frac{dw_j}{dt} = Q_{j-1} - Q_j + f_j$$

Energy balance:

$$C\frac{dT_j}{dt} = G_{j-1} - G_j - Lf_j$$

Parametrizations required for all flux and capacity terms

Parametrizations of snow compaction

Constant density (e.g. MOSES, SiB):

ρ = constant

Bulk compaction (e.g. CLASS, ISBA-FR, TESSEL):

$$\frac{d\rho}{dt} = \tau^{-1}(\rho_{\max} - \rho)$$

Overburden profile (e.g. CLM, ISBA-ES, HTESSEL, JULES):

$$\frac{1}{\rho}\frac{\partial \rho}{\partial t} = \frac{M(z)g}{\eta(\rho, T)}$$

Experimental snow data assimilation

Ensemble Kalman Filter

- Andreadis and Lettenmaier (2006)
- Dechant and Moradkhani (2011)
- Durand and Margulis (2006)
- Durand et al. (2009)
- Kumar et al. (2008)
- Slater and Clark (2006)
- Su et al. (2008)
- Toure et al. (2011)

Extended Kalman Filter

- Dong et al. (2007)
- Sun et al. (2004)

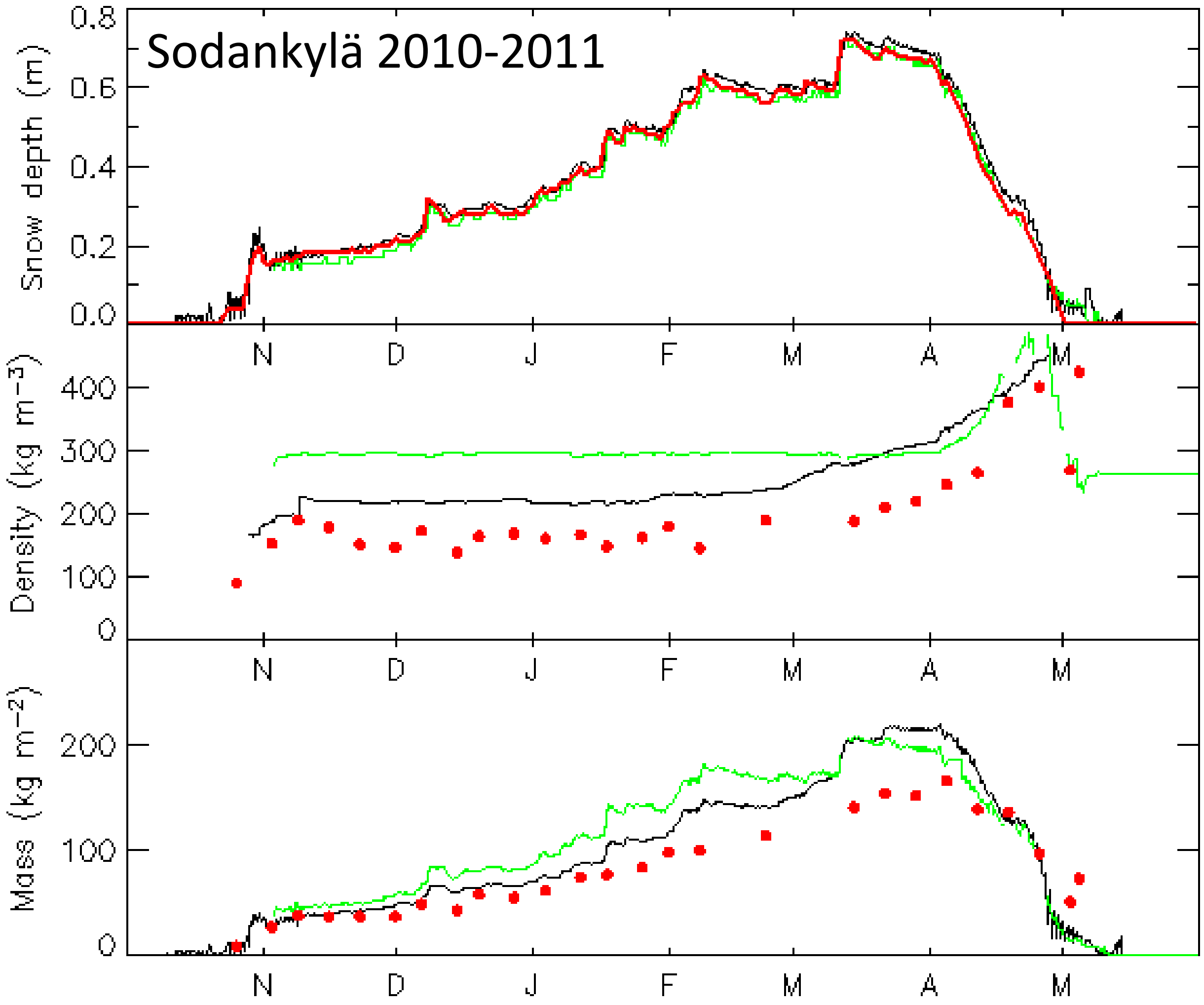
Particle Filter

- Dechant and Moradkhani (2011)

Operational snow analyses

Model	Observations	Assimilation	Operational
CMC	SYNOP	OI	1999
ECMWF	SYNOP	Cressman	1987
	IMS	Cressman	2004
		OI	2010
HARMONIE	SYNOP	OI	2010
HIRLAM	SYNOP	Cressman	1995
	SYNOP	OI	2004
	Globsnow	OI	Experimental
Met Office	IMS	Update	2009

Operational snow analyses



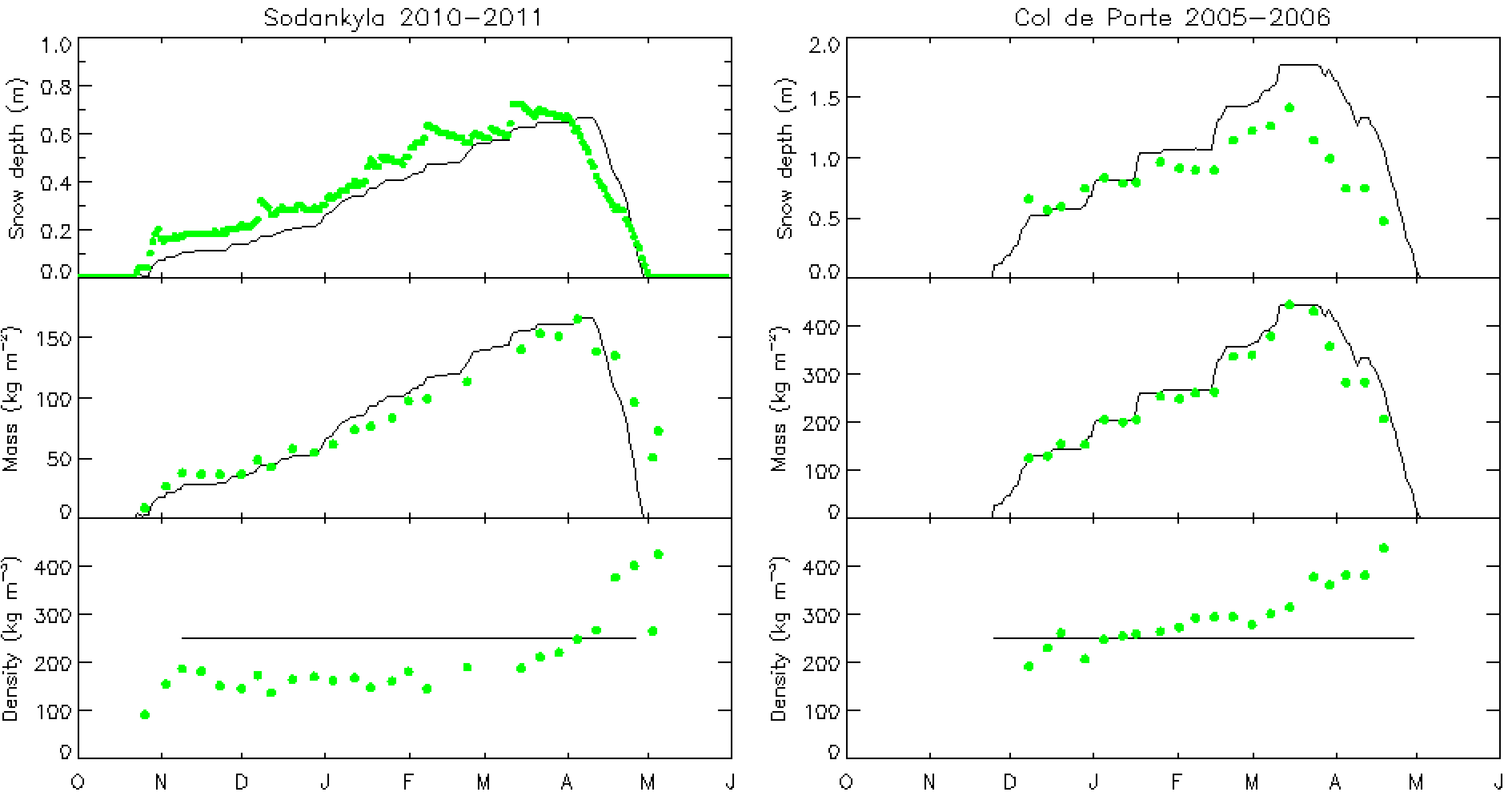
SYNOP snow depths and FMI snow pits (from Timo Ryyppö)

Hirlam snow analyses (from Laura Rontu)

ECMWF snow analyses (from Patricia de Rosnay)

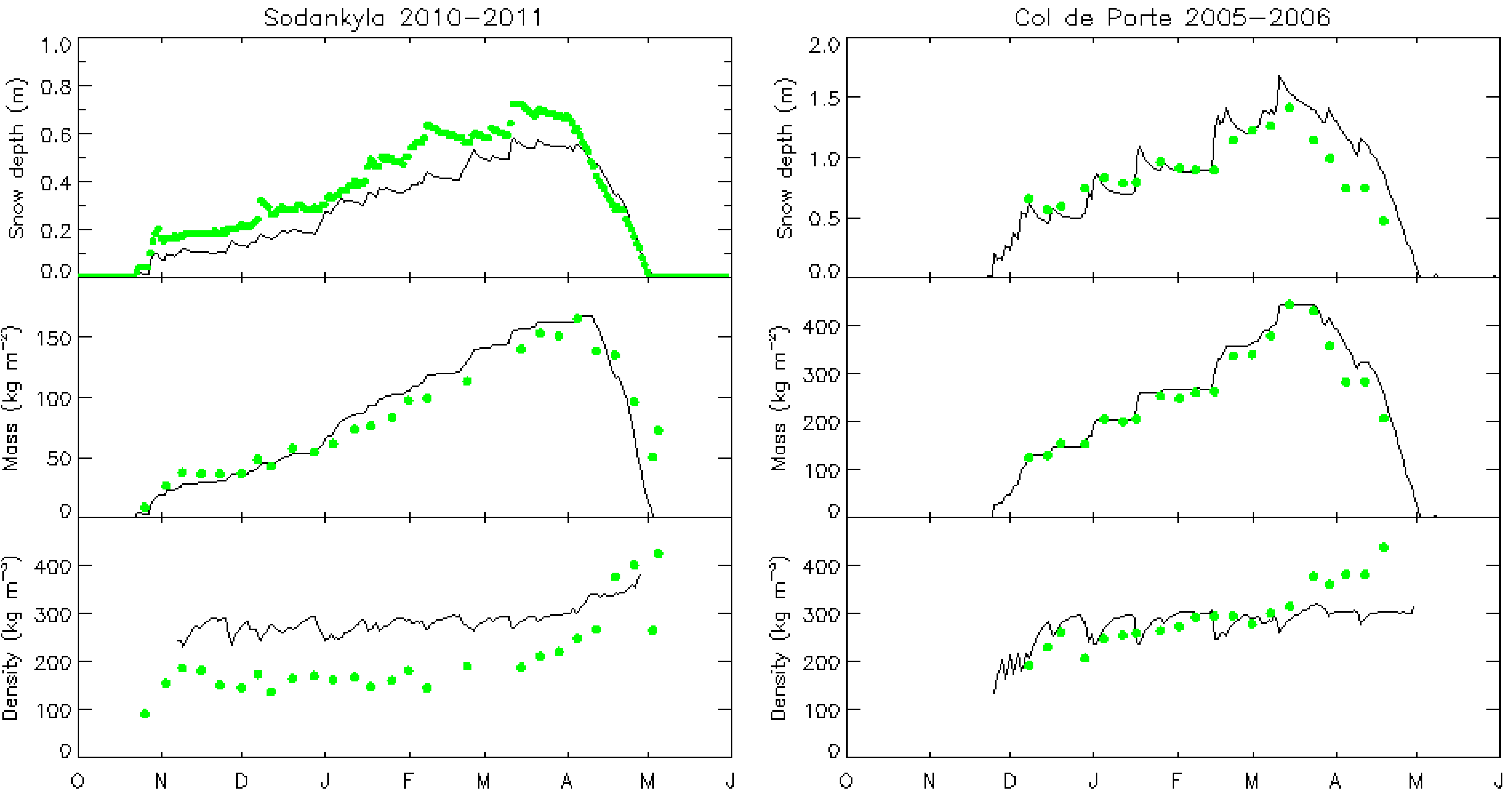
Open loop simulations with in situ driving data

Constant density



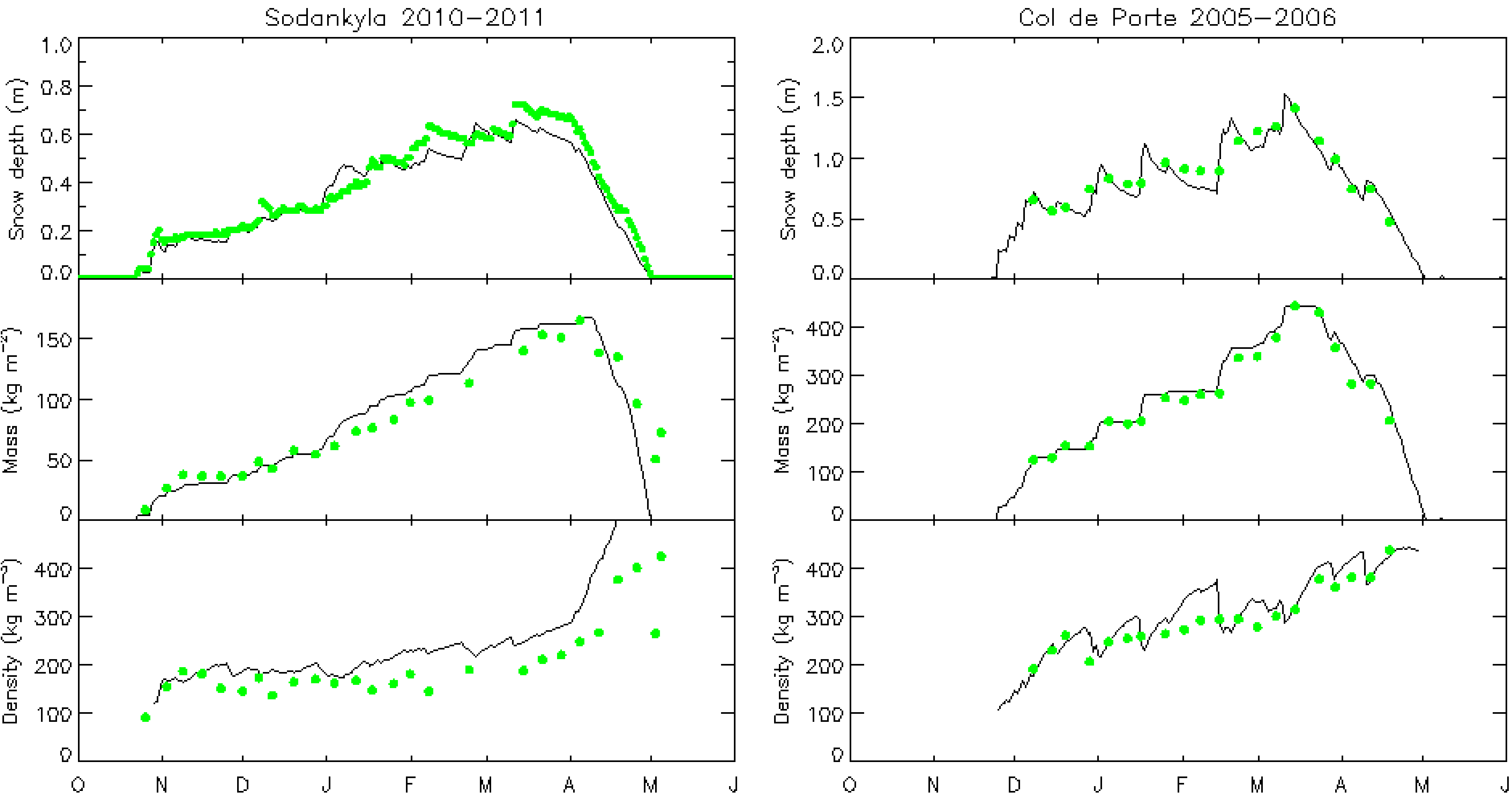
Open loop simulations with in situ driving data

Bulk density



Open loop simulations with in situ driving data

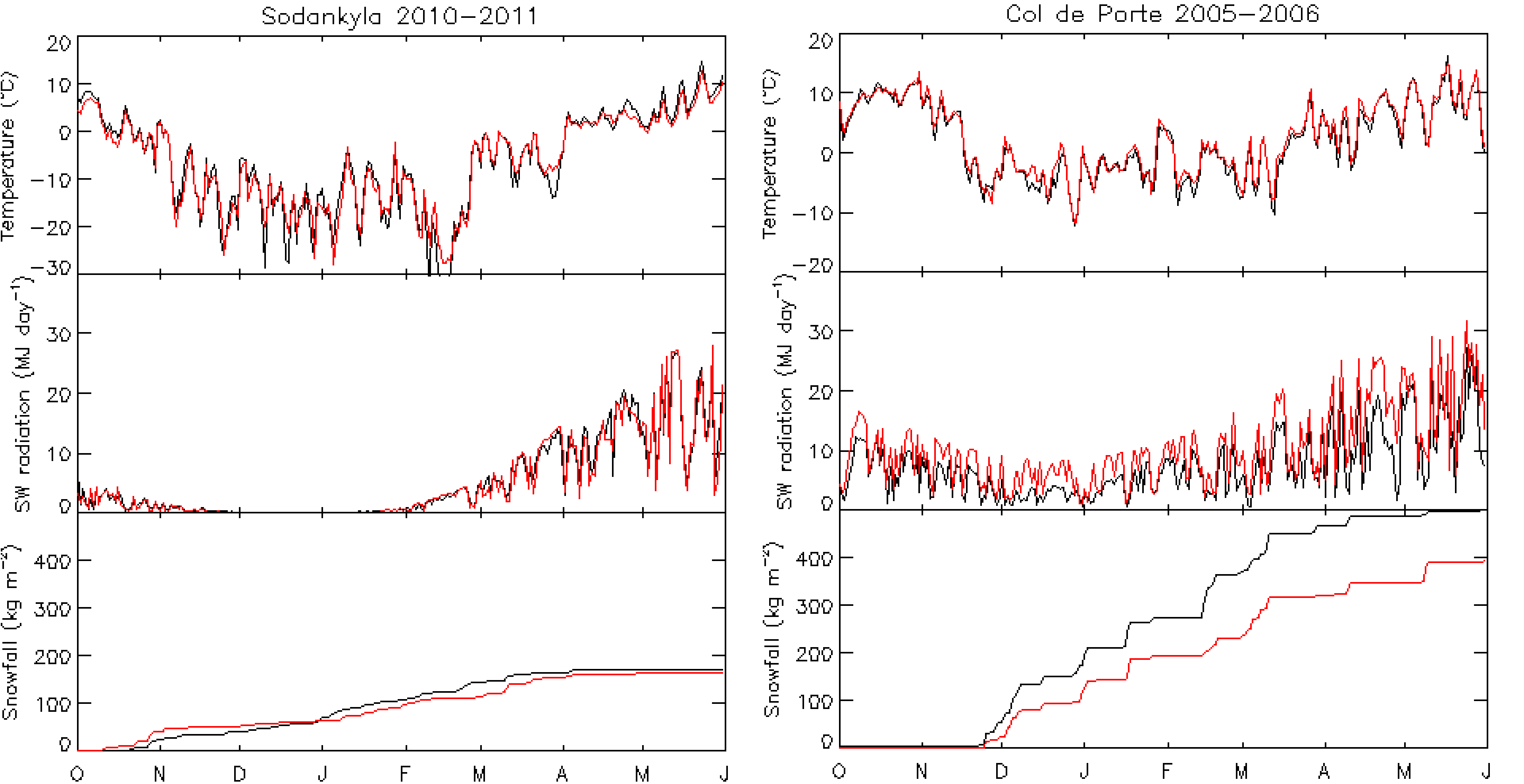
Density profile



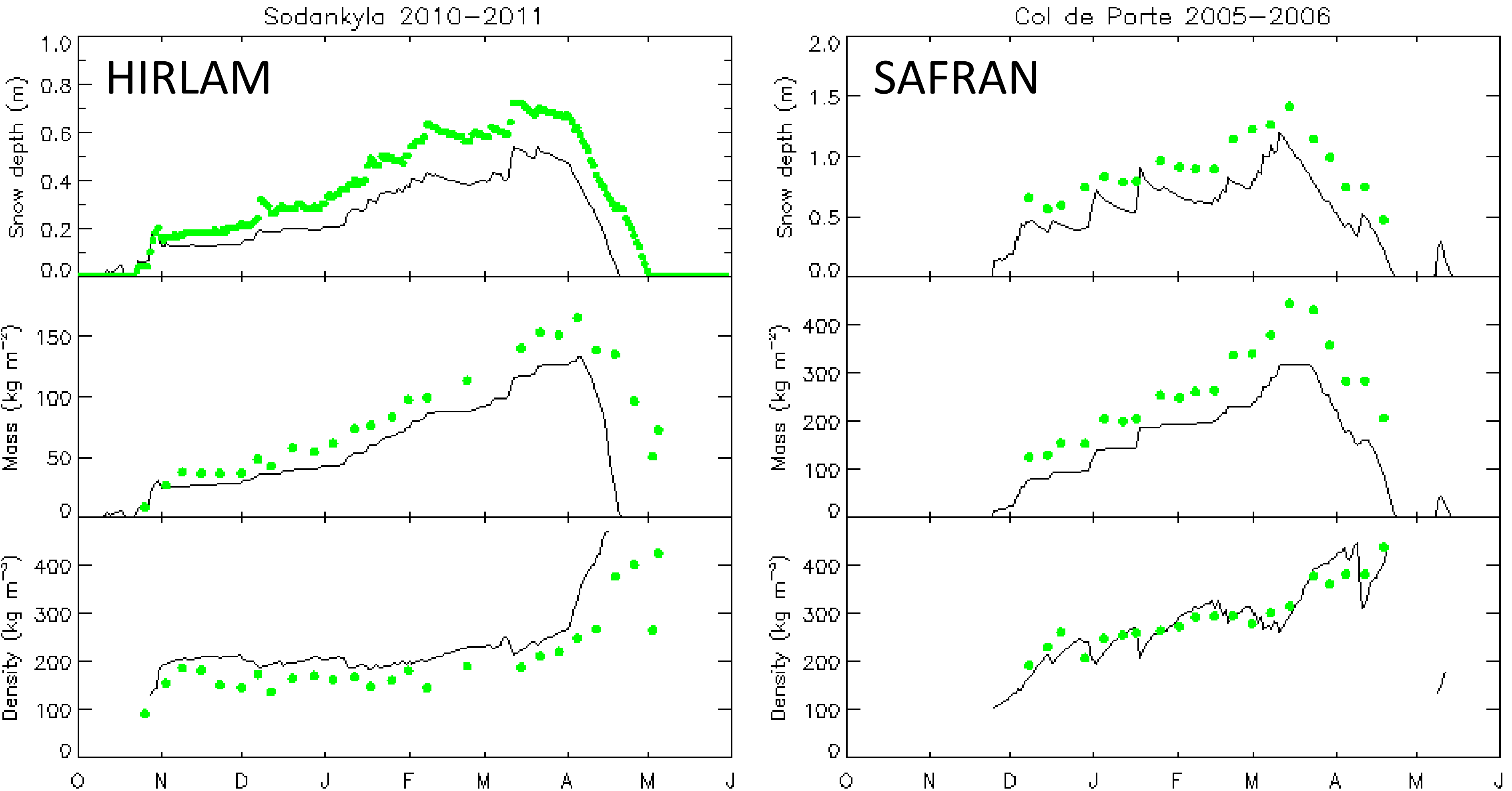
In situ and NWP driving data

HIRLAM

SAFRAN

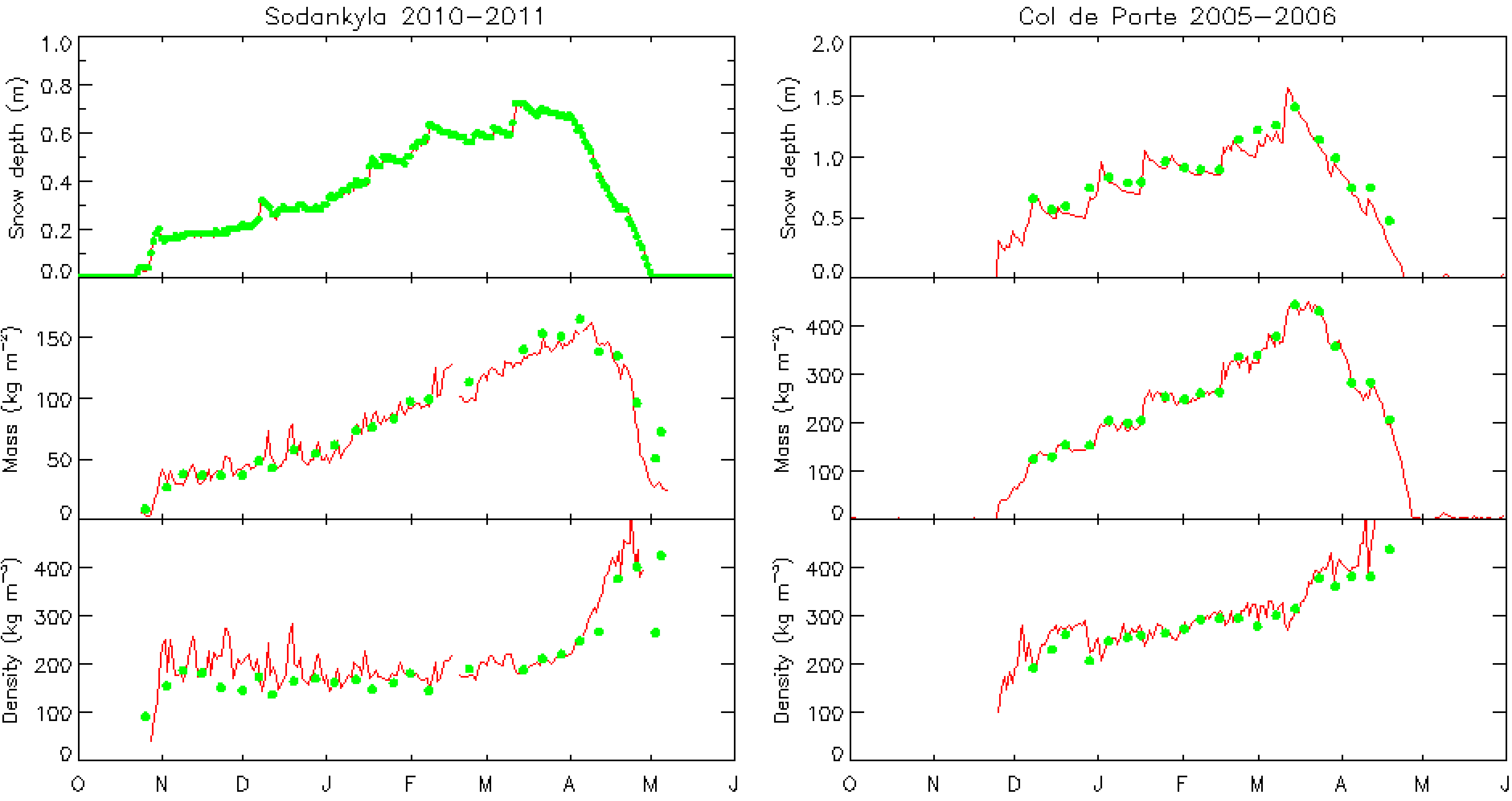


Open loop simulations with NWP driving data



rms errors	Sodankylä	Col de Porte
snow depth	13 cm	28 cm
snow mass	13 kg m^{-2}	86 kg m^{-2}

Independent snow measurements



rms differences	Sodankylä	Col de Porte
snow depth	0.7 cm	7.6 cm
snow mass	4.2 kg m^{-2}	8.8 kg m^{-2}

Data assimilation

Model state vector $\mathbf{x}_k$ and inputs $\mathbf{u}_k$ at time k

Model operator $\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k)$

Background state $\mathbf{x}_b$ estimates true state $\mathbf{x}_t$, error covariance matrix

$$\mathbf{B} = \overline{(\mathbf{x}_b - \mathbf{x}_t)^T(\mathbf{x}_b - \mathbf{x}_t)}$$

Model error covariance for forecast from true initial state

$$\mathbf{Q} = \overline{[\mathbf{f}(\mathbf{x}_{t,k}) - \mathbf{x}_{t,k+1}]^T[\mathbf{f}(\mathbf{x}_{t,k}) - \mathbf{x}_{t,k+1}]}$$

Observation operator $\mathbf{y} = \mathbf{h}(\mathbf{x})$

Observed value $\mathbf{y}_o$ estimates true value $\mathbf{y}_t$, error covariance matrix

$$\mathbf{R} = \overline{(\mathbf{y}_o - \mathbf{y}_t)^T(\mathbf{y}_o - \mathbf{y}_t)}$$

Find analysed state $\mathbf{x}_a$ to minimize quadratic cost function

$$J(\mathbf{x}_a) = (\mathbf{x}_a - \mathbf{x}_b)^T\mathbf{B}^{-1}(\mathbf{x}_a - \mathbf{x}_b) + [\mathbf{y}_o - \mathbf{h}(\mathbf{x}_a)]^T\mathbf{R}^{-1}[\mathbf{y}_0 - \mathbf{h}(\mathbf{x}_a)]$$

Analysis equation

Find $\mathbf{x}_a$ such that $$\frac{\partial J}{\partial \mathbf{x}} = 0$$

$$\Rightarrow \mathbf{x}_a = \mathbf{x}_b + \mathbf{B}\mathbf{H}^T(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R})^{-1}[\mathbf{y}_o - \mathbf{h}(\mathbf{x}_b)]$$

with Jacobian $\mathbf{H} = \frac{\partial \mathbf{h}}{\partial \mathbf{x}}$

Extended Kalman Filter (EKF):

Background error forecast $\mathbf{B}_{k+1} = \mathbf{F}\mathbf{B}_k\mathbf{F}^T + \mathbf{Q}$

and analysis $\mathbf{B}_a = \mathbf{B}_k + \mathbf{B}_k\mathbf{H}^T(\mathbf{H}\mathbf{B}_k\mathbf{H}^T + \mathbf{R})^{-1}\mathbf{H}\mathbf{B}_k$

using tangent linear model $\mathbf{F} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}$

Simplification (SEKF):

$$F_{ij} = \frac{\partial f_i}{\partial x_j} \approx \frac{f_i(\mathbf{x} + \delta x_j) - f_i(\mathbf{x})}{\delta x_j} \quad H_{ij} = \frac{\partial h_i}{\partial x_j} \approx \frac{h_i(\mathbf{x} + \delta x_j) - h_i(\mathbf{x})}{\delta x_j}$$

– control and one perturbed forecast for each state variable

Single observable state variable

$\mathbf{x} = [x]$, $\mathbf{y} = [x_o]$, $\mathbf{H} = [1]$

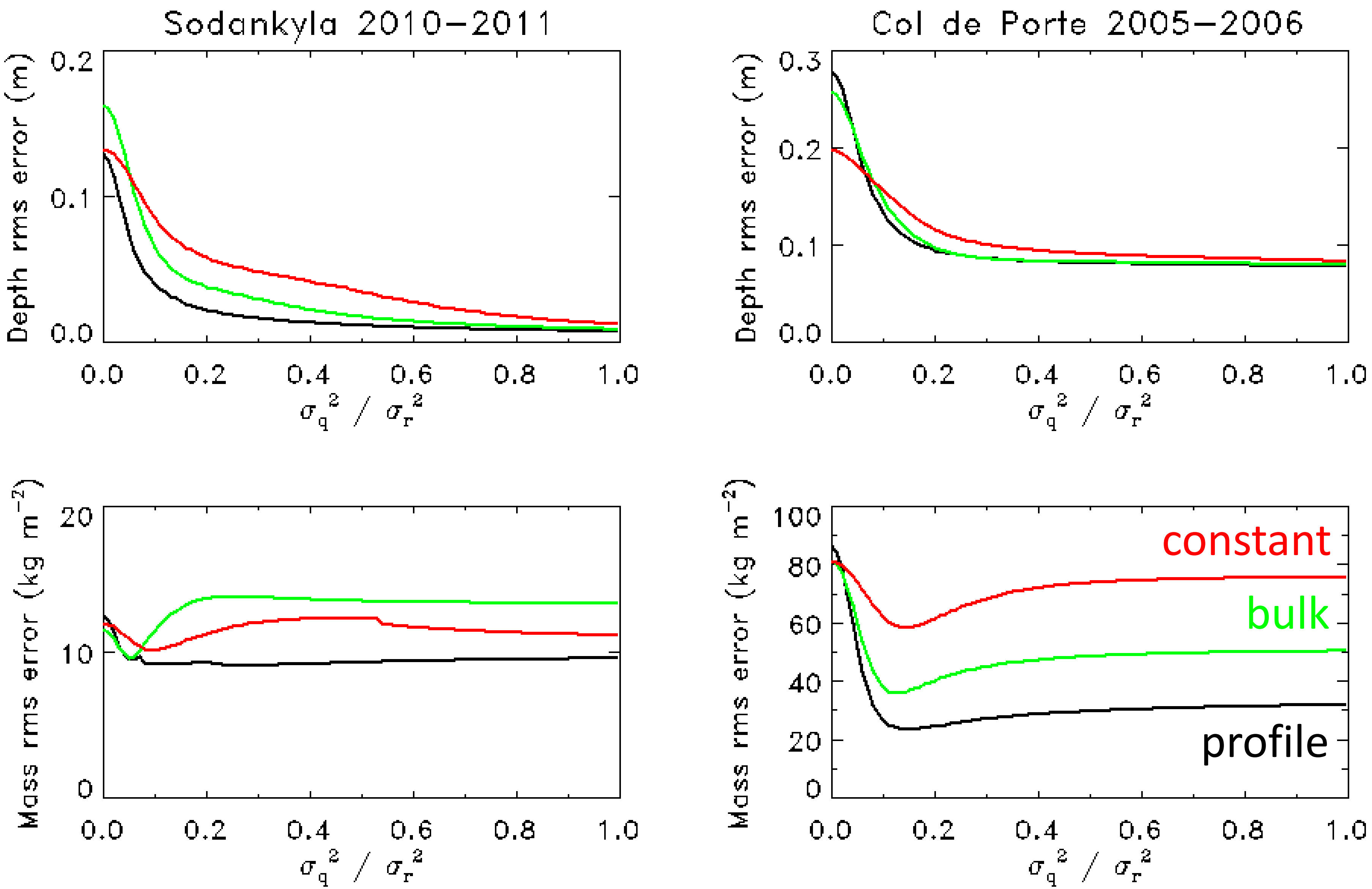
Analysis $$x_a = x_b + \frac{\sigma_b^2}{\sigma_b^2 + \sigma_r^2}[x_o - x_b]$$

$\sigma_b^2 \gg \sigma_r^2 \quad \Rightarrow x_a = x_o$ (direct insertion of observation)
$\sigma_b^2 \ll \sigma_r^2 \quad \Rightarrow x_a = x_b$ (open loop)

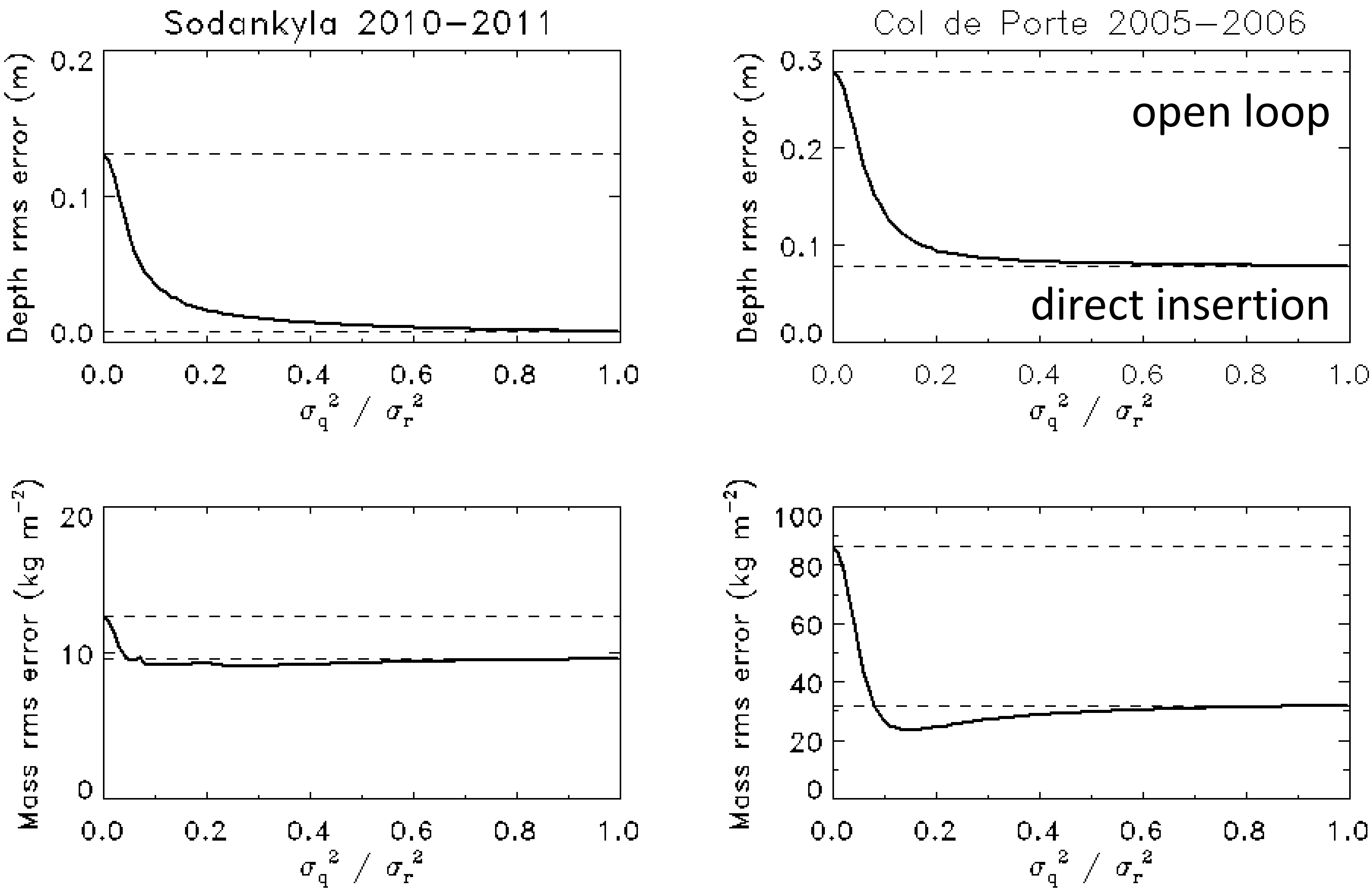
Background error forecast $\sigma_b^2 \rightarrow F^2\sigma_b^2 + \sigma_q^2$

and error analysis $$\sigma_b^2 \rightarrow \frac{\sigma_r^2}{\sigma_b^2 + \sigma_r^2}\sigma_b^2$$

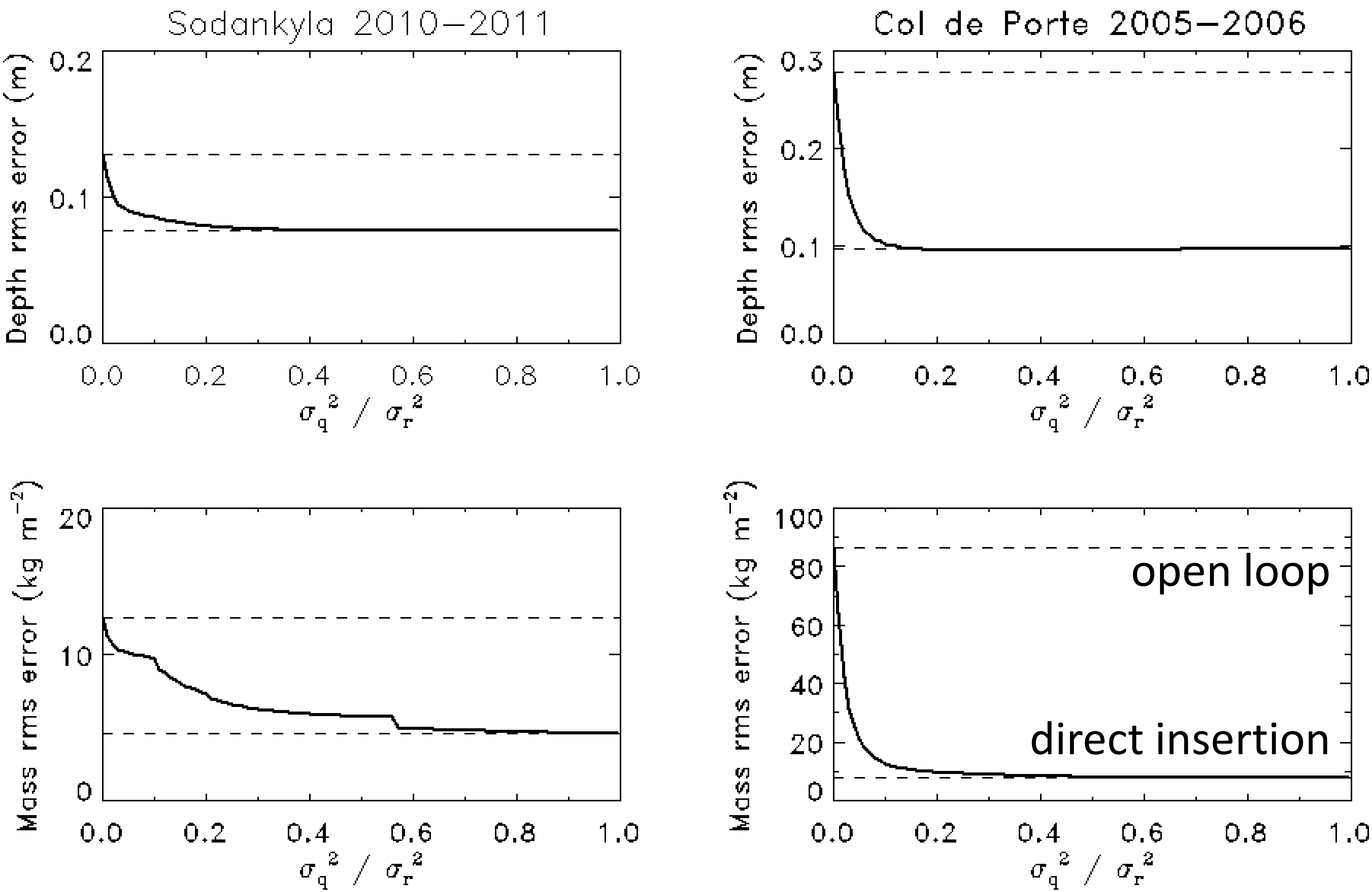
Daily assimilation of snow depth



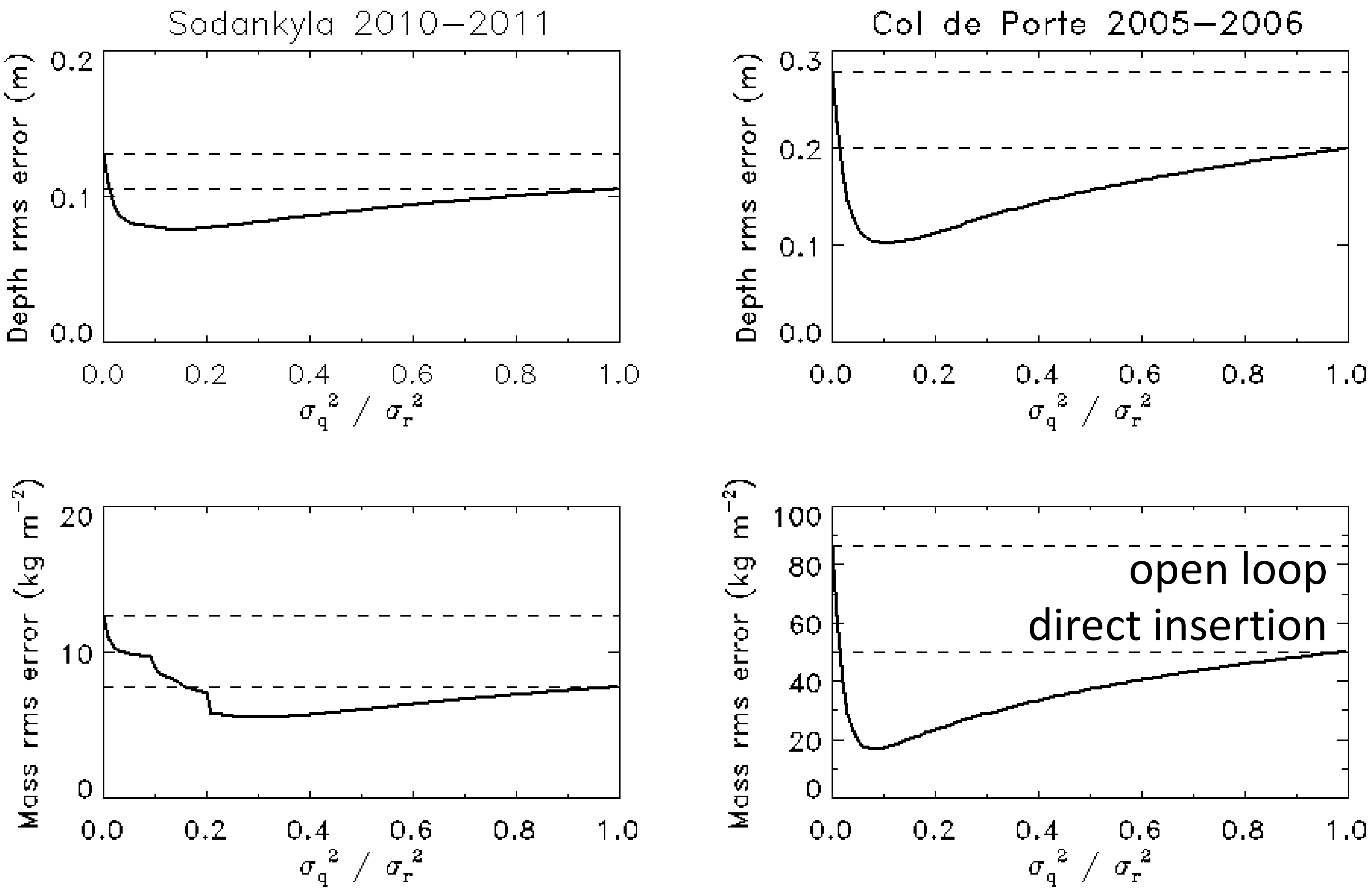
Daily assimilation of snow depth



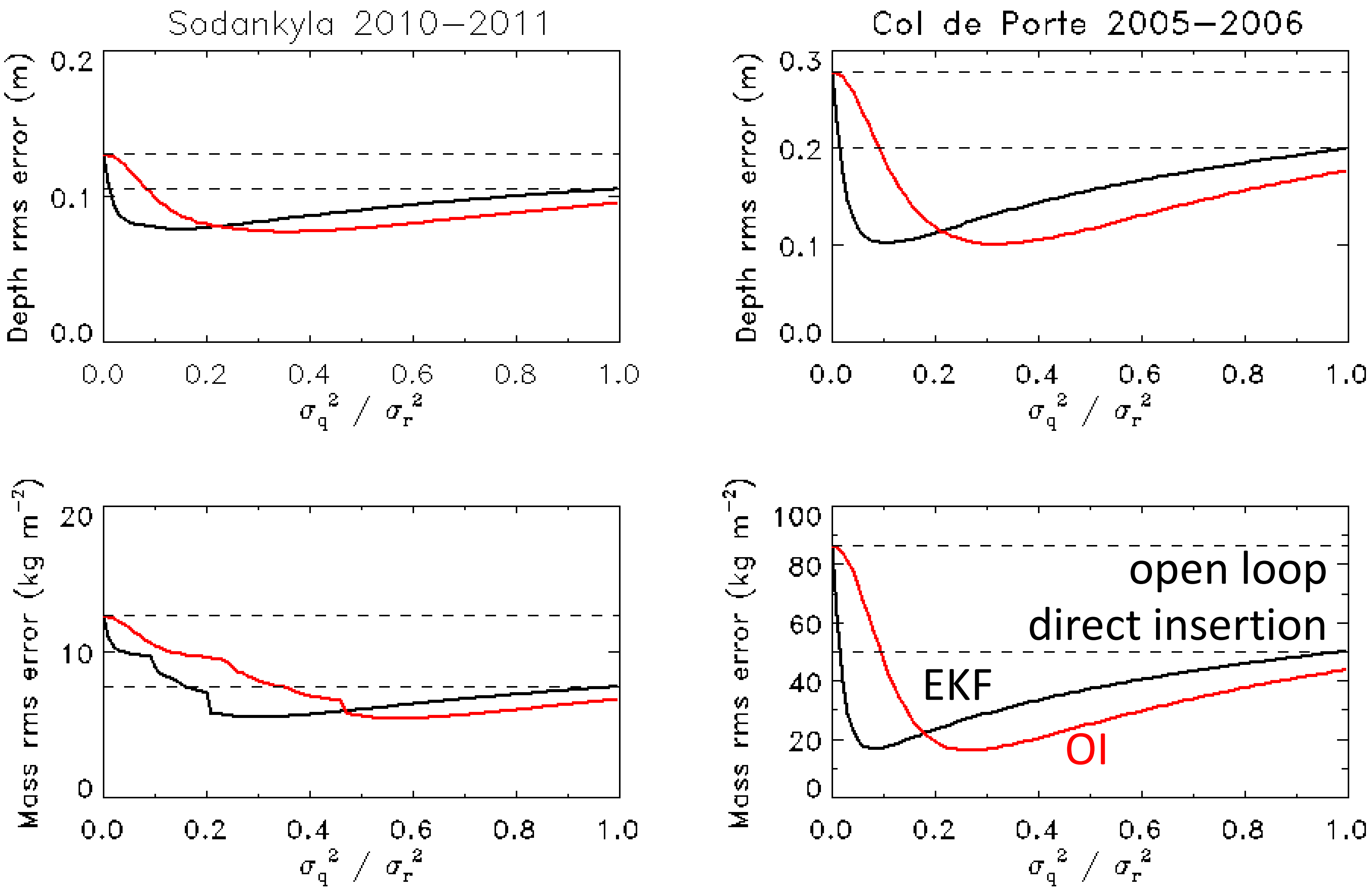
Daily assimilation of snow mass



Daily assimilation of degraded snow mass



Daily assimilation of degraded snow mass



Why is EKF no better than OI?

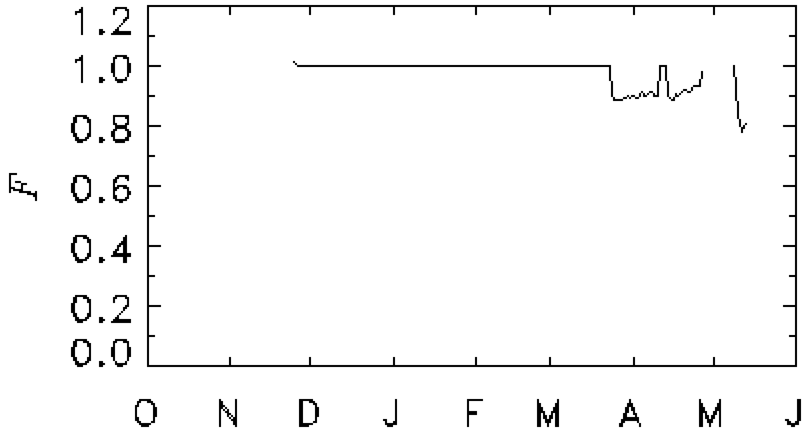
Mass balance $M_{k+1} = M_k + \left(S_f - E - R\right)\delta t$

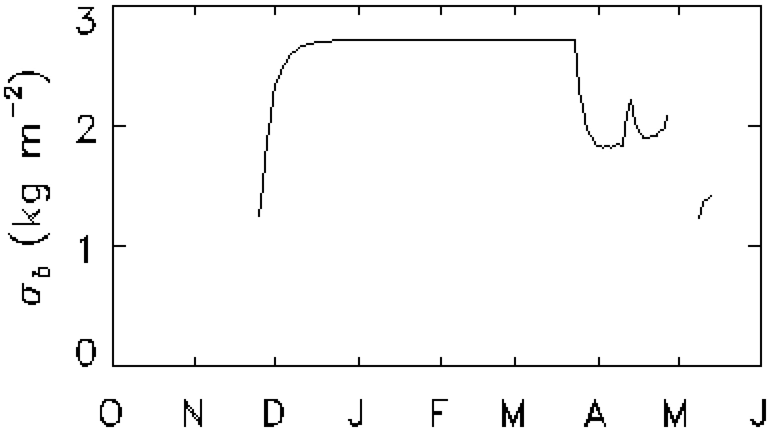
– almost linear for deep snow

$$F = \frac{dm_{k+1}}{dm_k} \approx 1$$

$\Longrightarrow \sigma_b^2 \rightarrow$ constant

$\Longrightarrow$ SEKF ≈ OI





Conclusions

Assimilation of ground-based snow data requires:

- good background estimate of snow density
- good estimates of observation and model errors (underestimation of model / observation error ratio is worse than overestimation)
- may not require advanced data assimilation techniques