

A PGAS implementation by co-design
of the ECMWF Integrated Forecasting
System

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Acknowledgements

Mats Hamrud

ECMWF

Nils Wedi

ECMWF

Jens Doleschal

Technische Universität Dresden

Harvey Richardson

Cray Research UK

And my other partners in the CRESTA Project

The CRESTA project has received funding from the EU Seventh Framework Programme (ICT-2011.9.13)



What is CRESTA - see <http://cresta-project.eu/>

- Collaborative Research into Exascale Systemware, Tools and Applications
- EU funded project, 3 years (year 1 just completed), ~ 50 scientists
- Six co-design vehicles (aka applications)
 - ELMFIRE (CSC, ABO, UEDIN) - fusion plasma
 - GROMACS (KTH) - molecular dynamics
 - HEMELB (UCL) - biomedical
 - IFS (ECMWF) - weather
 - NEK5000 (KTH) & OPENFOAM (USTUTT, UEDIN) - comp. fluid dynamics
- Two tool suppliers
 - ALLINEA (ddt : debugger) & TUD (vampir : performance analysis)
- Technology and system supplier – CRAY UK
- Many Others (mostly universities)
 - ABO, CRSA, CSC, DLR, JYU, KTH, UCL, UEDIN-EPCC, USTUTT-HRLS

IFS model: current and planned model resolutions

IFS model resolution	Envisaged Operational Implementation	Grid point spacing (km)	Time-step (seconds)	Estimated number of cores ¹
T1279 H²	2010 (L91)	16	600	1100
	2012 (L137)			1600
T2047 H	2014-2015	10	450	6K
T3999 NH³	2020-2021	5	240	80K
T7999 NH	2025-2026	2.5	30-120	1-4M

1 - a gross estimate for the number of 'Power7' equivalent cores needed to achieve a 10 day model forecast in under 1 hour (~240 FD/D), system size would normally be 10 times this number.

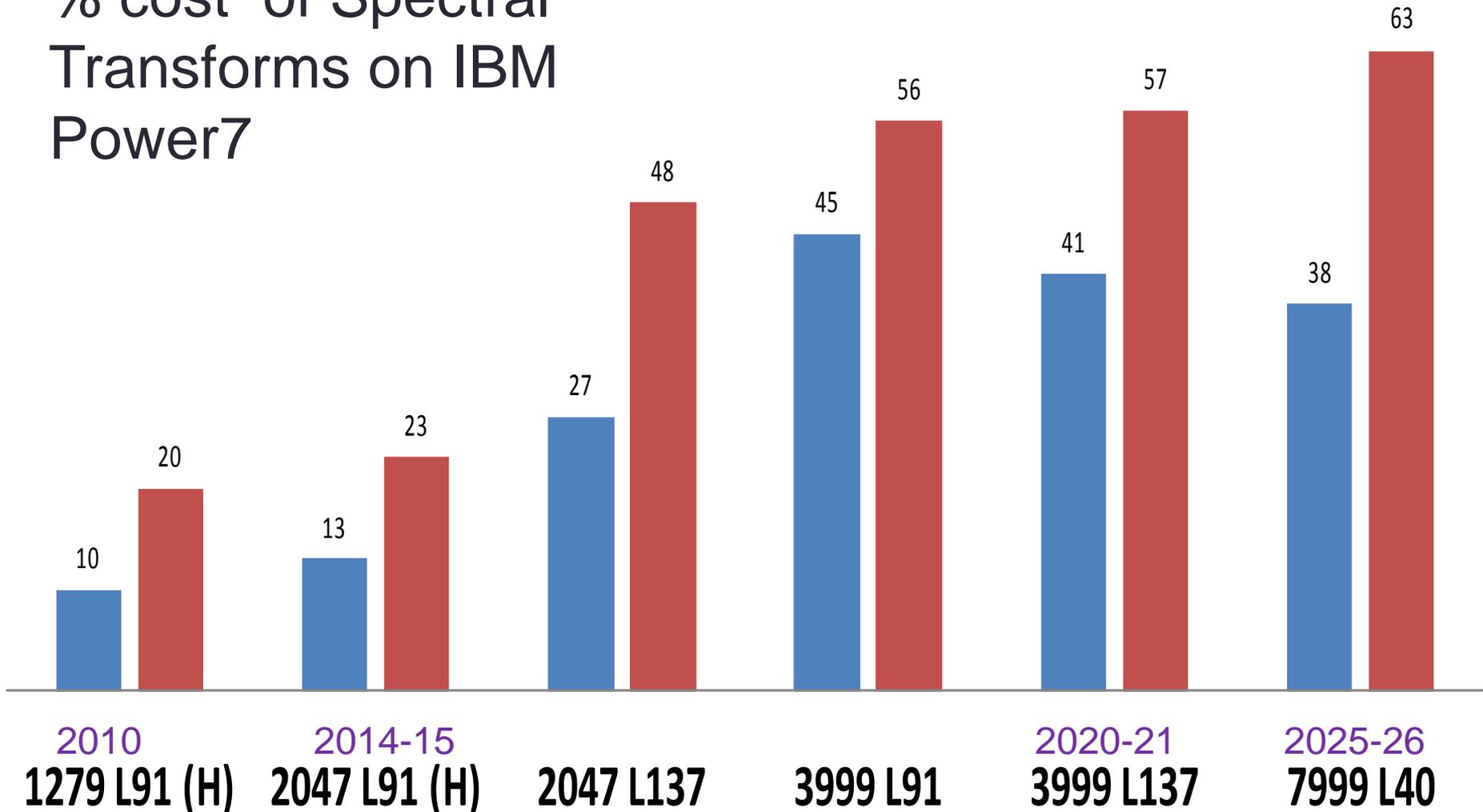
2 – Hydrostatic Dynamics

3 – Non-Hydrostatic Dynamics

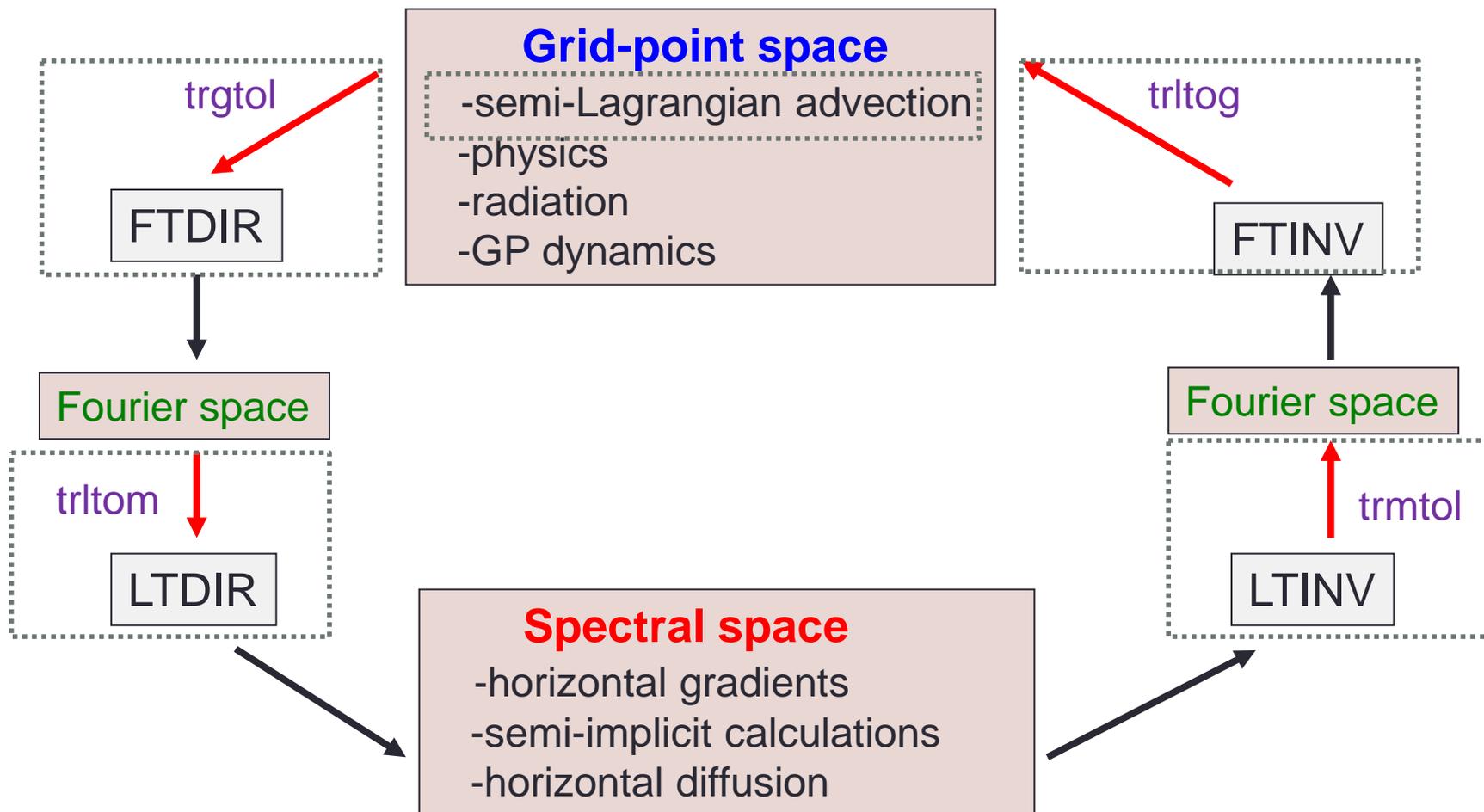
Thank you to Nils Wedi for providing this figure

■ Compute ■ Total

% cost of Spectral Transforms on IBM Power7

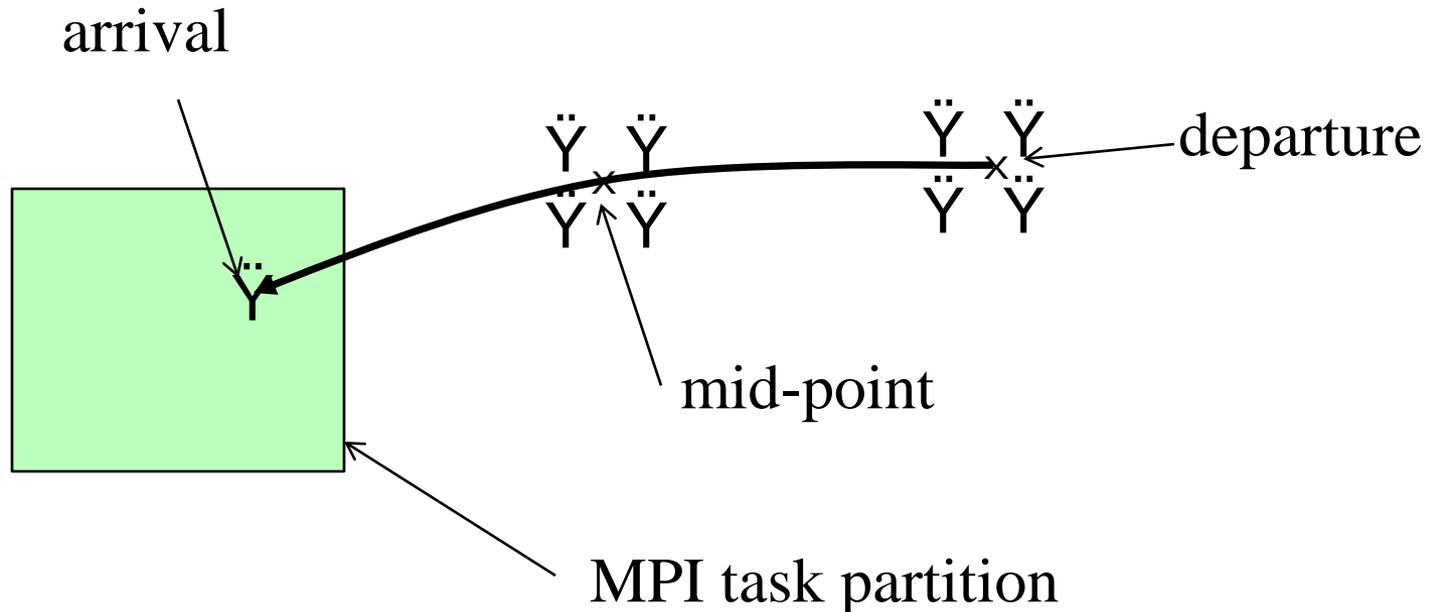


Planned IFS optimisations for [Tera,Peta,Exa]scale

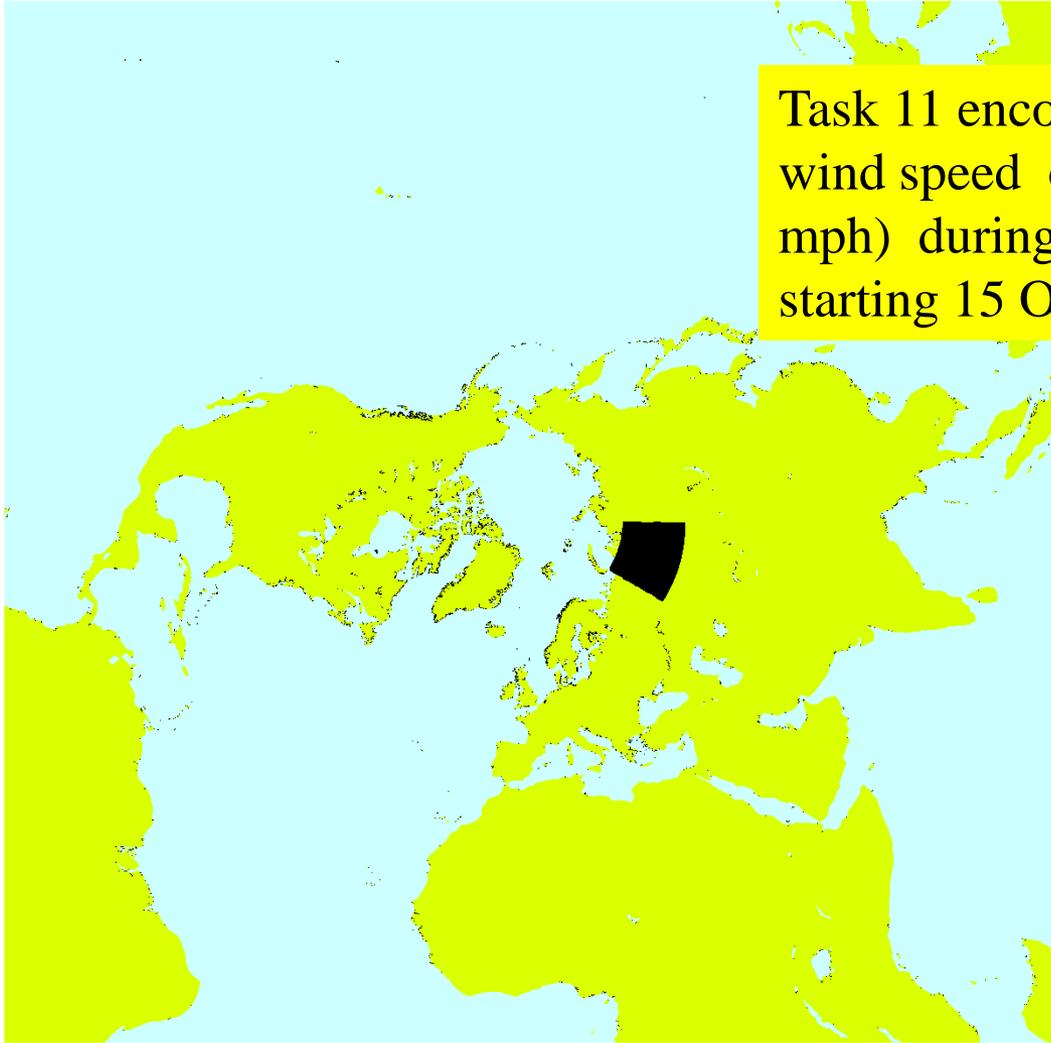


Semi-Lagrangian Transport

- Computation of a trajectory from each grid-point backwards in time, and
- Interpolation of various quantities at the departure and at the mid-point of the trajectory

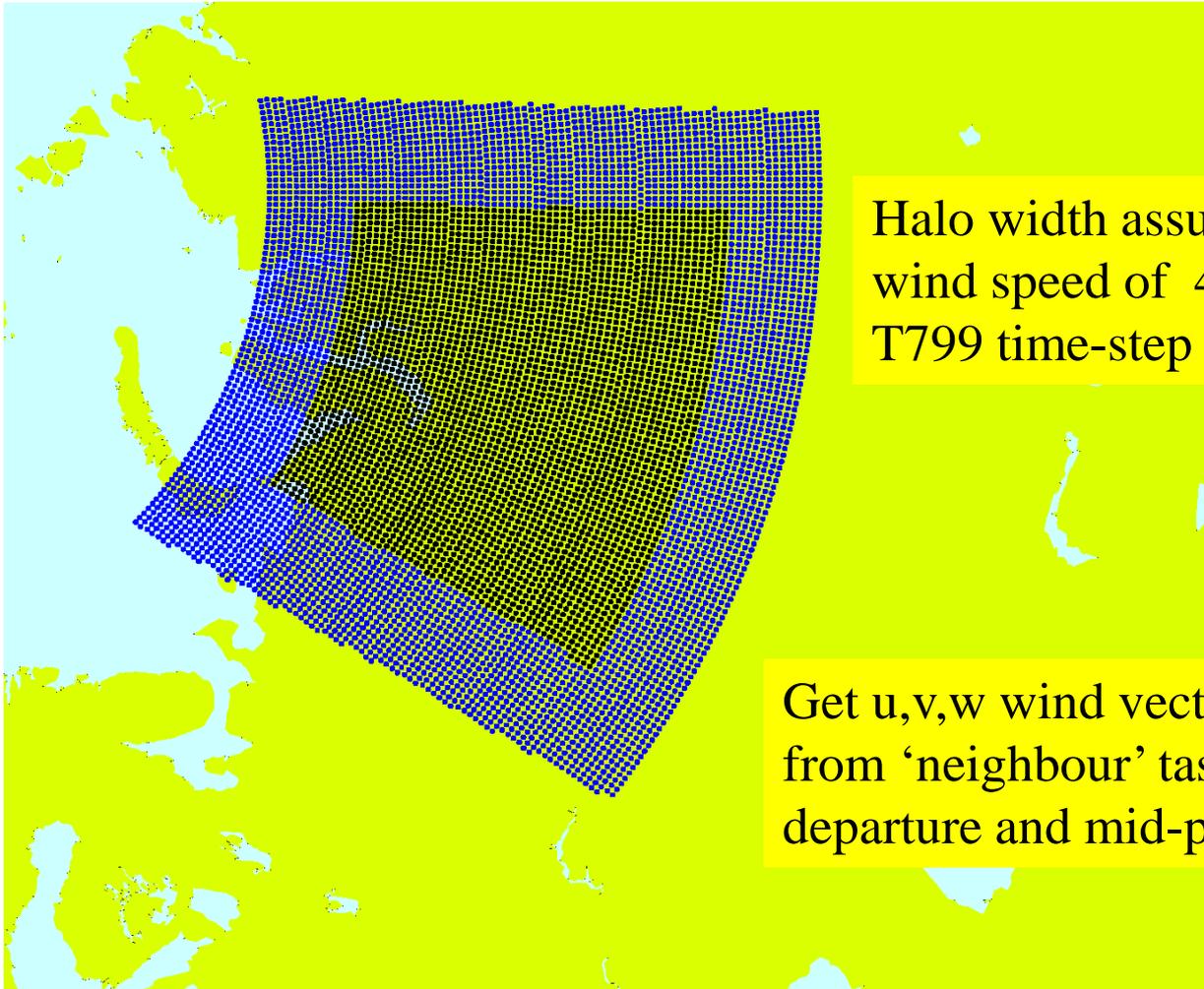


Semi-Lagrangian Transport: T799 model, 256 tasks



Task 11 encountered the highest wind speed of 120 m/s (268 mph) during a 10 day forecast starting 15 Oct 2004

blue: halo area



Halo width assumes a maximum wind speed of $400 \text{ m/s} \times 720 \text{ s}$
T799 time-step (288 km)

Get u, v, w wind vector variables (3)
from 'neighbour' tasks to determine
departure and mid-point of trajectory

red: halo points actually used



Get rest of the variables (26) from the red halo area and perform interpolations

Note that volume of halo data communicated is dependent on wind speed and direction in locality of each task

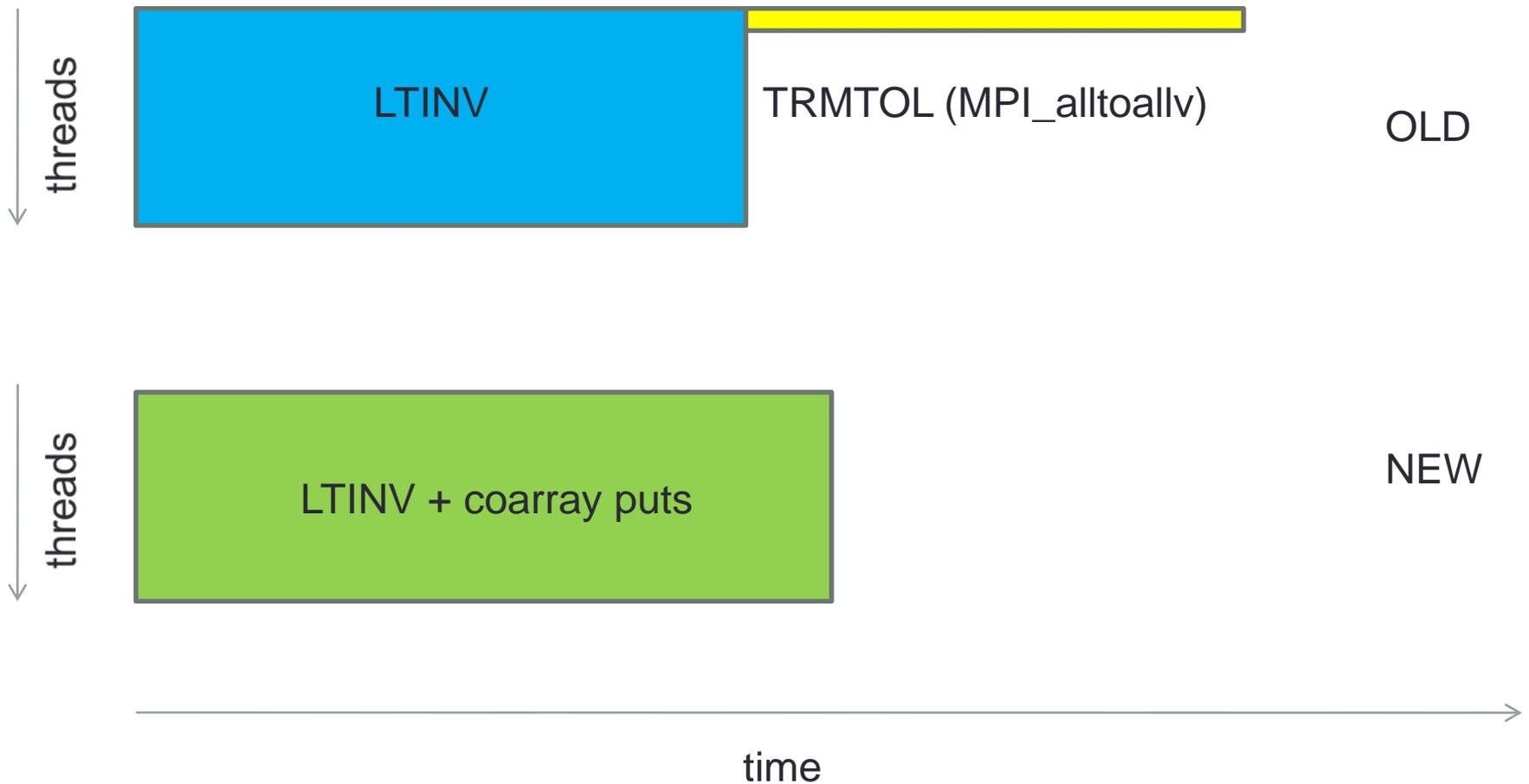
IFS Optimisations for ExaScale & Co-design

- All currently planned IFS optimisations in the CRESTA project
 - Involve use of Fortran2008 coarrays (CAF)
 - Used within context of OpenMP parallel regions
- Overlap Legendre transforms with associated transpositions
- Overlap Fourier transforms with associated transpositions
- Rework semi-Lagrangian communications
 - To substantially reduce communicated halo data
 - To overlap halo communications with SL interpolations
- CAF co-design team
 - caf-co-design@cresta-project.eu
 - ECMWF – optimise IFS as described above
 - CRAY – optimize DMAPP to be thread safe
 - TUD – visualize CAF operations in IFS with vampir
 - ALLINEA – debug IFS at scale with ddt (MPI/OMP/CAF)

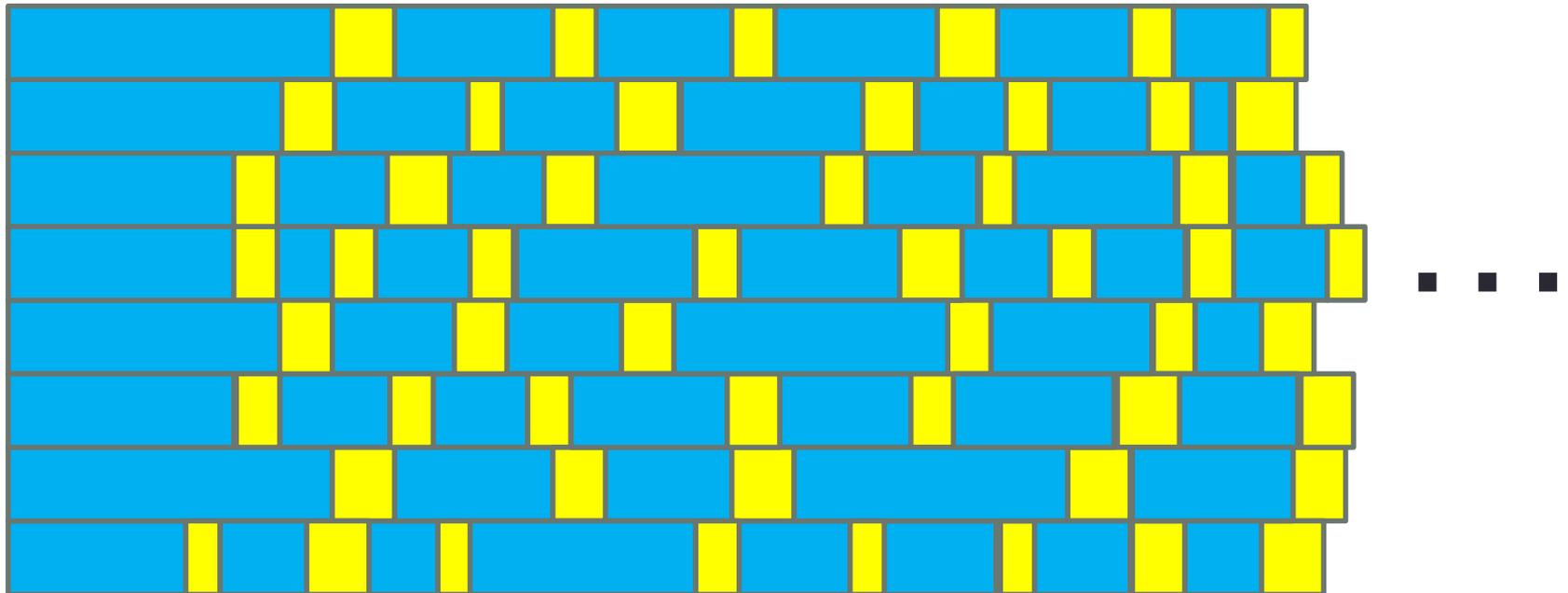


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Overlap Legendre transforms with associated transpositions



Overlap Legendre transforms with associated transpositions/3 (LTINV + coarray puts)



Expectation is that compute (LTINV-blue) and communication (coarray puts-yellow) overlap in time. We should be able to see this in the future with an extension to vampir being developed in CRESTA

Semi-Lagrangian – coarray implementation

red: only the halo points that are used are communicated

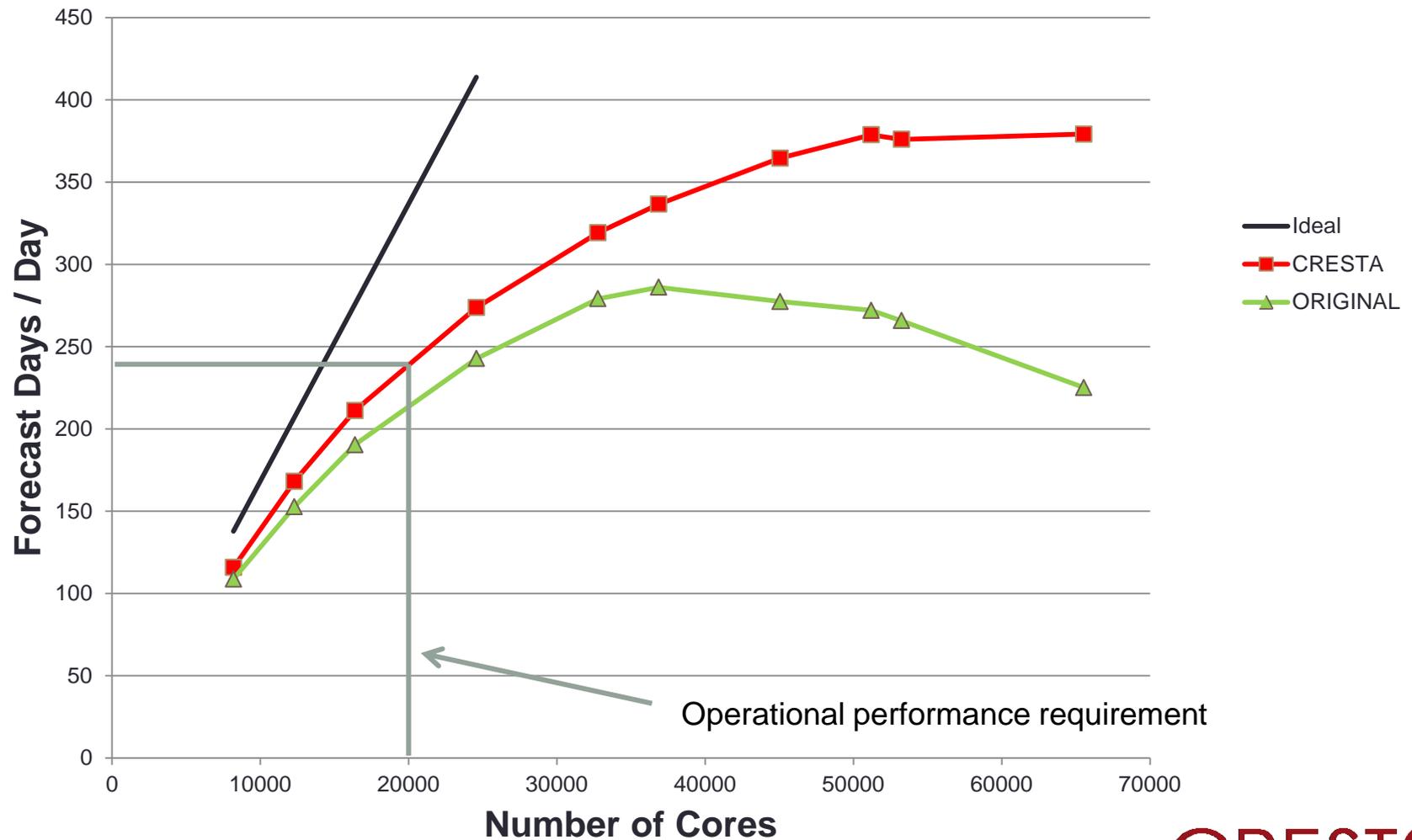


Note no more blue area (max wind halo) and associated overhead.

Also, halo coarray transfers take place in same OpenMP loop as the interpolations.

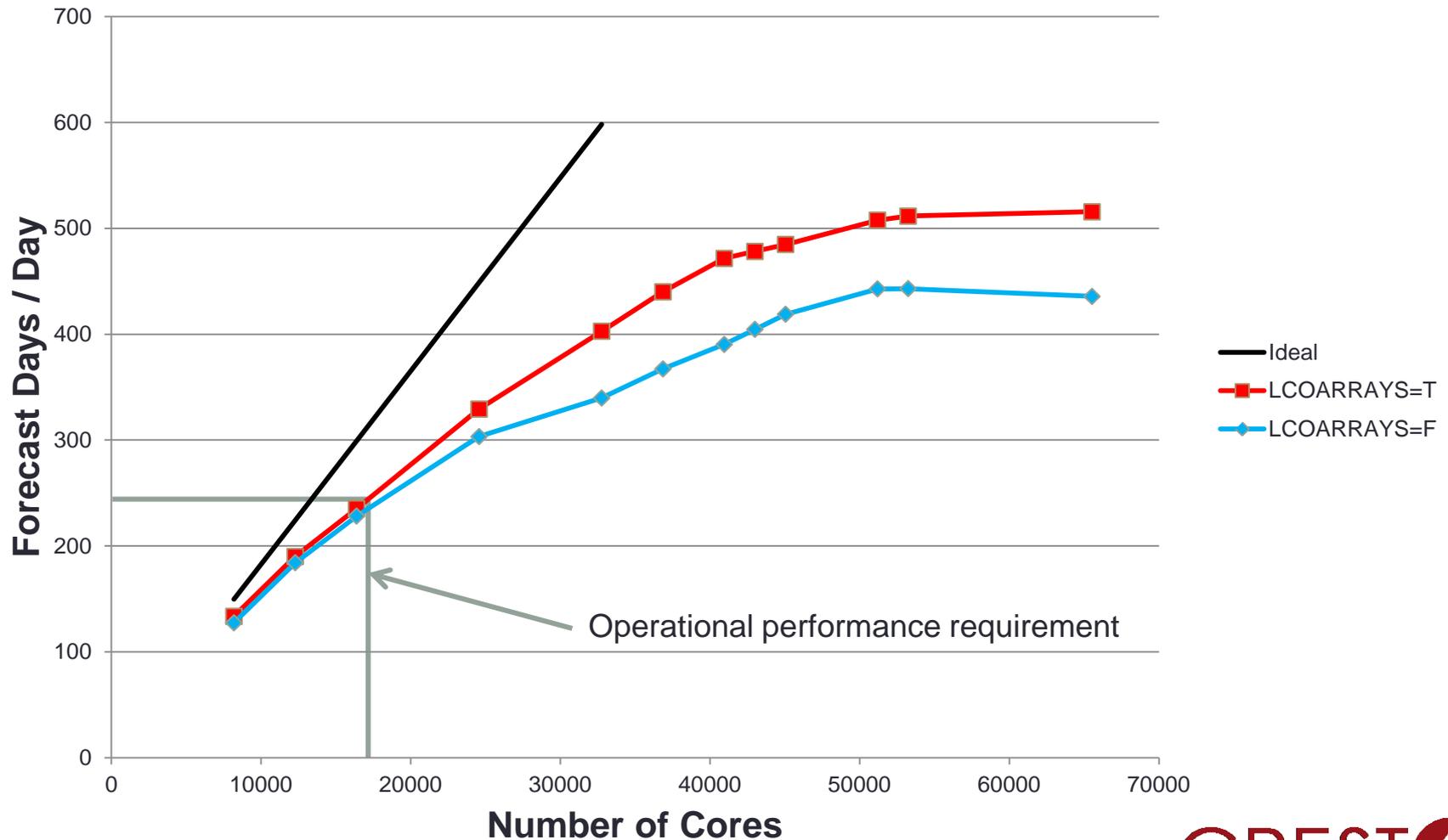
T2047L137 model performance on HECToR (CRAY XE6) RAPS12 IFS (CY37R3), cce=7.4.4

APRIL 2012

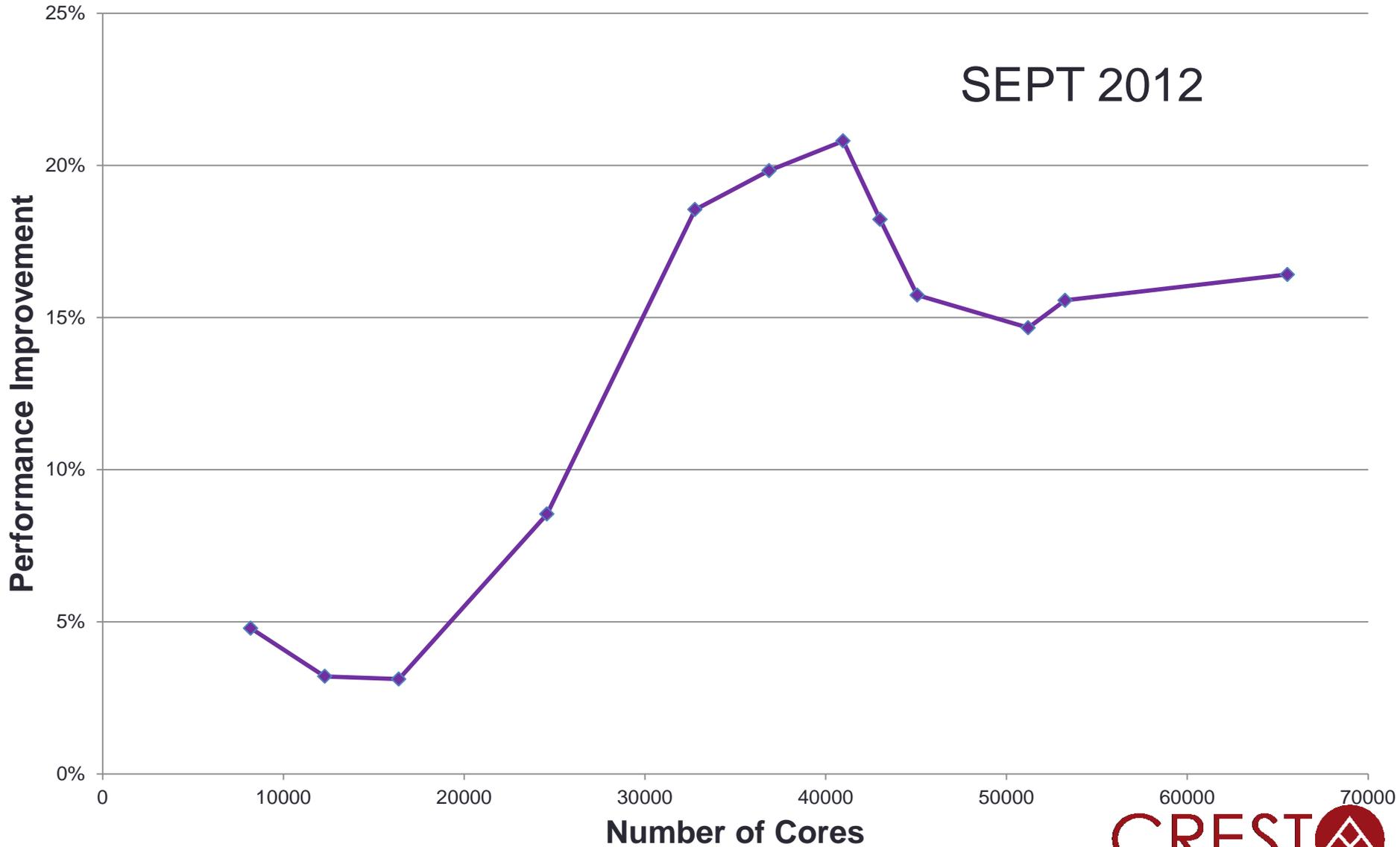


T2047L137 model performance on HECToR (CRAY XE6) RAPS12 IFS (CY37R3), cce=8.0.6 -hflex_mp=intolerant

SEPT 2012

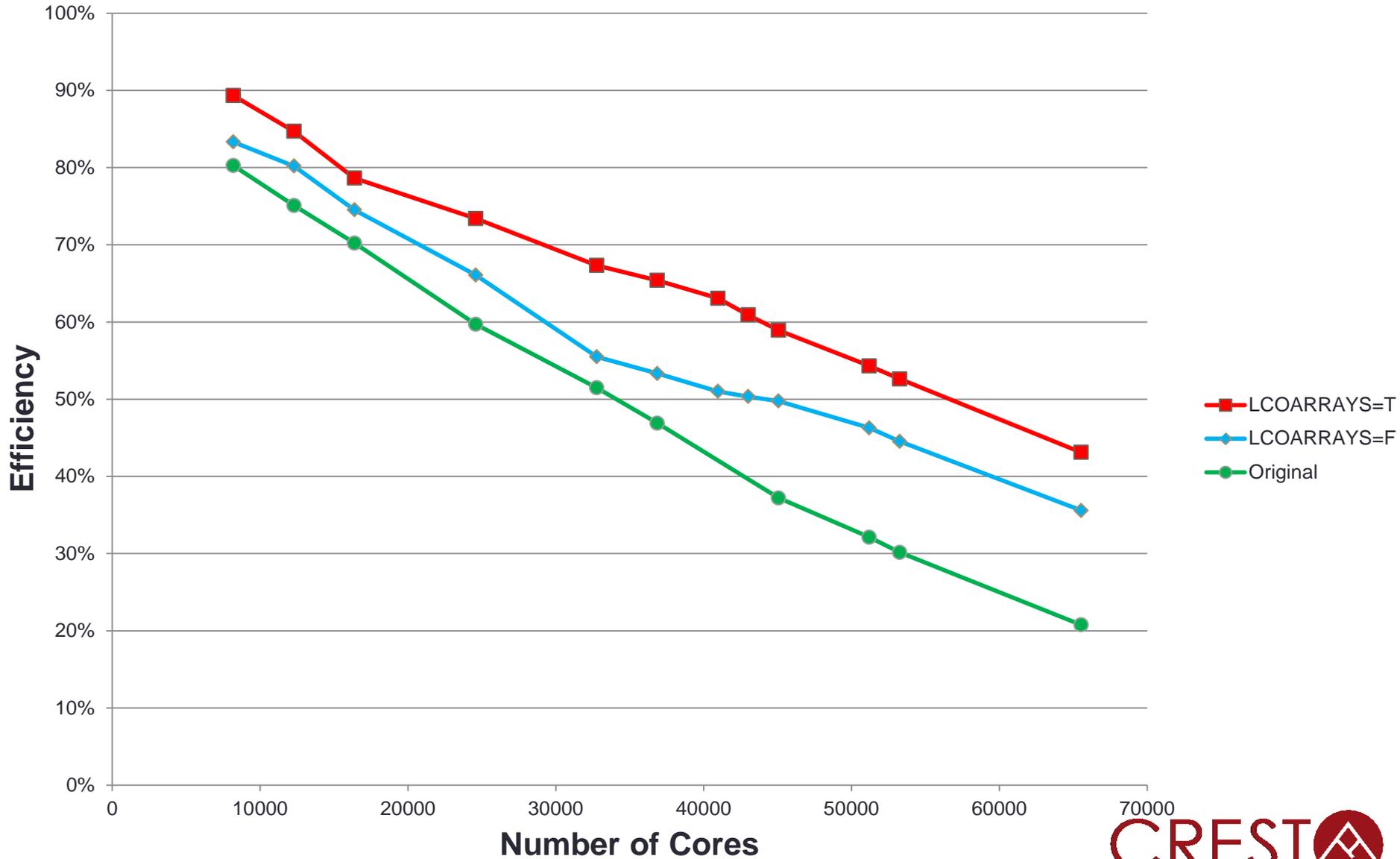


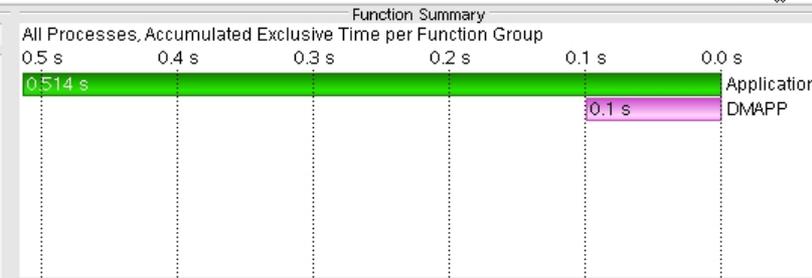
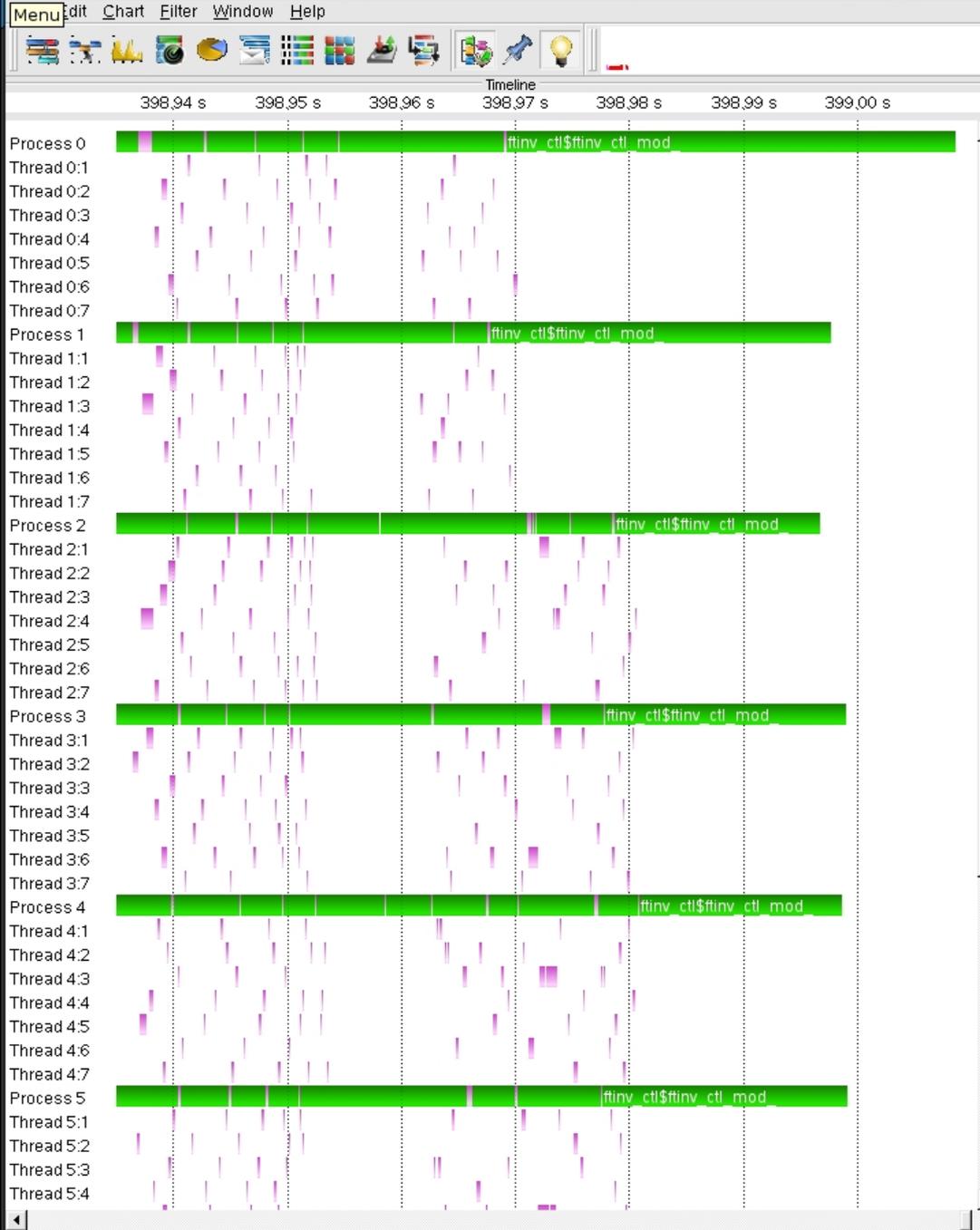
T2047L137 IFS model performance improvement by using Fortran2008 coarrays on HECToR (CRAY XE6)



SEPT 2012

T2047L137 model Efficiency on HECToR (CRAY XE6) RAPS12 IFS (CY37R3)

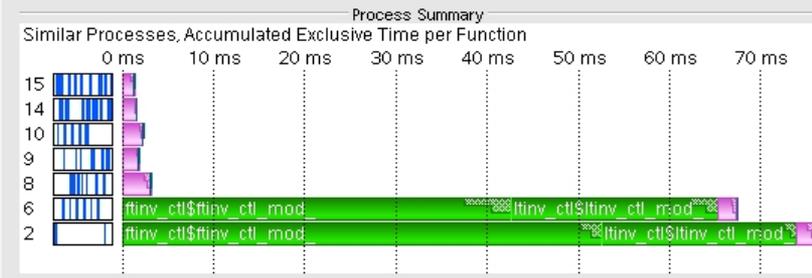




Context View

Master Timeline

Property	Value
Display	Master Timeline
Type	Function
Location	Process 0
Function	ftinv_ctl\$ftinv_ctl_mod
Function Group	Application
Interval Begin	398.938043 s
Interval End	398.942646 s
Duration	4.6023 ms
Source Loca	ftinv_ctl_mod.F90:3



Call Tree

All Processes

Function	Min Inclusive Time	Max Inclusive Time
ftinv_ctl\$ftinv_ctl_mod	0.000 s	26.442 ms
ftinv_ctl\$ftinv_ctl_mod	0.000 s	52.853 ms
dmapp_syncid_wait	0.000 s	5.590 µs
dmapp_set_rma_attrs	0.000 s	5.777 µs
dmapp_put	0.000 s	2.995 ms
dmapp_get_rma_attrs	0.000 s	12.411 µs
dmapp_get_nb	0.000 s	84.420 µs



Schedule for IFS optimisations in CRESTA

When	Activity
4Q2011-1Q2012	Coarray Kernel ✓
1Q2012	IFS CY37R3 port to HECToR ✓ Run T2047 model at scale and analyze performance ✓
2Q2012	Scalability improvements arising from T2047 analyses (“low hanging fruit”) ✓ Overlap Legendre transform computations with associated TRMTOL & TRLTOM transpositions ✓
3Q2012	Semi-Lagrangian optimisation ✓ Overlap TRGTOL & TRLTOM transpositions with associated Fourier transforms ✓
4Q2012	RAPS13 IFS CY38R2 port to HECToR (contains Fast Legendre Transform) T3999 model runs on HECToR Test with IBM F2008 ‘coarray technology preview’ compiler on Power7 at ECMWF
1Q2013	Use coarrays to optimise TRMTOS/TRSTOM transpositions Initial use of GPUs for IFS (targeting LTDIR/LTDIR dgemm’s)
2013-2014	Other IFS scalability optimisations (transpose SL data, physics load balancing, +++) Development & testing of a future solver for IFS (Plan B) Following closely developments in GungHO! project (MetOffice, NERC, STFC) GungHO= <u>G</u> lobally <u>U</u> niform <u>N</u> ext <u>G</u> eneration <u>H</u> ighly <u>O</u> ptimised

A wide-angle photograph of a snowy mountain range under a clear blue sky. The central focus is a sharp, snow-covered peak. In the foreground, a cable car station is partially buried in deep snow, with a brown cable car visible. A person in a red jacket stands near the station. The surrounding slopes are covered in snow and patches of dark rock.

Thank you for your
attention

QUESTIONS?

IFS model coarray developments

Compile with `-DCOARRAYS`

for compilers that support Fortran2008 coarray syntax

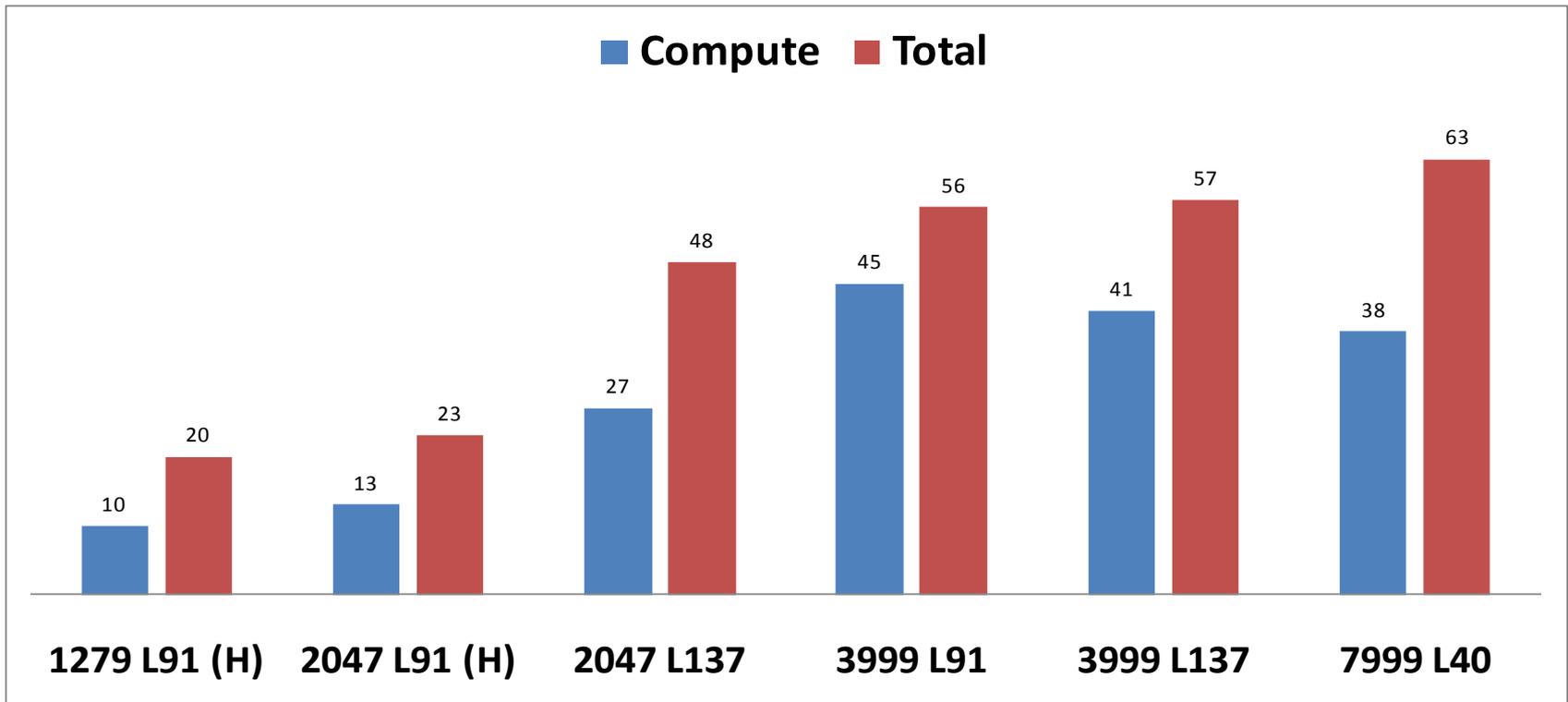
Run with,

`&NAMPAR1`
`LCOARRAYS=true,` to use coarray optimizations

`&NAMPAR1`
`LCOARRAYS=false,` to use original MPI implementation

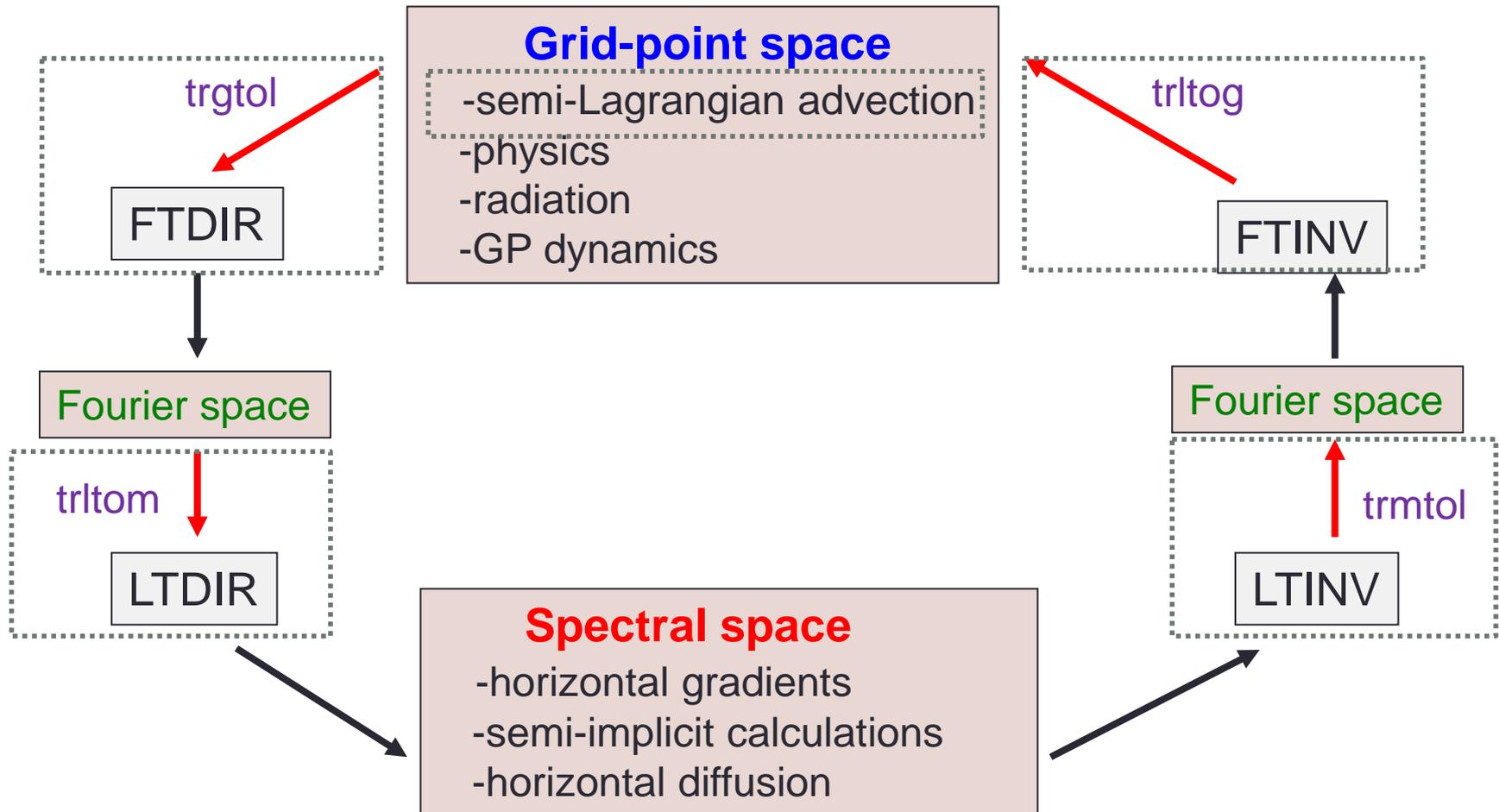
Motivation – T2047 and T3999 costs on IBM Power7 (percentage of wall clock time)

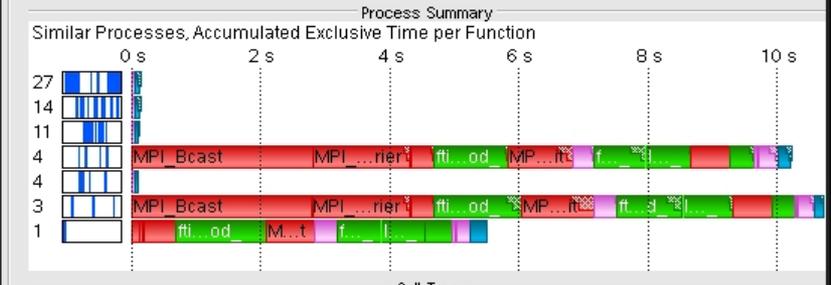
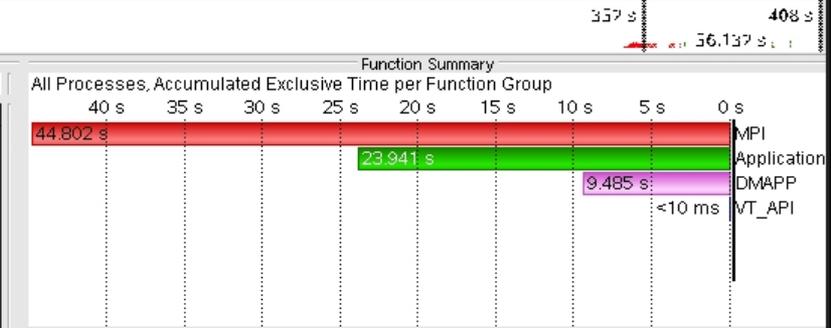
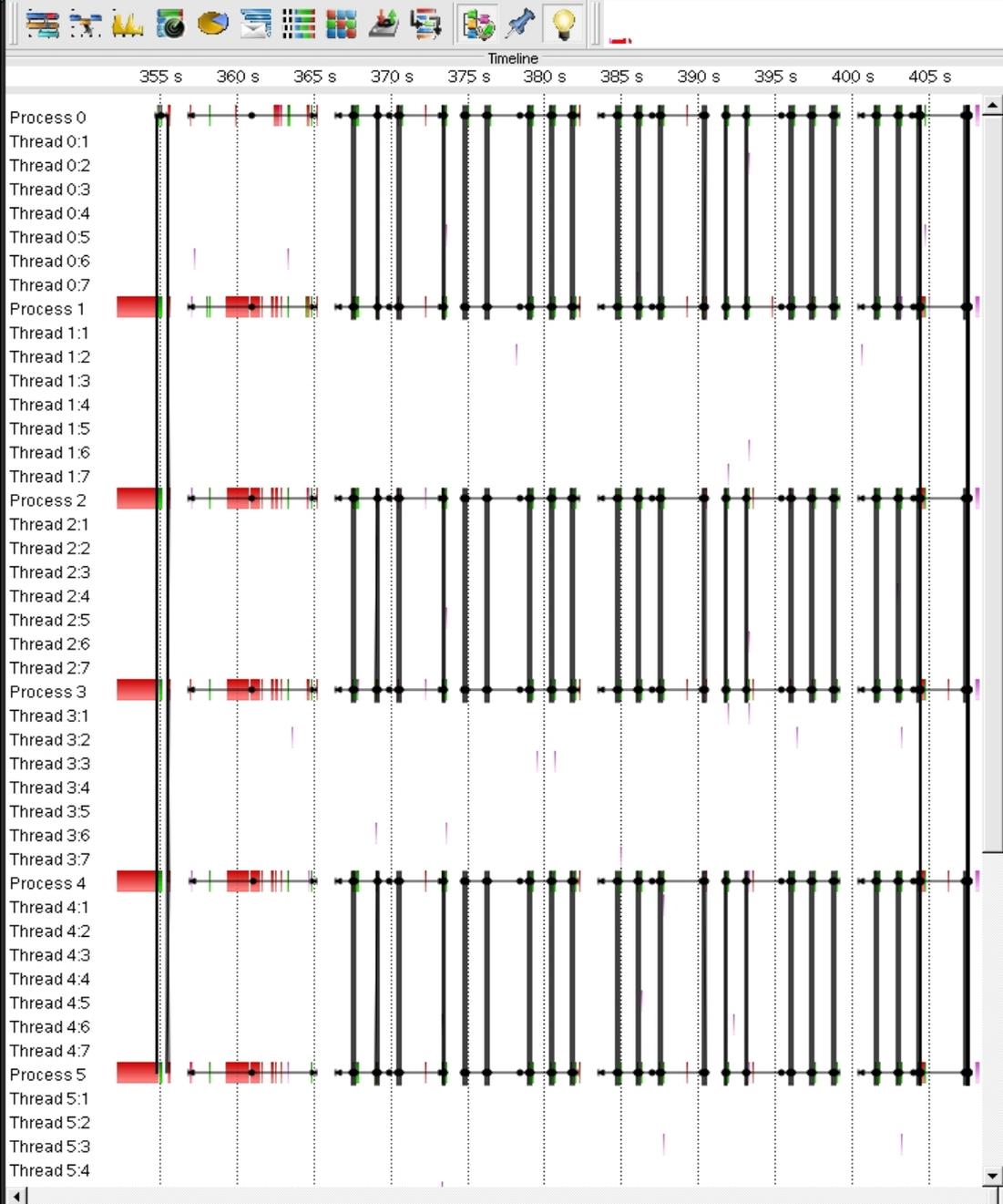
		T2047 (%) 2014-15	T3999 (%) 2020-21
LTINV_CTL	- INVERSE LEGENDRE TRANSFORM	3.30	8.40
LTINV_CTL	- M TO L TRANSPOSITION	5.37	5.24
LTDIR_CTL	- DIRECT LEGENDRE TRANSFORM	3.56	5.30
LTDIR_CTL	- L TO M TRANSPOSITION	2.84	3.14
FTDIR_CTL	- DIRECT FOURIER TRANSFORM	0.20	1.07
FTDIR_CTL	- G TO L TRANSPOSITION	2.85	2.21
FTINV_CTL	- INVERSE FOURIER TRANSFORM	0.72	3.76
FTINV_CTL	- L TO G TRANSPOSITION	4.47	7.36
	SUM(%)	23.4	36.5
		L137/LT	L91/FLT
		4224Tx8t	1024Tx16t
		528 Nodes	256 Nodes
		470 FD/D	28 FD/D



Relative **computational** cost of the spherical harmonics transforms plus the spectral computations (solving the Helmholtz equation) as a percentage of the overall model cost for various configurations. Red bars indicate the total cost **including** the global communications involved. Percentages have been derived considering all gridpoint dynamics and physics computations but without considering IO, synchronization costs (barriers), and any other ancillary costs. All runs are non-hydrostatic unless indicated with (H). All runs further show that the communications cost is less than or equal to the compute cost on the IBM Power7 and have good potential for “hiding” this overhead. However, communication cost is likely to increase with the number of cores.

Planned IFS optimisations for [Tera,Peta,Exa]scale

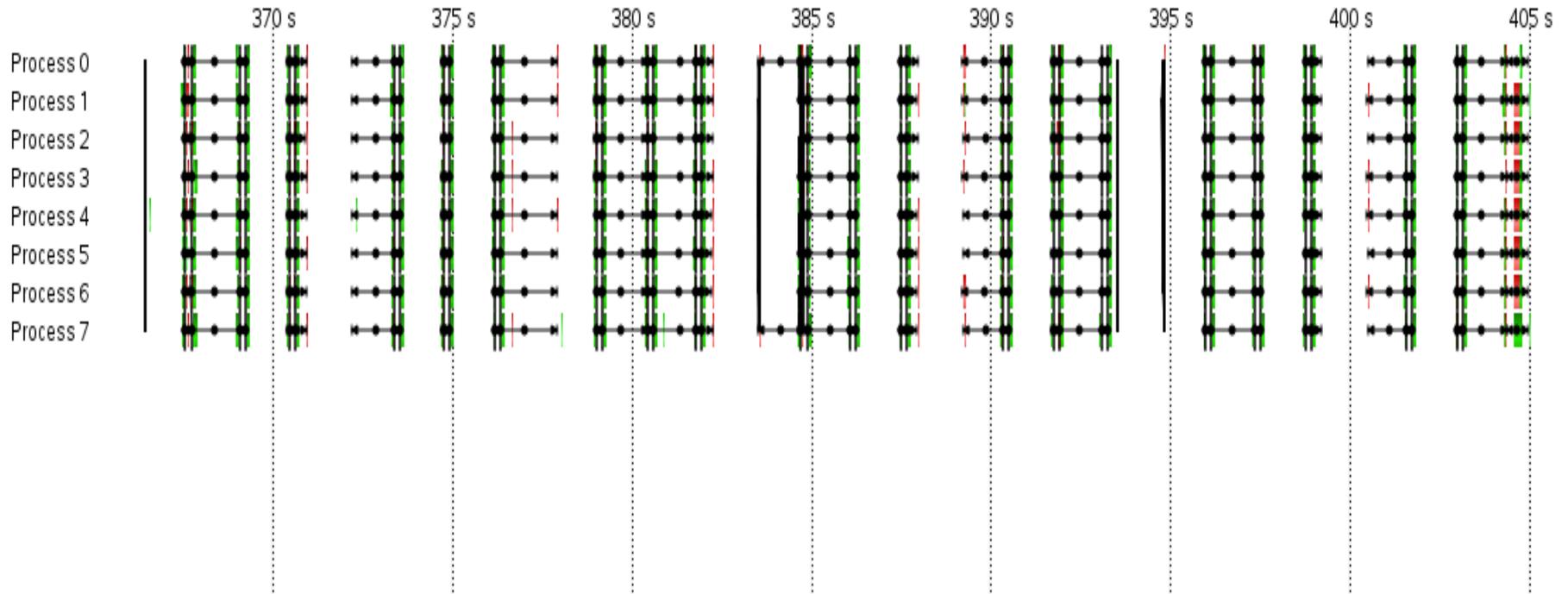




All Processes

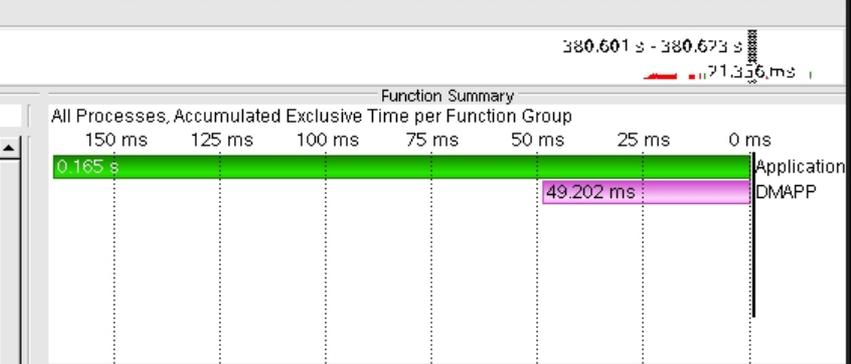
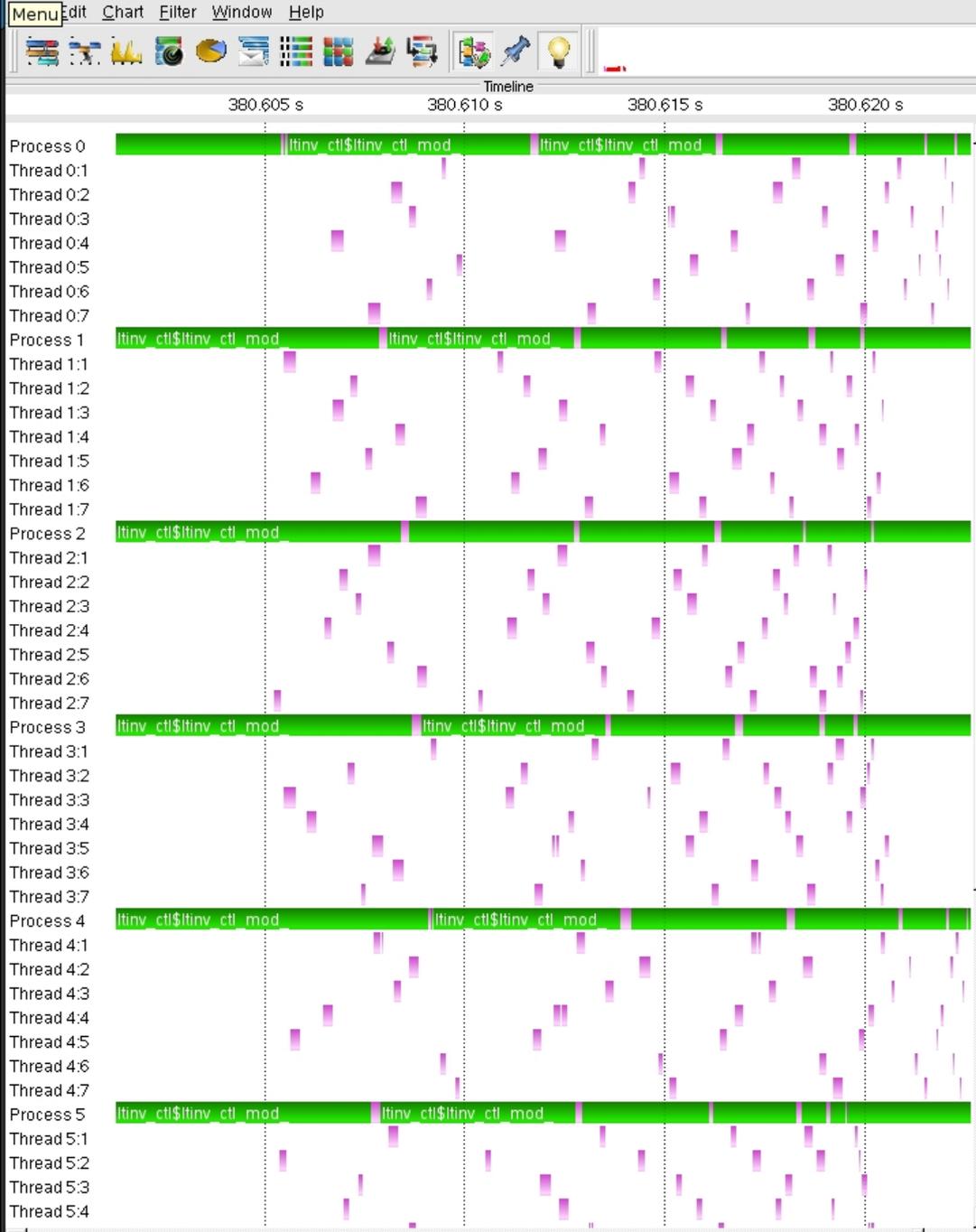
Function	Min Inclusive Time	Max Inclusive Time
ltnv ctl\$ltinv ctl mod	0.000 s	0.927 s
ltdir ctl\$ltidir ctl mod	0.000 s	0.466 s
ftinv ctl\$ftinv ctl mod	0.000 s	1.504 s
ftdir ctl\$ftidir ctl mod	0.000 s	1.087 s
dmapp syncid wait	0.000 s	341.146 µs
dmapp sheap malloc	0.000 s	2.197 µs
dmapp sheap free	0.000 s	3.279 µs
dmapp set rma attrs	407.629 µs	3.488 ms
dmapp put ixpe	0.000 s	35.447 µs





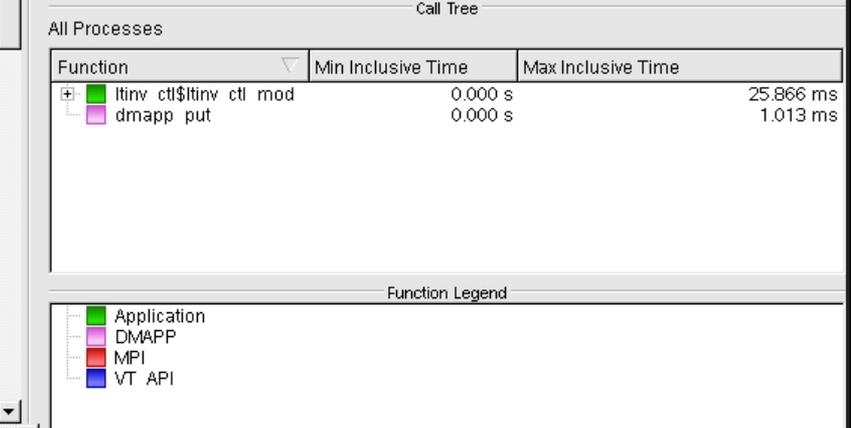
Similar Processes, Accumulated Exclusive Time per Function

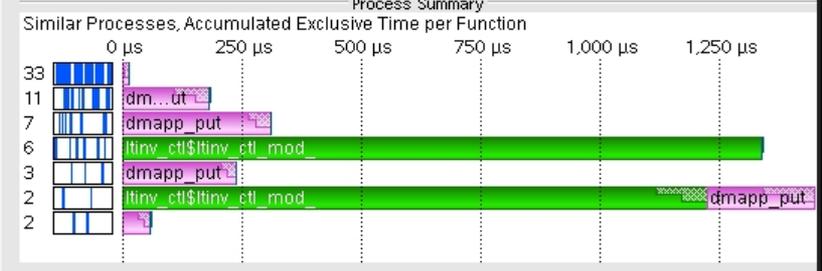
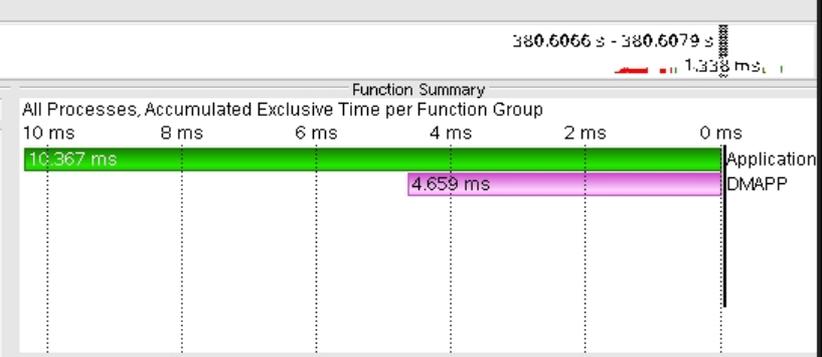
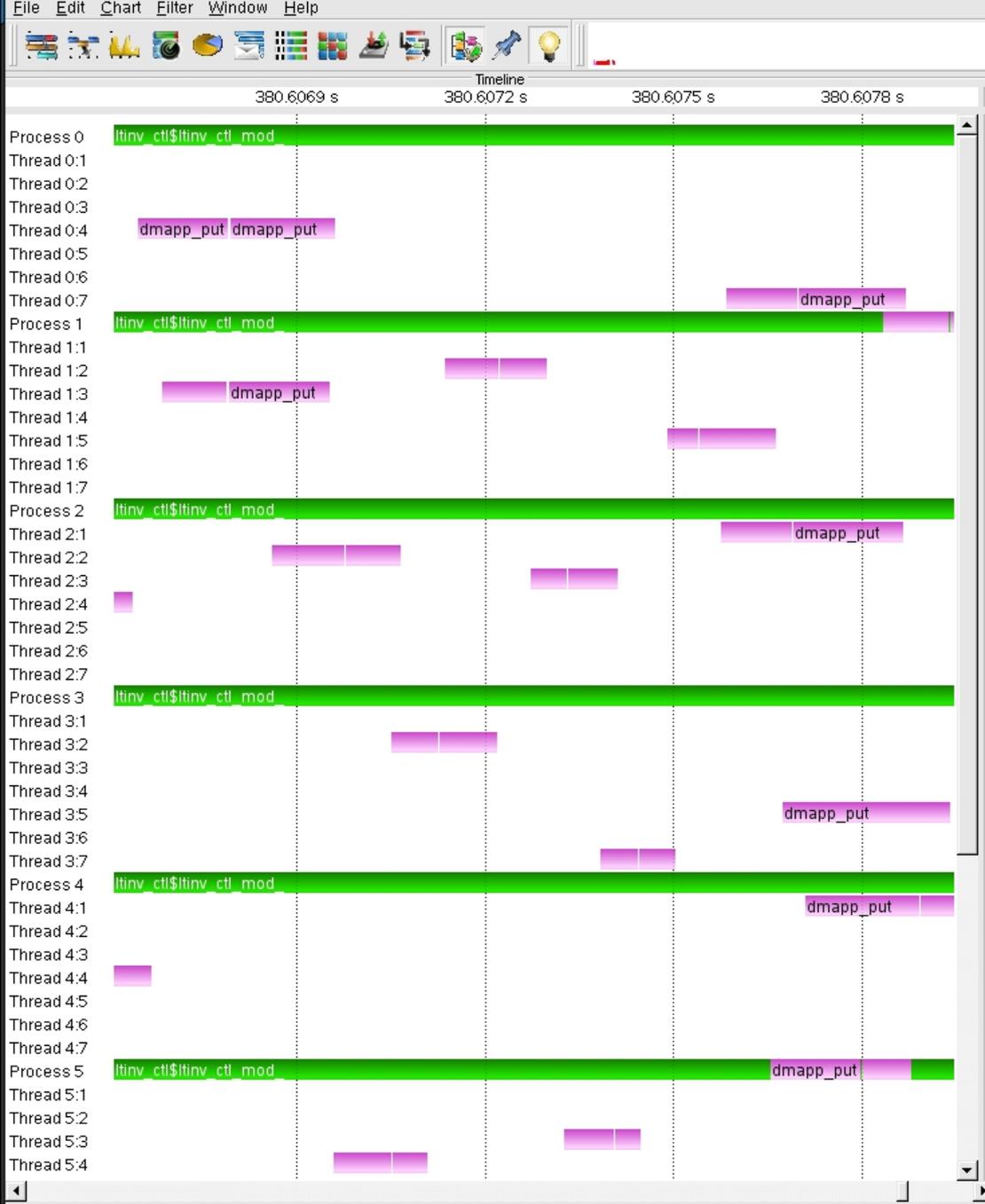




All Processes

Function	Min Inclusive Time	Max Inclusive Time
Application	0.000 s	25.866 ms
DMAPP	0.000 s	1.013 ms



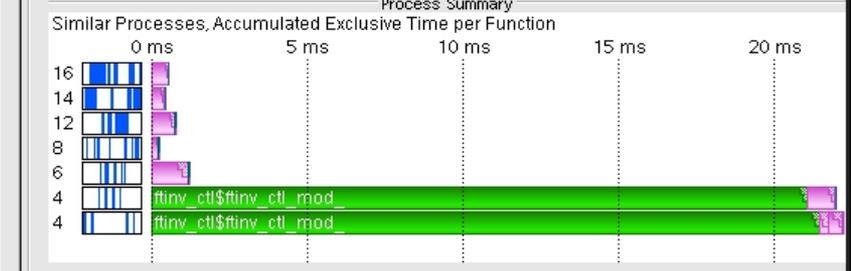
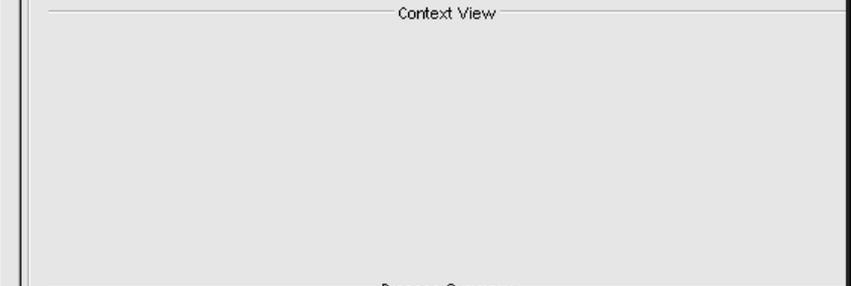
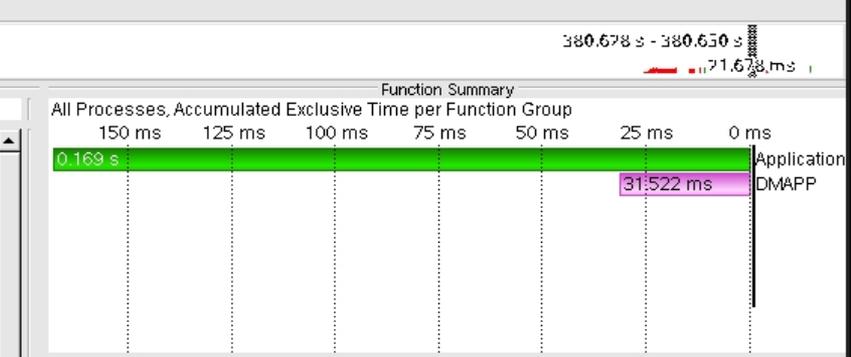
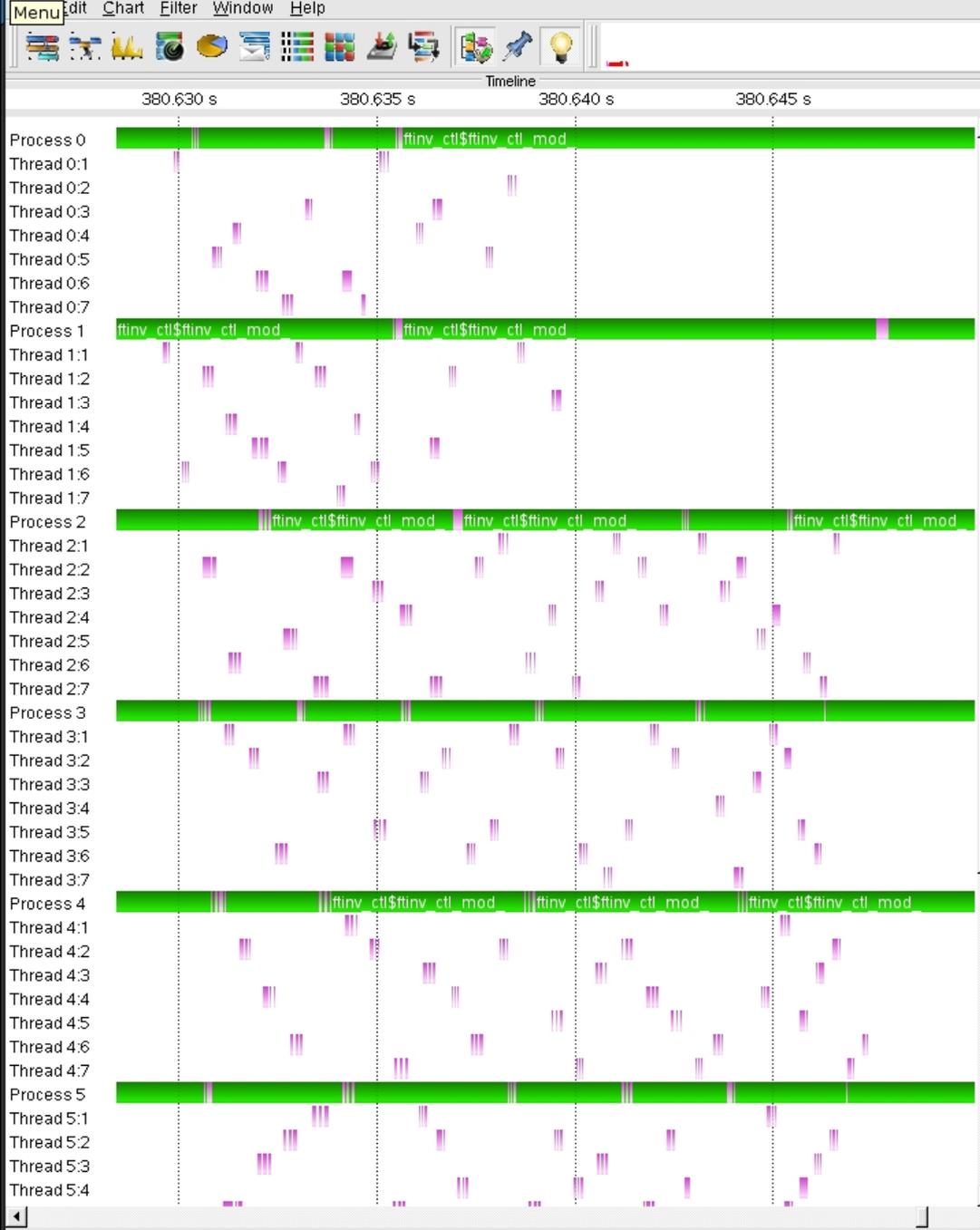


Call Tree

All Processes

Function	Min Inclusive Time	Max Inclusive Time
ltinv_ctl\$ltinv_ctl_mod	0.000 s	12.193 ms
dmapp_put	0.000 s	308.858 μs



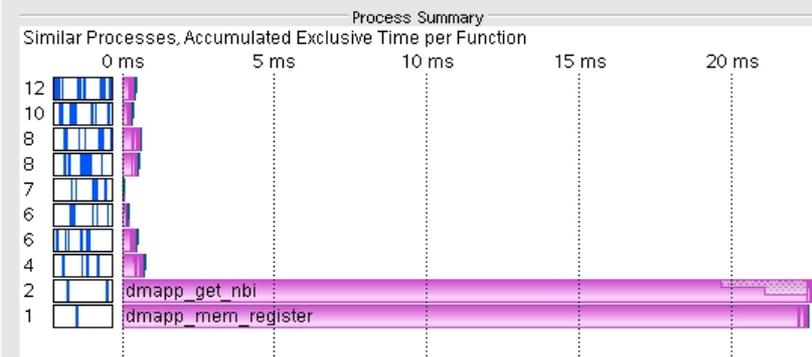
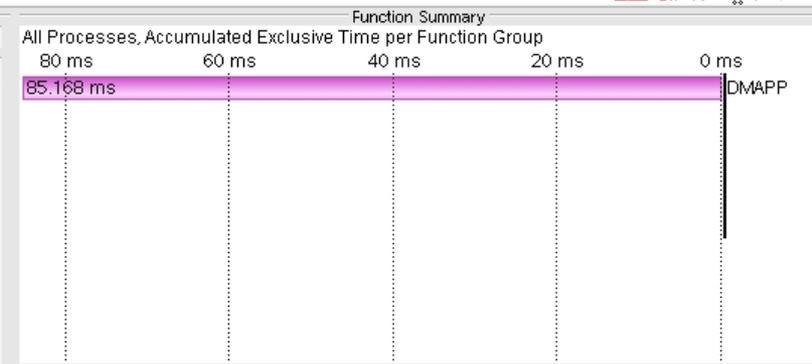
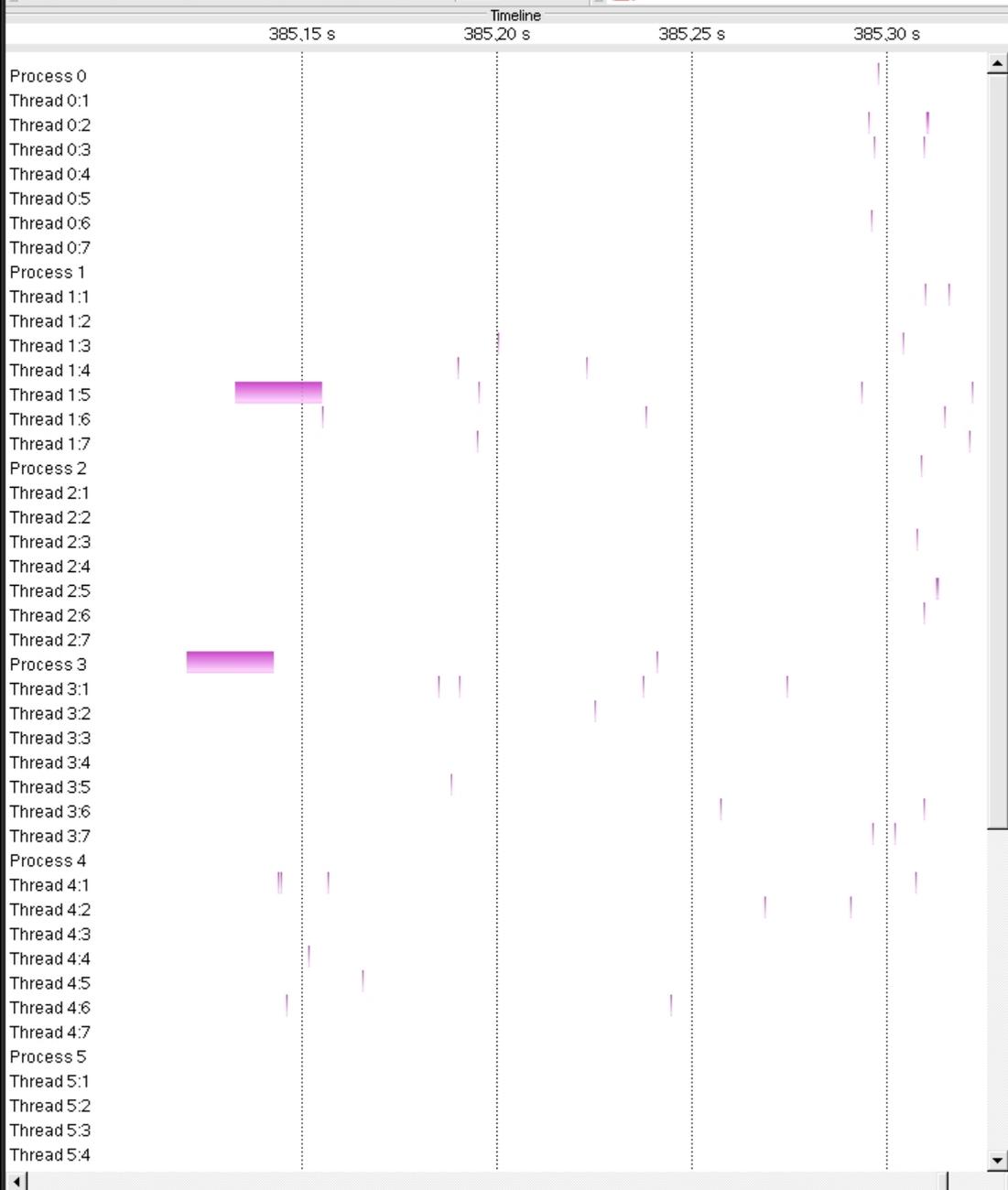


Call Tree

All Processes

Function	Min Inclusive Time	Max Inclusive Time
ftinv_ctl\$ftinv_ctl_mod	0.000 s	50.984 ms
dmapp syncid wait	0.000 s	5.377 µs
dmapp set rma attrs	0.000 s	6.327 µs
dmapp put	0.000 s	1.158 ms
dmapp get rma attrs	0.000 s	11.596 µs
dmapp get nb	0.000 s	10.102 µs

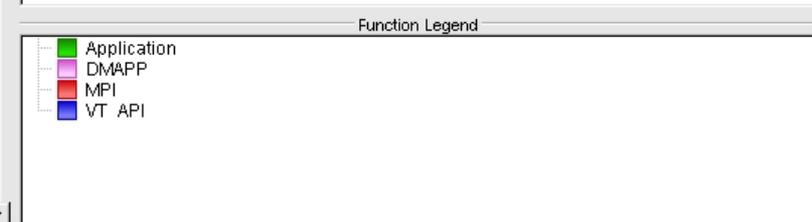




Call Tree

All Processes

Function	Min Inclusive Time	Max Inclusive Time
dmapp set rma attrs	0.000 s	109.670 µs
dmapp mem register	0.000 s	22.190 ms
dmapp get rma attrs	0.000 s	175.998 µs
dmapp get nbi	0.000 s	22.464 ms
dmapp afadd qw nbi	0.000 s	16.133 µs



Other Fortran 2008 compilers

- License finally agreed with IBM
 - ECMWF will install xlf v14 compiler on Power7
 - Only took 1 year from first inquiry (pre-CRESTA)
 - Subject to non-disclosure
 - Am sure we will be granted permission to present and publish results if they are good
 - Plan is first to test IFS RAPS12 with this compiler
- Promoting need for Fortran 2008 to vendors is important
- Intel ?
- Fujitsu ?
- gfortran ?
- PGI ?

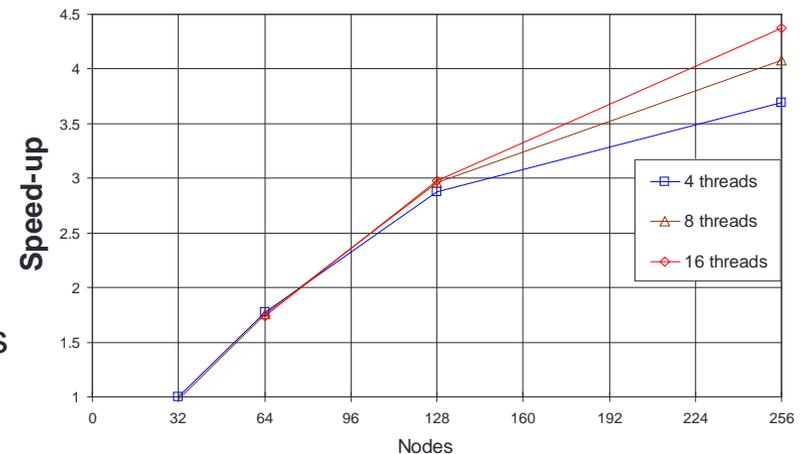
Using HECToR

- Moved IFS from cce=8.0.3 to cce=8.0.6
 - To pick up fix to random hangs at start of job
 - Job would run to cp time limit without executing a single application statement
 - Refunded lost KAu's
 - 8.0.6 also fixed a couple of random coarray runtime failures
 - Thanks to CRAY for providing a good compiler release
- Multiple aprun's in high core count jobs (10K to 64K cores)
 - To improve overall system resource utilization
 - Small, medium and large batched jobs
 - Some waste due to unused cores in each job
 - Promise of refund (more KAUs) at some time in future

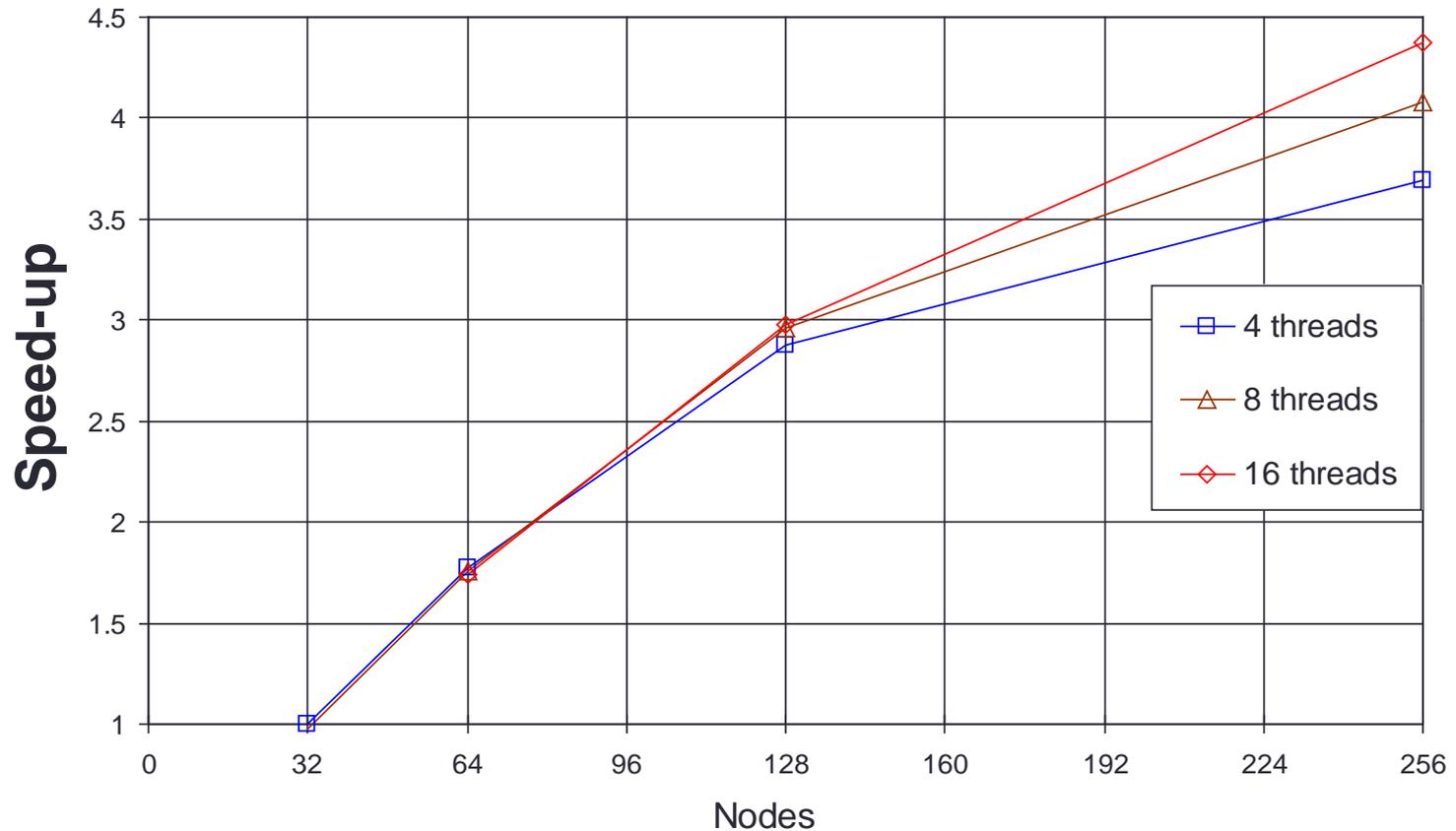
Hybrid runtime support - IFS

- Initial IFS MPI implementation 1994-1996
- Hybrid MPI/OpenMP implementation ~1999
 - OpenMP implementation at highest level
 - Single parallel regions for each of physics, radiation scheme, dynamics, Legendre transforms, Fourier transforms and Fourier space computations
 - Schedule dynamic used in most parallel regions
- Hybrid implementation benefits
 - About 20 percent performance improvement at scale
 - Huge memory savings, memory use reduces linearly with number of OpenMP threads
- Next evolutionary step: use of Fortran 2008 coarrays to
 - Overlap computation with communication in transpositions
 - Fourier space <-> Spectral space comms, overlapped with Legendre transforms
 - grid point space <-> Fourier space comms, overlapped with FFTs and Fourier space computations
 - Reduce total halo communication in semi-Lagrangian scheme
 - Dominant coarray communications in OpenMP parallel regions

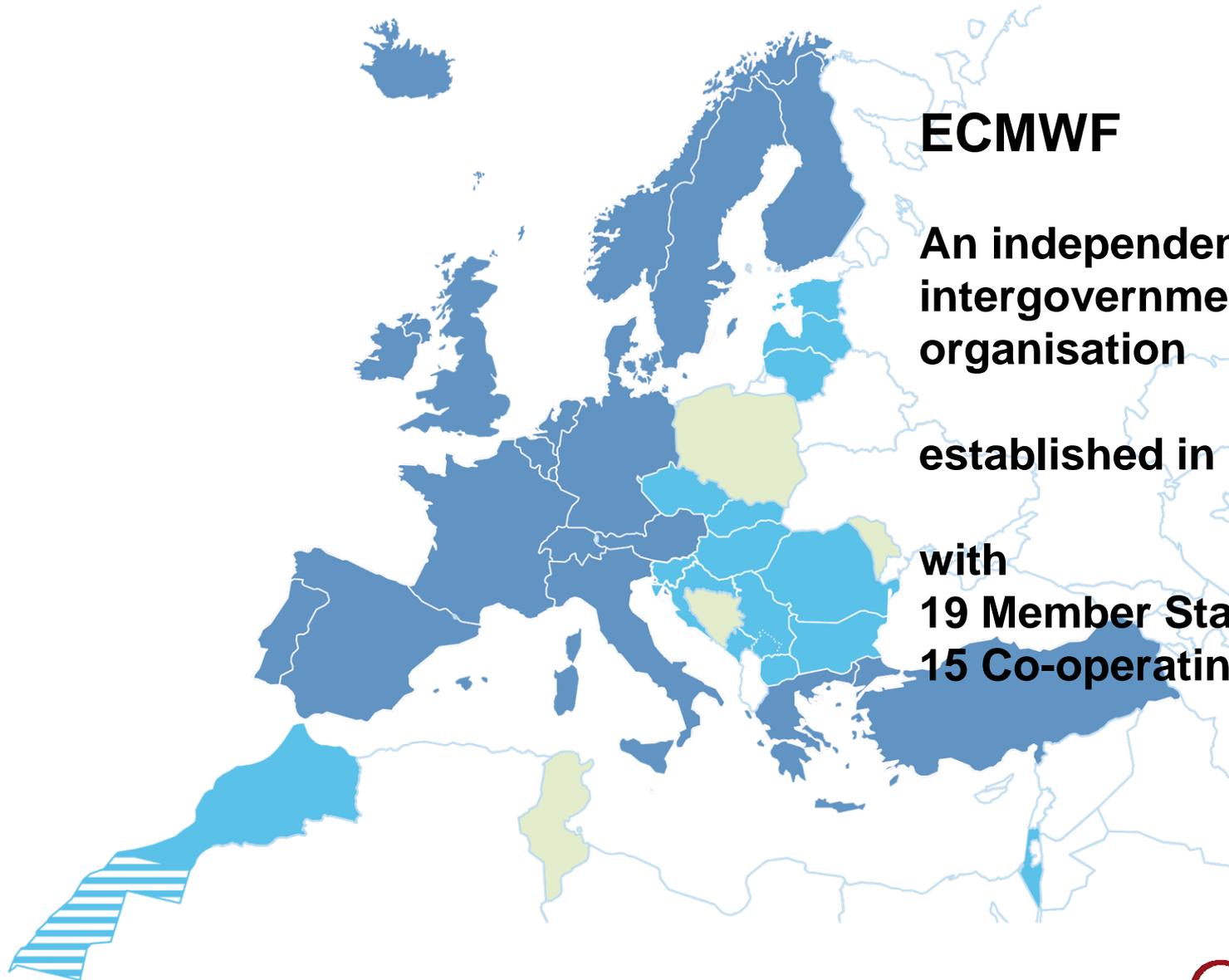
OpenMP for IFS T1279L91 model
on IBM Power6 (~2009)



OpenMP for IFS T1279L91 model on IBM Power6 (~2009)



■ Member States ■ Co-operating States ■ Under negotiation



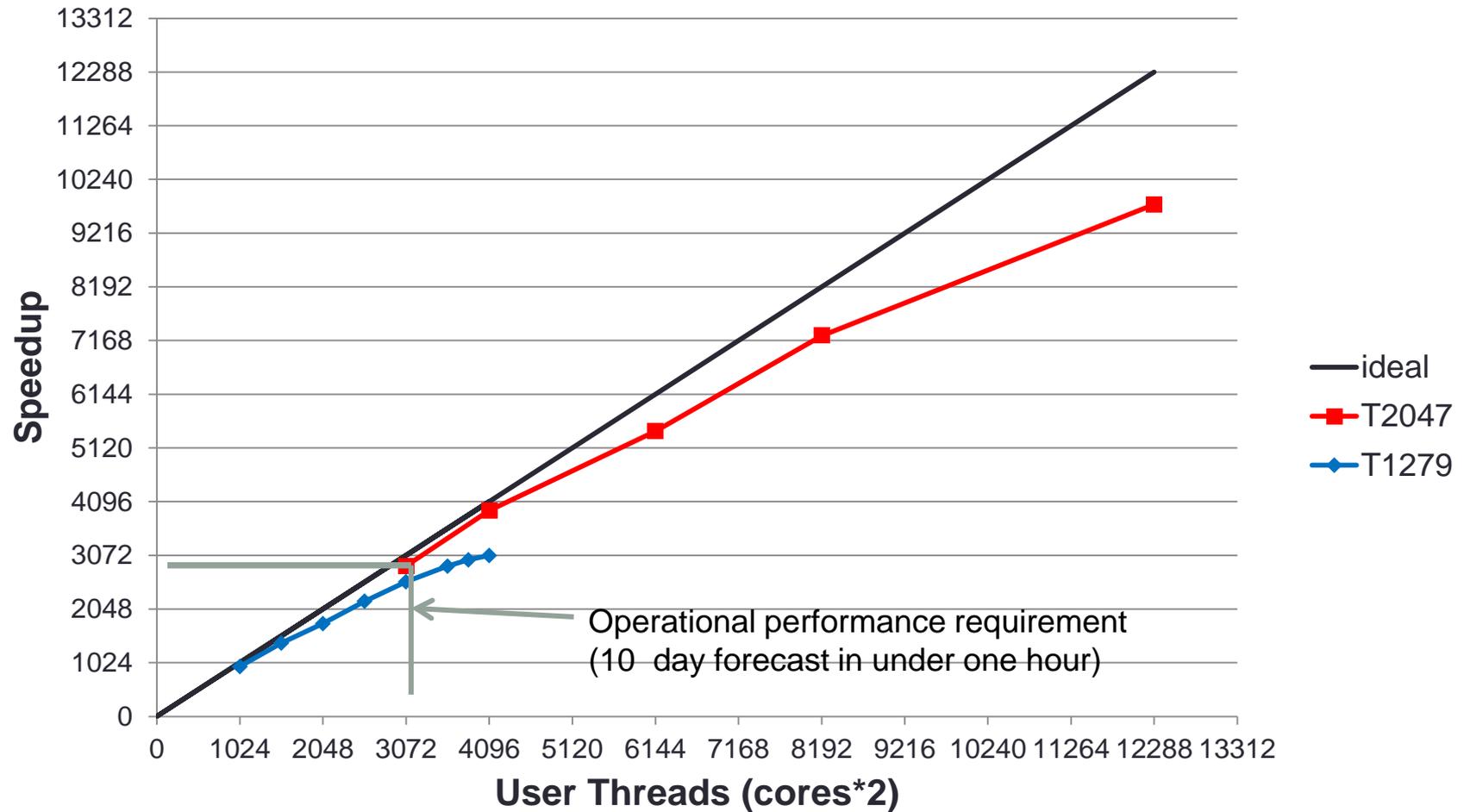
ECMWF

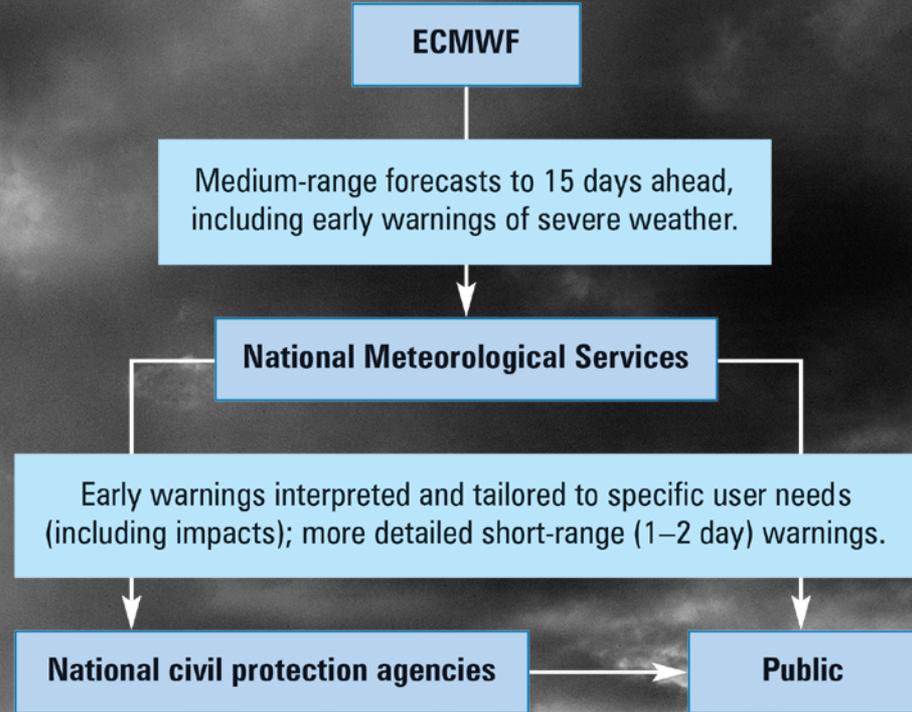
An independent
intergovernmental
organisation

established in 1975

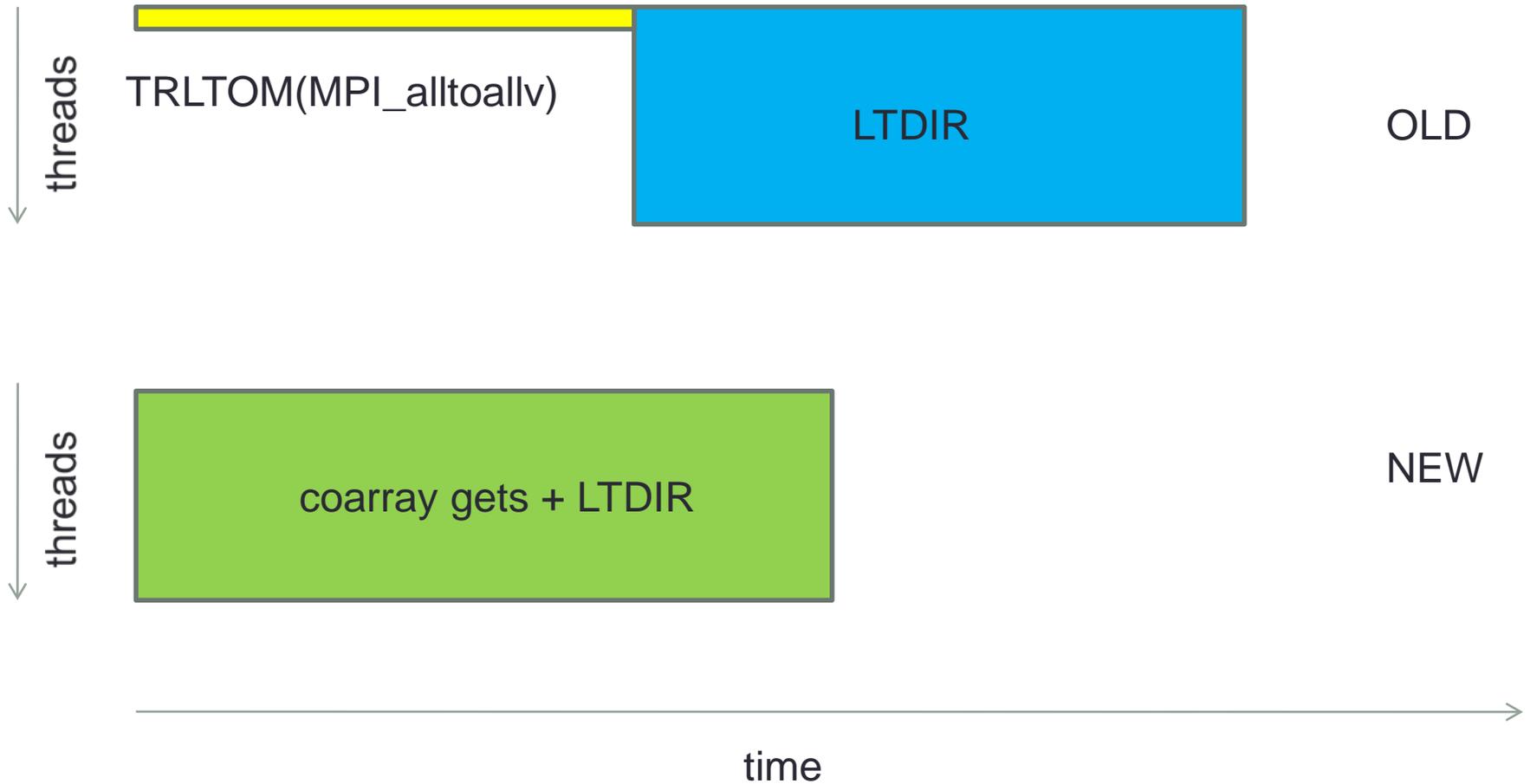
with
19 Member States
15 Co-operating States

IFS model speedup on IBM Power6 (~2010)



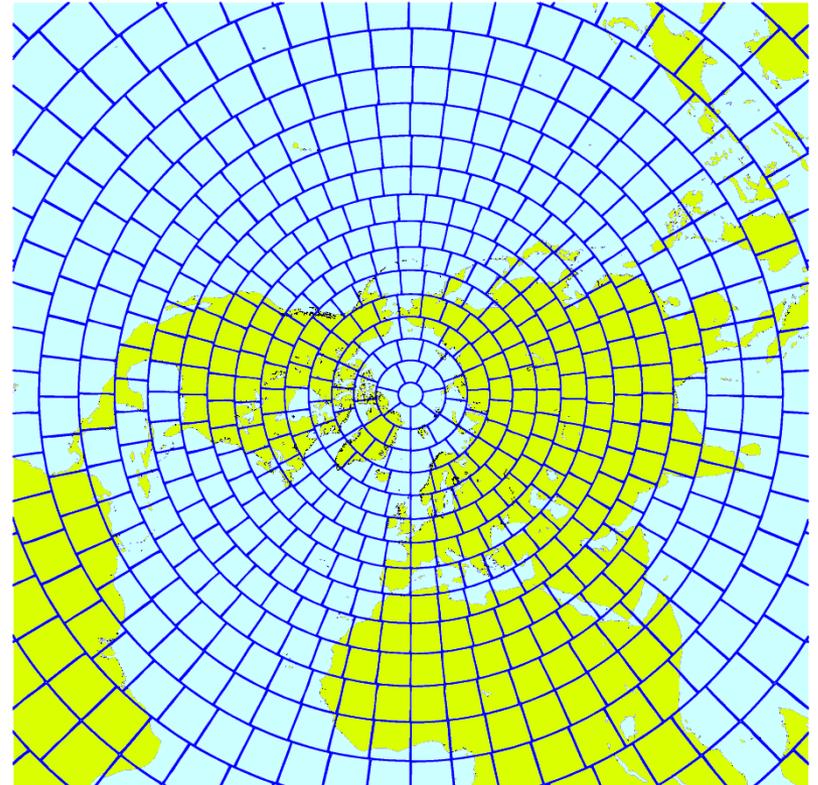


Overlap Legendre transforms with associated transpositions/2



IFS grid point space: “EQ_REGIONS” partitioning for 1024 MPI tasks

Each MPI task has an equal number of grid points



LTINV recoding

COMPUTE COMMUNICATION

```

!$OMP PARALLEL DO SCHEDULE(DYNAMIC,1) PRIVATE(JM,IM)
DO JM=1,D%NUMP
  IM = D%MYMS(JM)
  CALL LTINV(IM,JM,KF_OUT_LT,KF_UV,KF_SCALARS,KF_SCDERS,ILEI2, IDIM1 , &
    & PSPVOR,PSPDIV,PSPSCALAR , &
    & PSPSC3A,PSPSC3B,PSPSC2 , &
    & KFLDPTRUV,KFLDPTRSC,FSPGL_PROC)
ENDDO
!$OMP END PARALLEL DO
DO J=1,NPRTRW
  ILENS(J) = D%NLTSFTB(J)*IFIELD
  IOFFS(J) = D%NSTAGT0B(J)*IFIELD
  ILENR(J) = D%NLTSGTB(J)*IFIELD
  IOFFR(J) = D%NSTAGT0B(D%MSTABF(J))*IFIELD
ENDDO
CALL MPL_ALLTOALLV(PSENBUFF=FOUBUF_IN,KSENDCOUNTS=ILENS, &
  & PRECVBUF=FOUBUF,KRECVCOUNTS=ILENR, &
  & KSENDDISPL=IOFFS,KRECVDISPL=IOFFR, &
  & KCOMM=MPL_ALL_MS_COMM,CDSTRING='TRMTOL:')

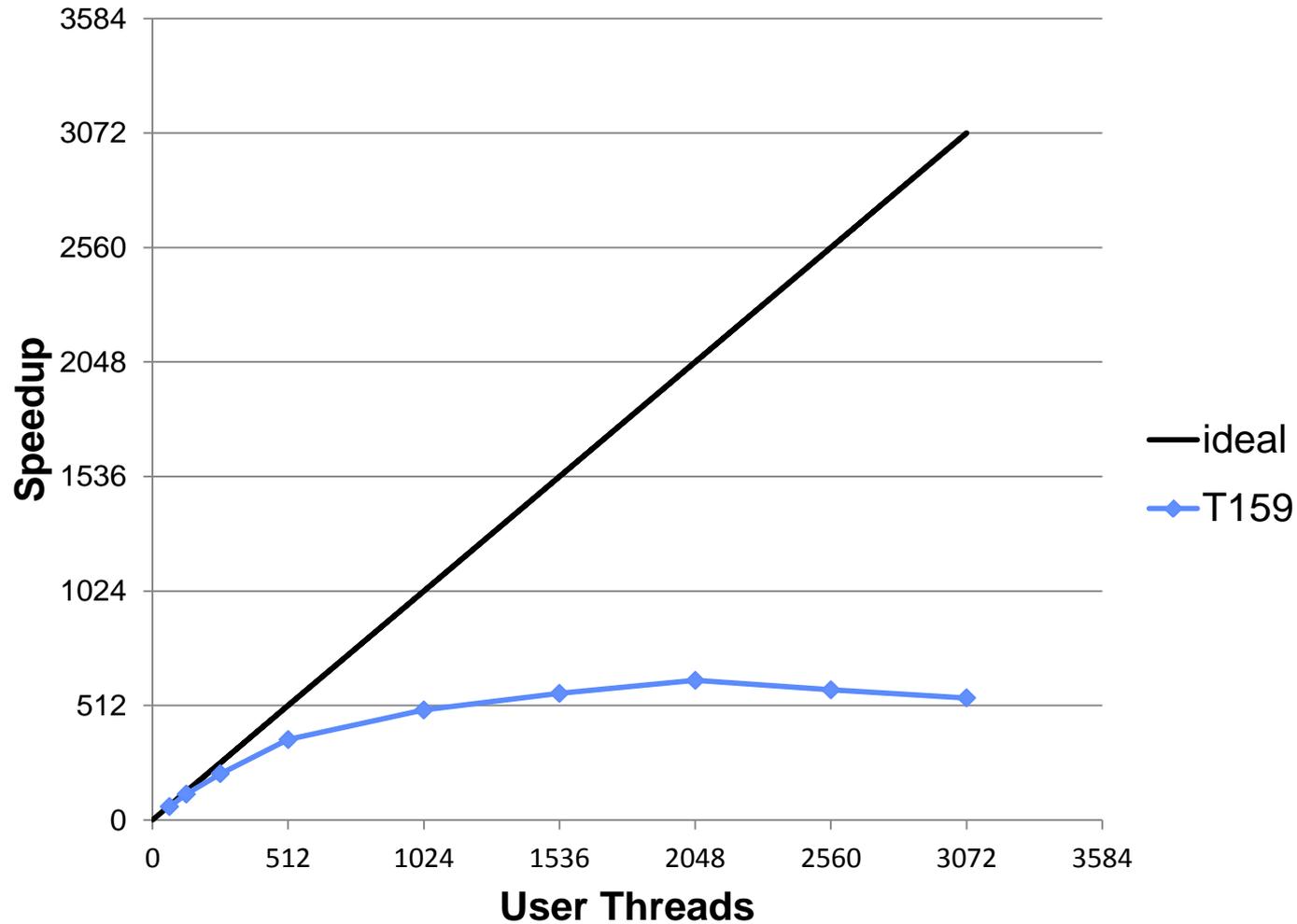
!$OMP PARALLEL DO SCHEDULE(DYNAMIC,1) PRIVATE(JM,IM,JW,IPE,ILEN,ILENS,IOFFS,IOFFR)
DO JM=1,D%NUMP
  IM = D%MYMS(JM)
  CALL LTINV(IM,JM,KF_OUT_LT,KF_UV,KF_SCALARS,KF_SCDERS,ILEI2, IDIM1 , &
    & PSPVOR,PSPDIV,PSPSCALAR , &
    & PSPSC3A,PSPSC3B,PSPSC2 , &
    & KFLDPTRUV,KFLDPTRSC,FSPGL_PROC)
DO JW=1,NPRTRW
  CALL SET2PE(IPE,0,0,JW,MYSETV)
  ILEN = D%NLEN_M(JW,1,JM)*IFIELD
  IF(ILEN > 0)THEN
    IOFFS = (D%NSTAGT0B(JW)+D%NOFF_M(JW,1,JM))*IFIELD
    IOFFR = (D%NSTAGT0BW(JW,MYSETW)+D%NOFF_M(JW,1,JM))*IFIELD
    FOUBUF_C(IOFFR+1:IOFFR+ILEN)[IPE]=FOUBUF_IN(IOFFS+1:IOFFS+ILEN)
  ENDIF
  ILENS = D%NLEN_M(JW,2,JM)*IFIELD
  IF(ILENS > 0)THEN
    IOFFS = (D%NSTAGT0B(JW)+D%NOFF_M(JW,2,JM))*IFIELD
    IOFFR = (D%NSTAGT0BW(JW,MYSETW)+D%NOFF_M(JW,2,JM))*IFIELD
    FOUBUF_C(IOFFR+1:IOFFR+ILENS)[IPE]=FOUBUF_IN(IOFFS+1:IOFFS+ILENS)
  ENDIF
ENDDO
ENDDO
!$OMP END PARALLEL DO
SYNC IMAGES(D%NMYSETW)
FOUBUF(1:IBLEN)=FOUBUF_C(1:IBLEN)[MYPROC]

```

ORIGINAL
code

NEW
code

T159 model scaling: small model with 'large' number of user threads (4 threads per task)



IFS Semi-Lagrangian Comms

- **SL comms scaling limited by**

- constant width halo for u,v,w (400 m/s x time step)
- Halo volume communicated, which is a function of wind speed and direction in locality of each task

- **'Halo-lite' approach tested (2010)**

- Only get (using MPI) grid columns from neighbouring tasks that your task needs, i.e. only the red points
- Requires more MPI communication steps (e.g. mid-point, departure point)
- No faster than original approach due to overheads of above

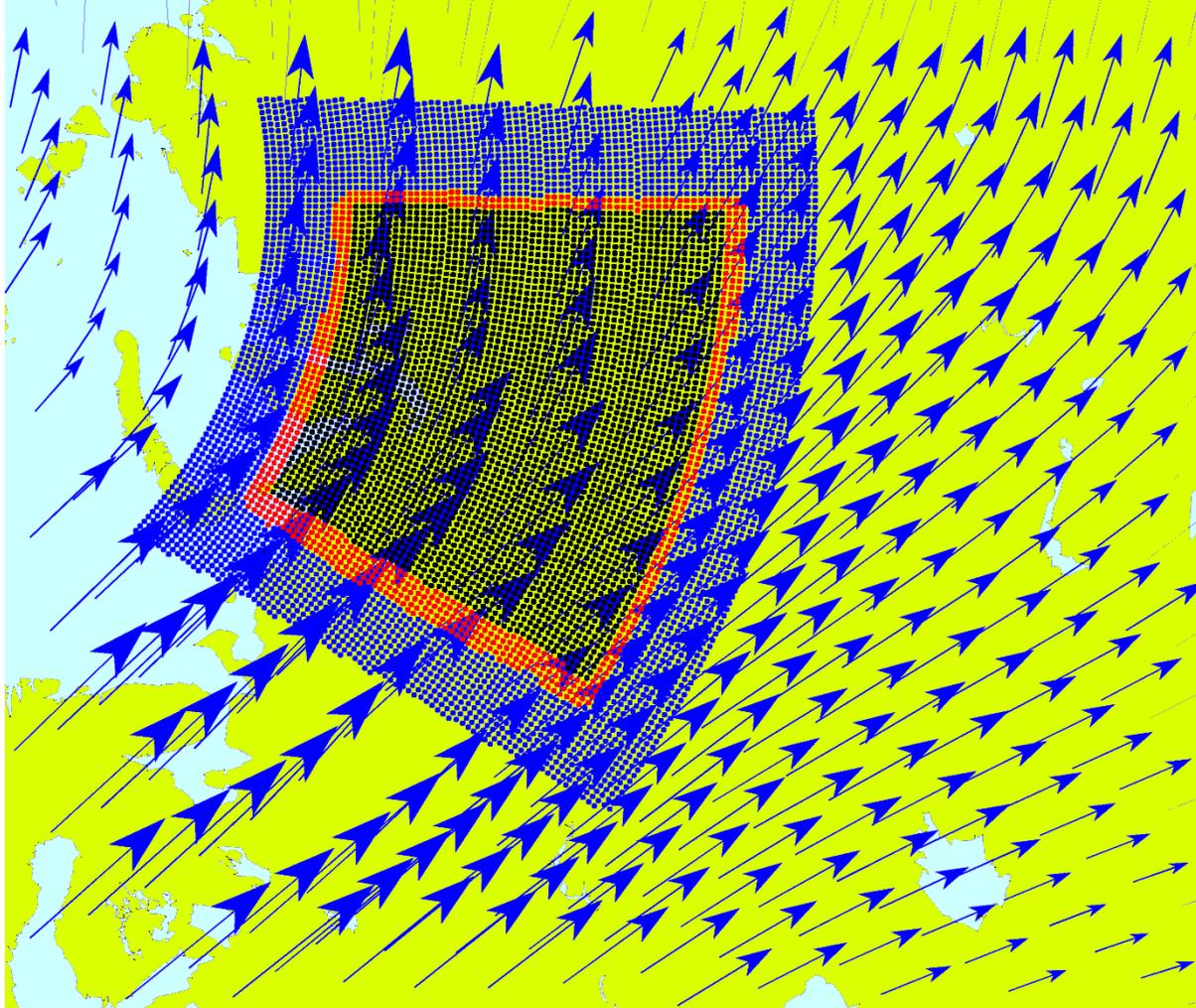
- **CRESTA optimisation using F2008 coarrays (2012)**

- Only get grid columns from neighbouring tasks that your task needs, i.e. only the red points
- Do the above in the context of an OpenMP parallel region; overlapping interpolations for determining the departure point & mid-point and interpolations at these points

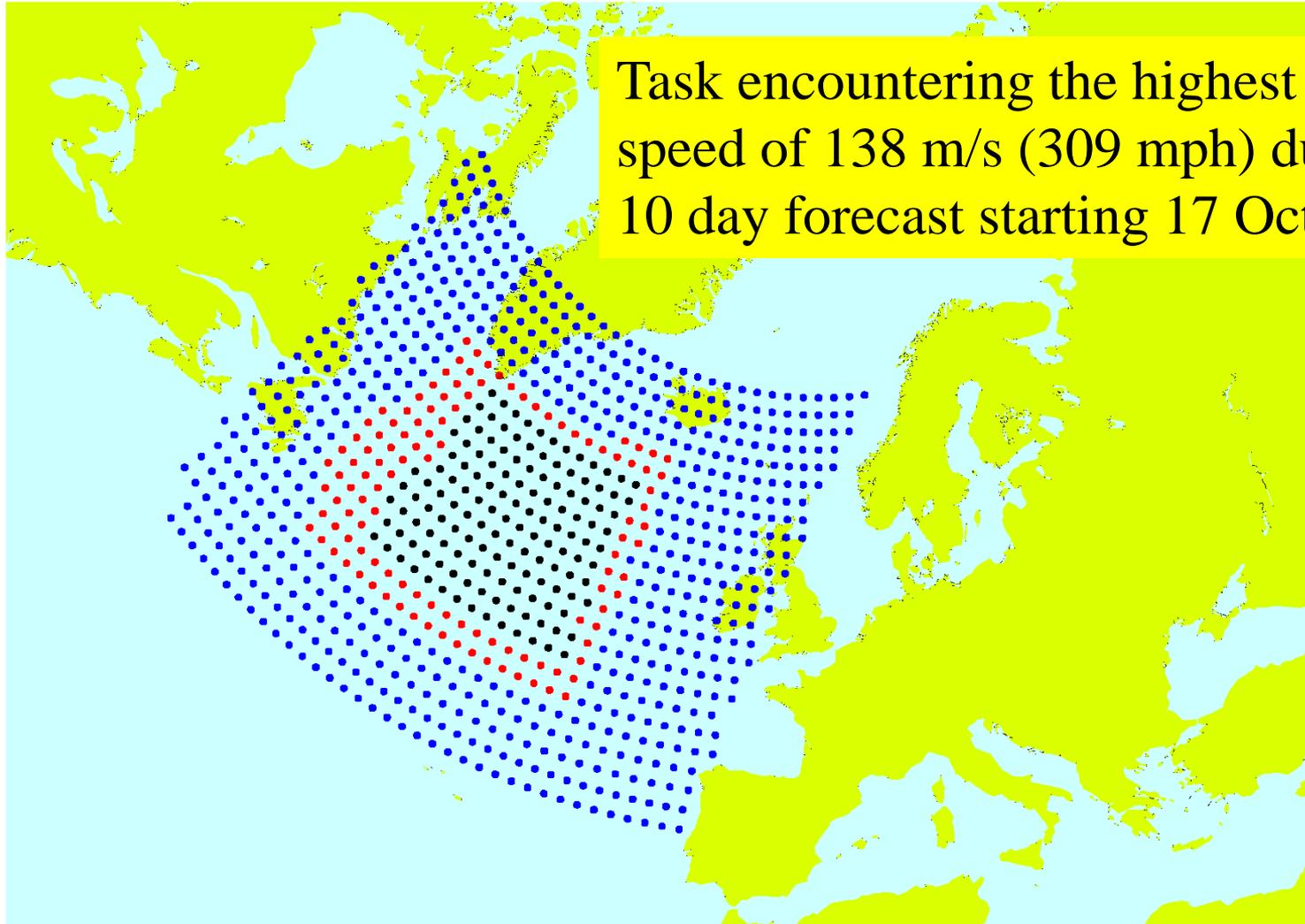
wind plot

Friday 15 October 2004 12UTC ECMWF Forecast t+0 VT: Friday 15 October 2004 12UTC Model Level 1 U velocity/V velocity

25.0m/s



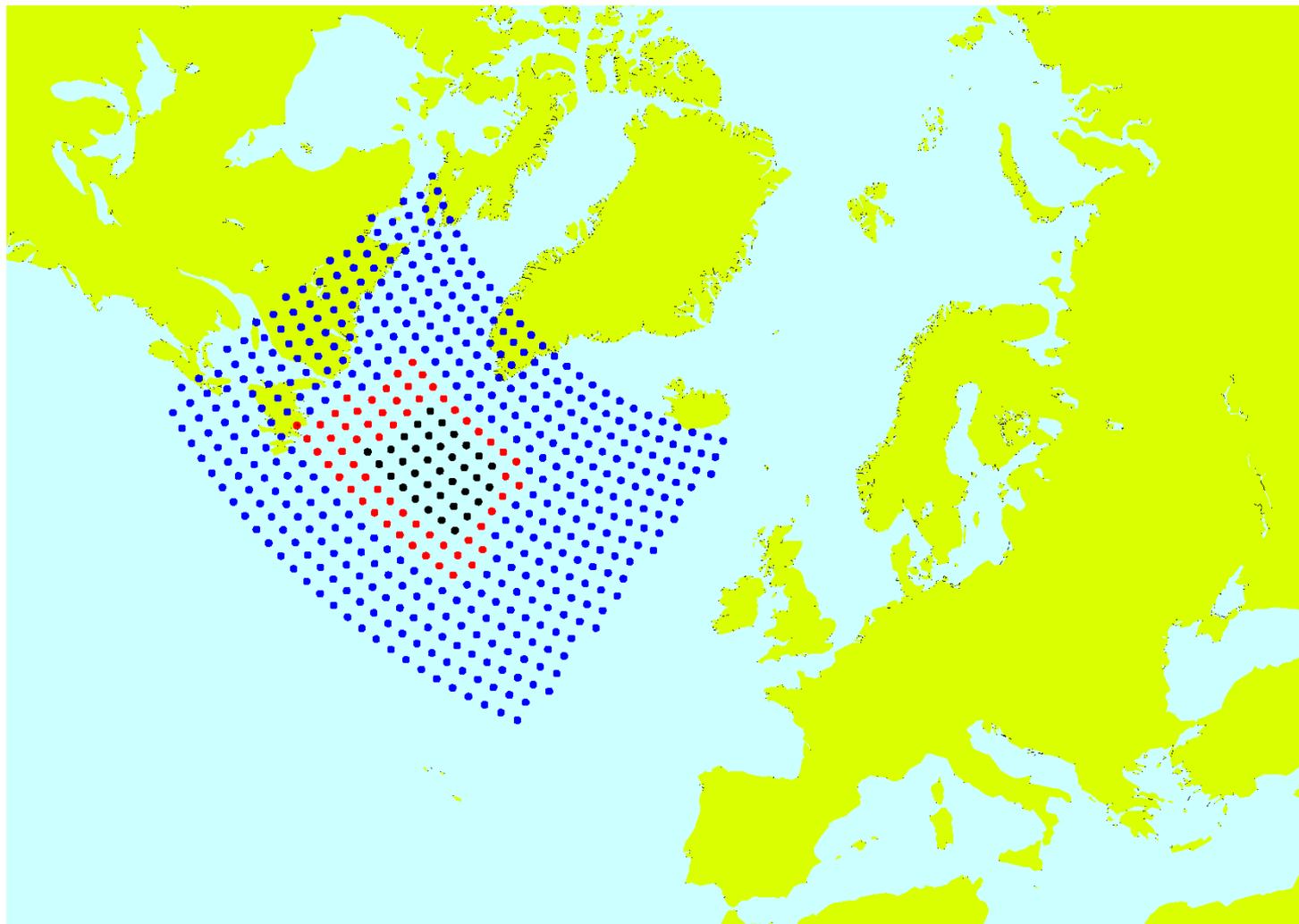
T159 model task 37 of 256 tasks



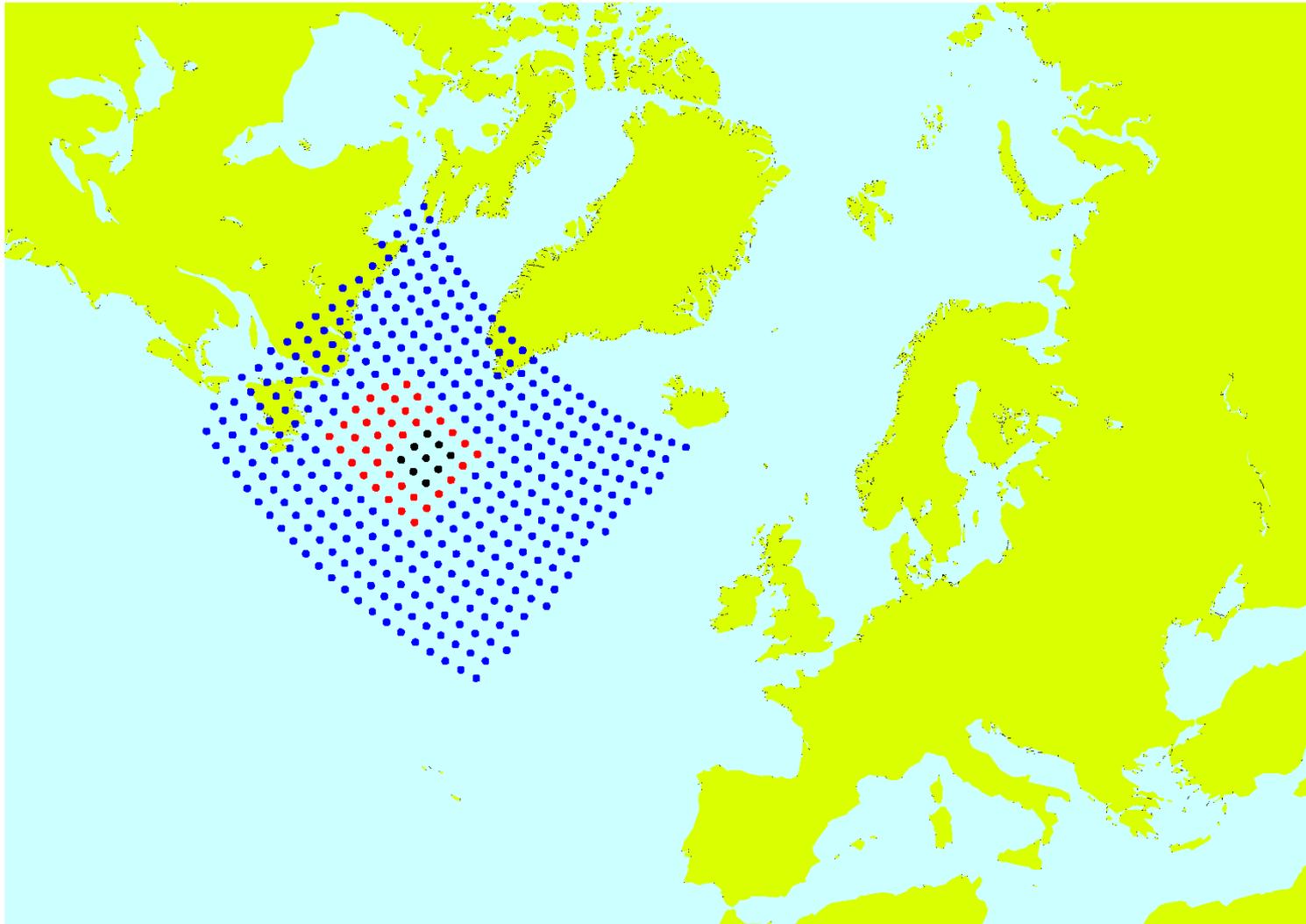
Task encountering the highest wind speed of 138 m/s (309 mph) during a 10 day forecast starting 17 Oct 2010



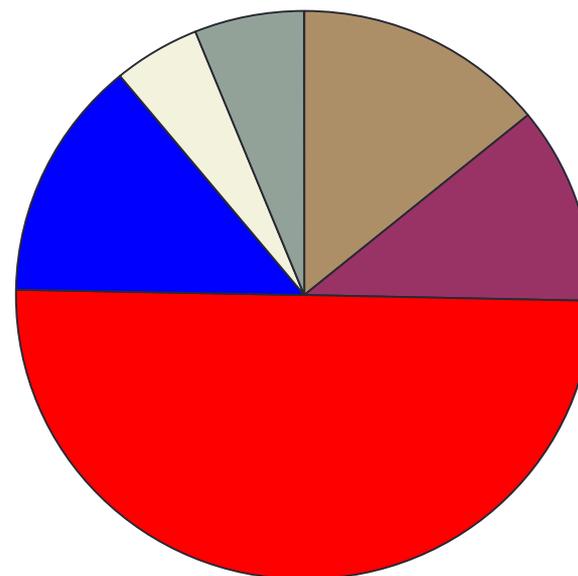
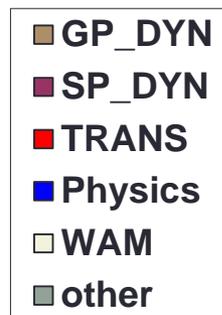
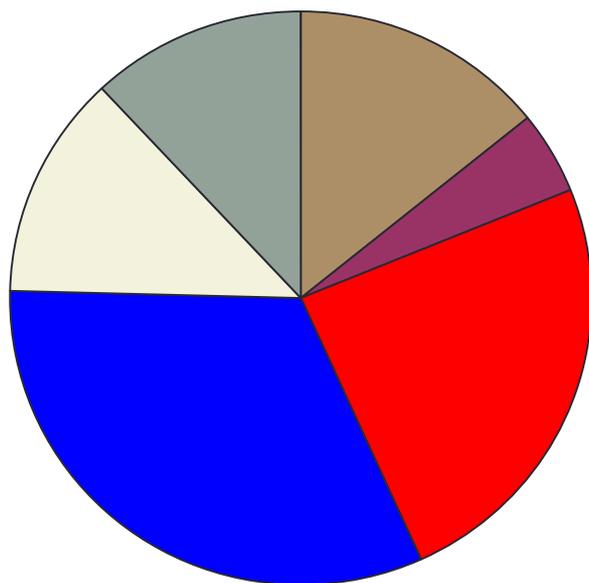
T159 model task 128 of 1024 tasks



T159 model task 462 of 4096 tasks



Computational Cost at T2047 and T3999



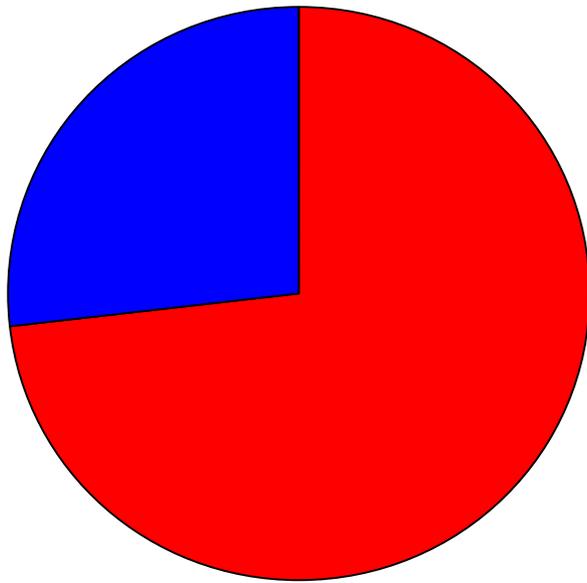
Hydrostatic T_{L2047}

Tstep=450s, 5.8s/Tstep
With 256x16 ibm_power6

Non-Hydrostatic T_{L3999}

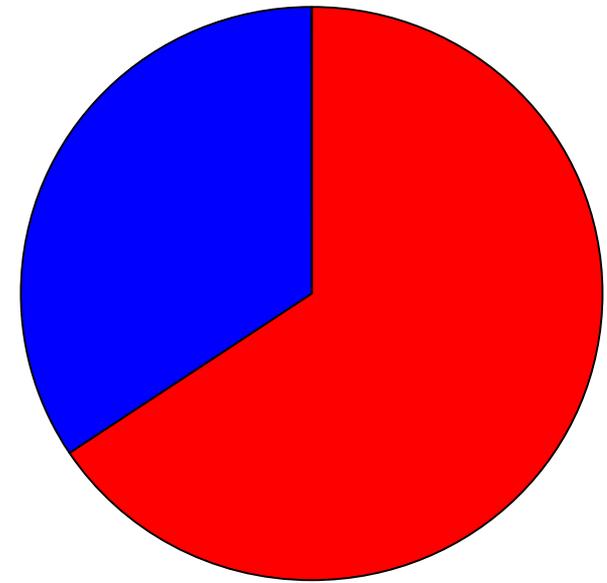
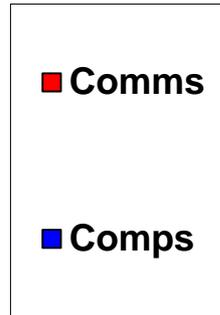
Tstep=240s, 13.6s/Tstep
With 512x16 ibm_power6

Breakdown of TRANS cost: Computations vs. Communications



HT_L2047 ~2015

Data sent/received: 117.8GB/s



NHT_L3999 ~2020

Data sent/received: 289.6GB/s

IFS Introduction – A history

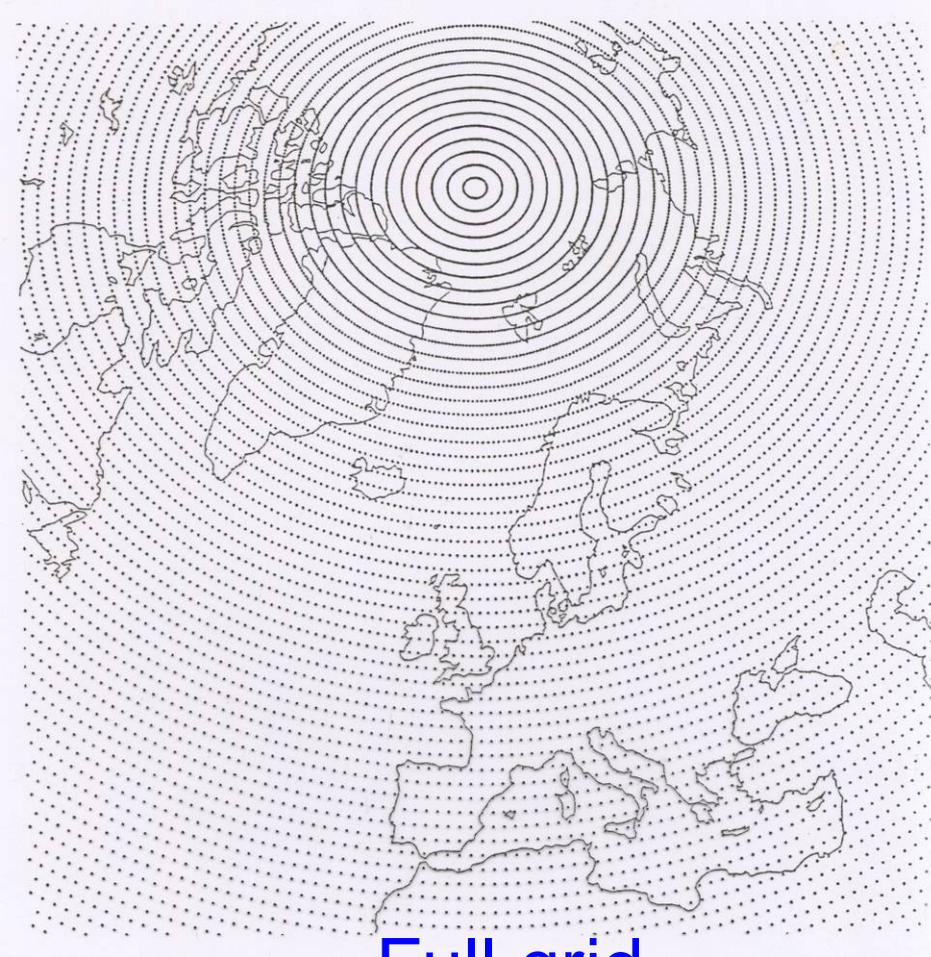
- **Resolution increases of the deterministic 10-day medium-range Integrated Forecast System (IFS) over ~25 years at ECMWF:**
 - 1987: T 106 (~125km)
 - 1991: T 213 (~63km)
 - 1998: T_L 319 (~63km)
 - 2000: T_L 511 (~39km)
 - 2006: T_L 799 (~25km)
 - 2010: T_L 1279 (~16km)

Introduction – A history

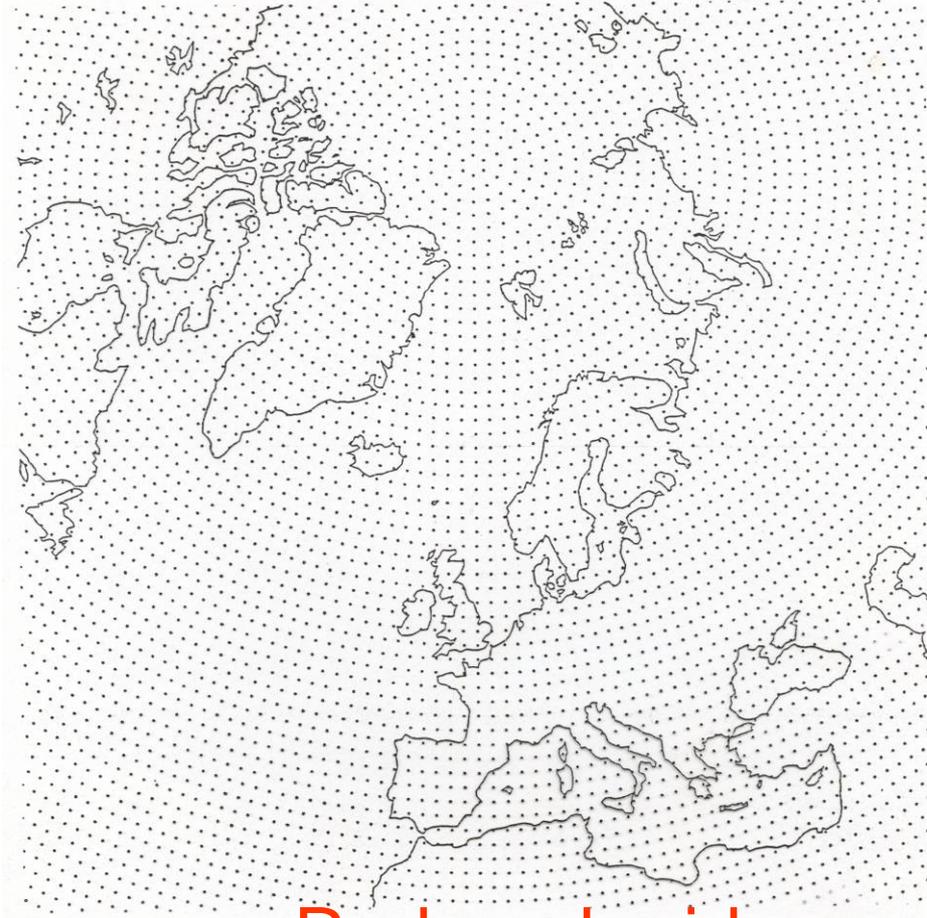
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 - 1998: T_L319 (~63km)
 - 2000: T_L511 (~39km)
 - 2006: T_L799 (~25km)
 - 2010: T_L1279 (~16km)
 - **2015?:** T_L2047 (~10km)
 - **2020-???:** (~1-10km) Non-hydrostatic, cloud-permitting, substantially different cloud-microphysics and turbulence parametrization, substantially different dynamics-physics interaction ?

The Gaussian grid

About 30% reduction in number of points



Full grid



Reduced grid

Reduction in the number of Fourier points at high latitudes is possible because associated Legendre functions are very small near the poles for large m .



(Adaptive) Mesh Refinement

- The IFS model is **inherently based on a fixed structured mesh** due to the link between the spectral representation and the position of the grid-points (zero's of the ordinary Legendre polynomials), which makes selective mesh refinement (adaptive or not) difficult to achieve.
- “AMR” possibilities: coexisting global multigrids, physics/ dynamics on different grids, wavelet-collocation methods, ...: Costly investment both in RD and computational cost
- Hence it is of strategic importance to **understand the added-value of adaptive or static mesh refinement** for multiscale global NWP and climate prediction !

Nonhydrostatic IFS (NH-IFS)

Bubnová et al. (1995); Bénard et al. (2004), Bénard et al. (2005), Bénard et al. (2010), Wedi et al. (2009), Yessad and Wedi (2011)

- **Arpégé/ALADIN/Arome/HIRLAM/ECMWF nonhydrostatic dynamical core, which was developed by Météo-France and their ALADIN partners and later incorporated into the ECMWF model and also adopted by HIRLAM.**

Numerical solution

- Two-time-level, semi-implicit, semi-Lagrangian.
- Semi-implicit procedure with two reference states, with respect to gravity and acoustic waves, respectively.
- The resulting **Helmholtz equation** can be solved (subject to some constraints on the vertical discretization) with a **direct spectral method**, that is, a mathematical separation of the horizontal and vertical part of the linear problem in spectral space, with the remainder representing at most a pentadiagonal problem of dimension $NLEV^2$. Non-linear residuals are treated explicitly (or iteratively implicitly)!

(Robert, 1972; Bénard et al 2004,2005,2010)

The spectral transform method

Eliassen et. al (1970), Orszaag (1970)

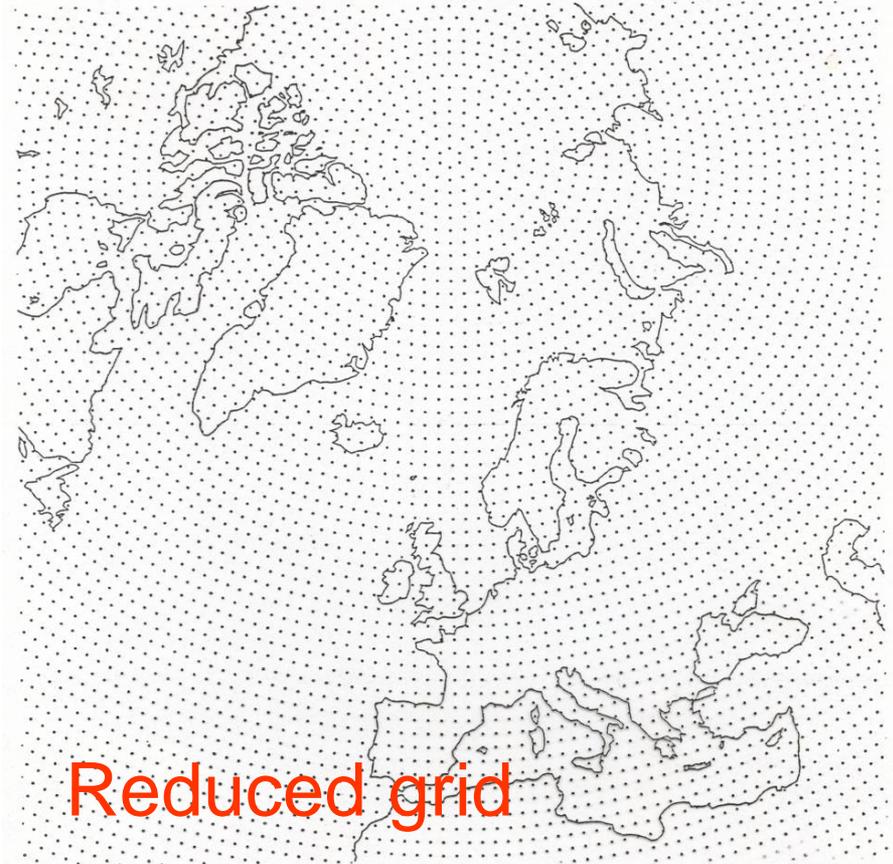
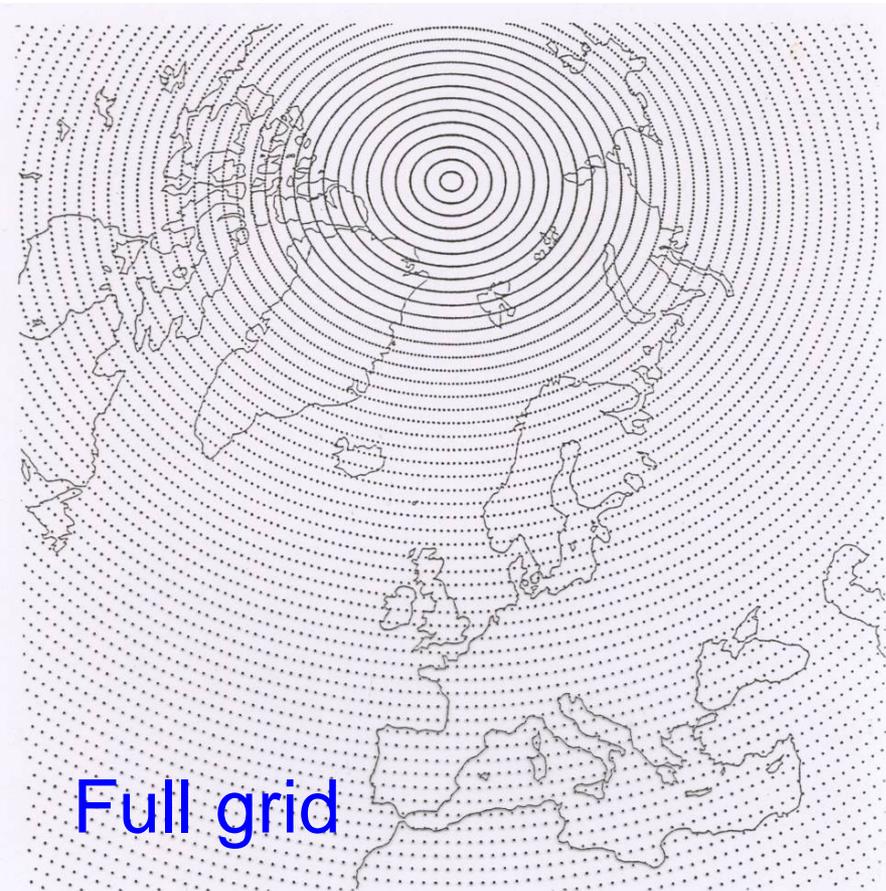
Applied at ECMWF for the last 30 years ...

**Spectral semi-Lagrangian semi-implicit
(compressible) a viable option ?**

- Computational efficiency** on future MPP architectures ?
- Accuracy** at cloud-resolving scales ?
- Suitability for** the likely mixture of medium and high resolution ensembles and **ultra-high resolution forecasts** ?

The Gaussian grid

About 30% reduction in number of points

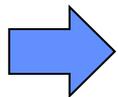


Reduction in the number of Fourier points at high latitudes is possible because associated Legendre polynomials are very small near the poles for large m .

Note: number of points nearly equivalent to quasi-uniform icosahedral grid cells of the ICON model.

Cost of the spectral transform method

- FFT can be computed as $C*N*\log(N)$ where C is a small positive number and N is the cut-off wave number in the triangular truncation.
- Ordinary Legendre transform is $O(N^2)$ but can be combined with the fields/levels such that the arising matrix-matrix multiplies make use of the highly optimized BLAS routine DGEMM.
- But overall cost is $O(N^3)$ for both memory and CPU time requirements.



Desire to use a fast Legendre transform where the cost is proportional to $C*N*\log(N)$ with $C \ll N$

and thus overall cost $N^2*\log(N)$

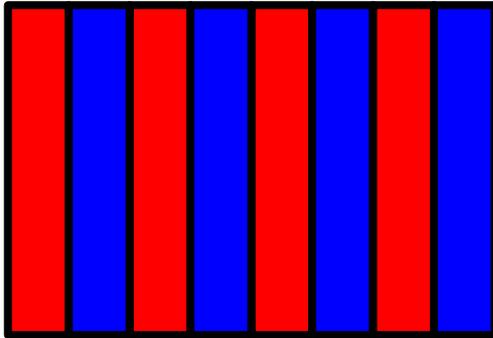
Fast Legendre Transform (FLT)

- The algorithm proposed in [\(Tygert, 2008,2010\)](#) suitably fits into the IFS transform library by simply replacing the matrix-matrix multiply DGEMM call with a BUTTERFLY_MATRIX_MULT call plus slightly more expensive pre-computations.
- (1) Instead of the recursive [Cuppen divide-and-conquer algorithm \(Tygert, 2008\)](#) we use the so called [butterfly algorithm \(O'Neil et al, 2009; Tygert, 2010\)](#) based on a matrix compression technique via rank reduction with a specified accuracy to accelerate the arising [matrix-vector/matrix multiplies \(sub-problems still use DGEMM\)](#).
- (2) We apply the matrix compression directly on the matrix of the associated polynomials, which reduces the required precomputations and **eliminates the need** to apply [FMM \(fast multipole method\)](#) accelerated interpolations. Notably, the latter were an essential part of the proposed FLT in [Suda and Takami \(2001\)](#).

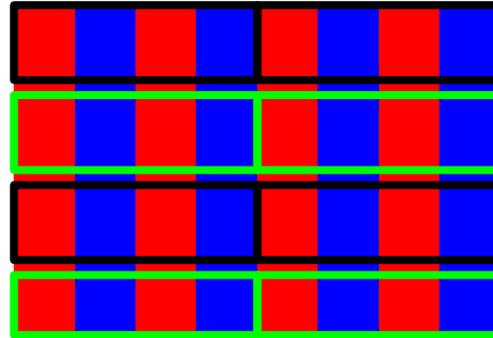
The butterfly compression

(O'Neil, Woolfe, Rokhlin, 2009; Tygert 2010)

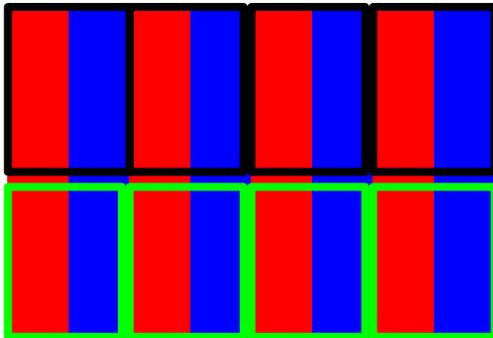
With each level l ,
double the columns
and half the rows



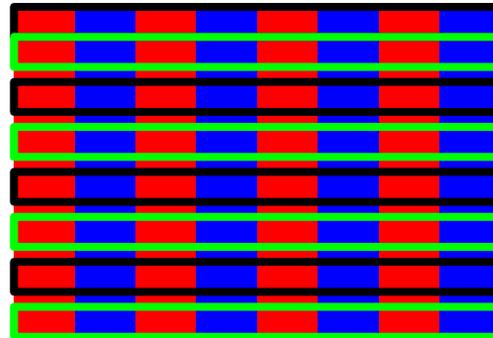
$l=0$



$l=2$



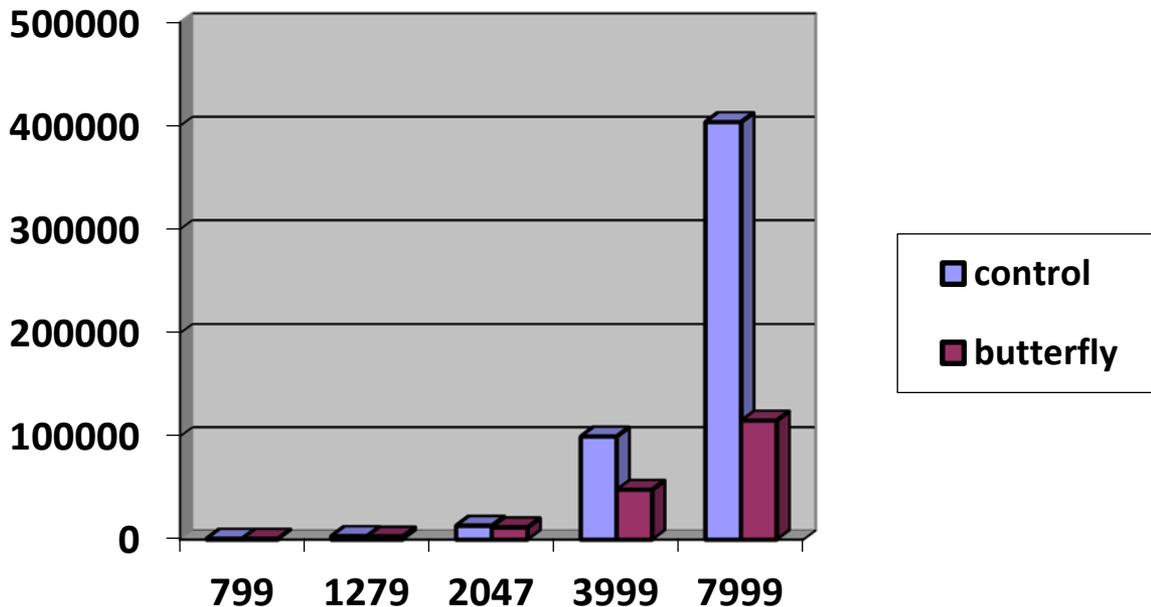
$l=1$



$l=3$

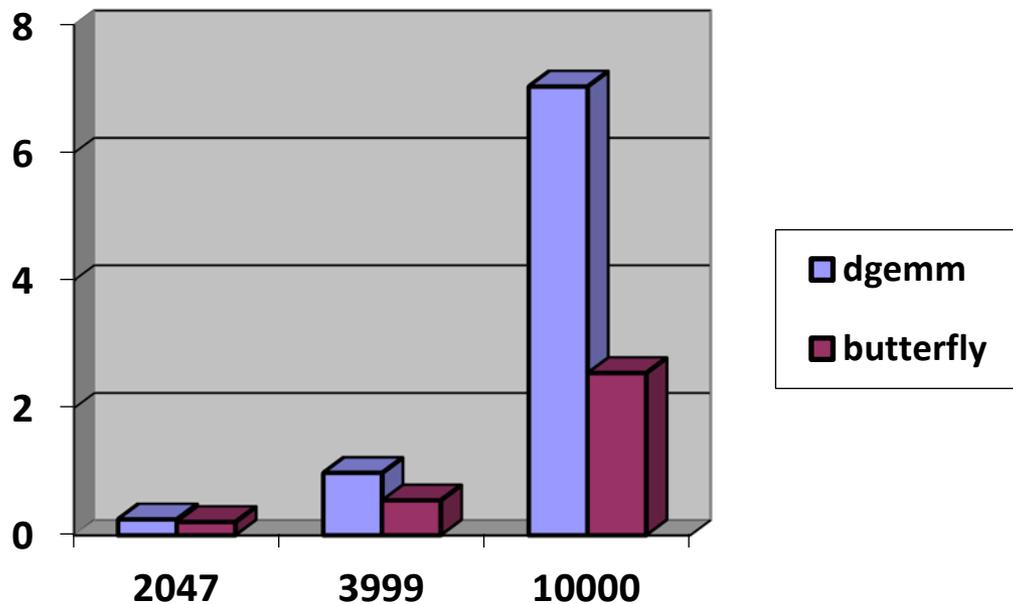
Floating point operations
per time-step in Gflop

**Inverse transform of
single field/level**



Wallclock time in seconds

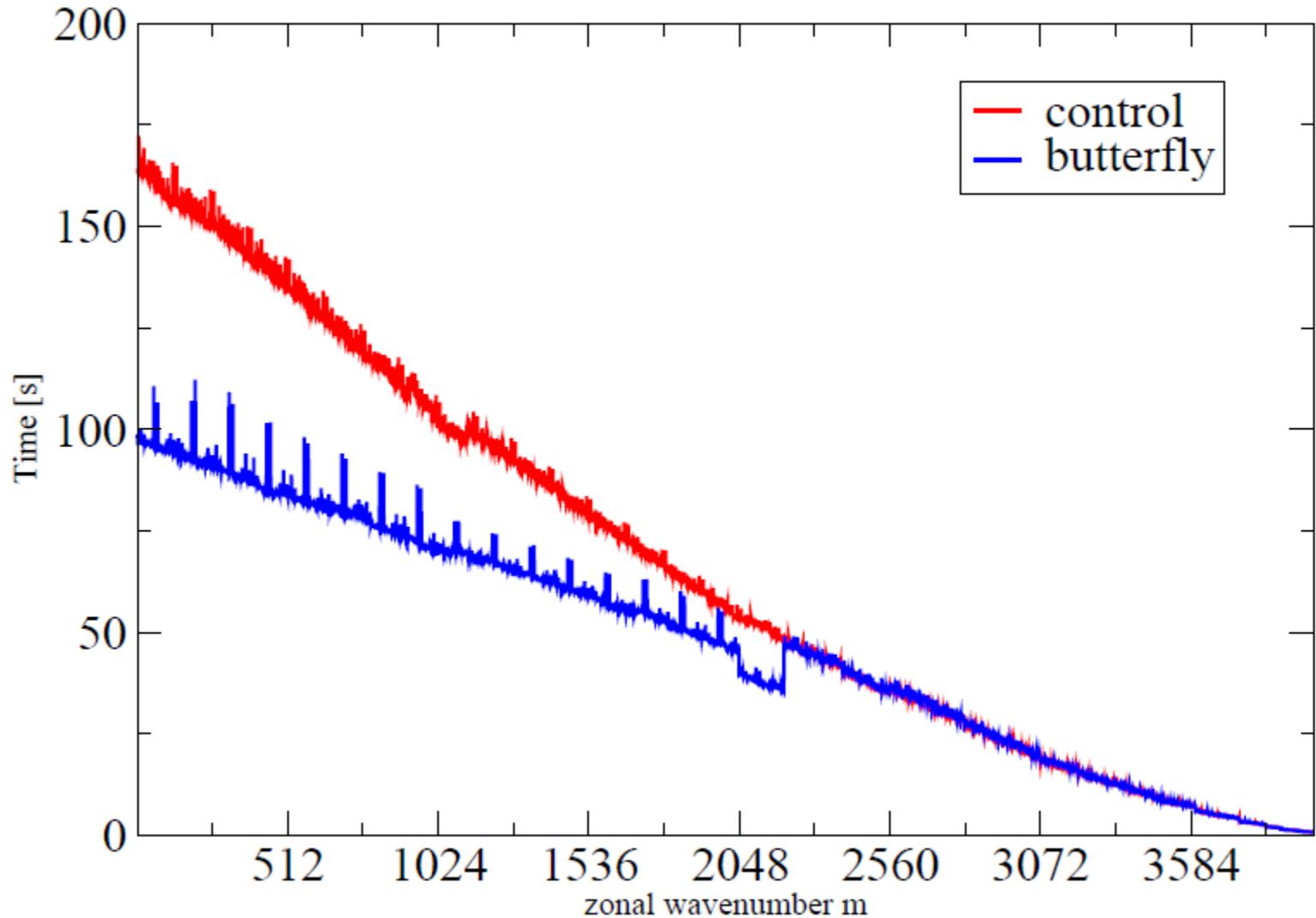
**inverse transform of 10 fields,
offline test environment**



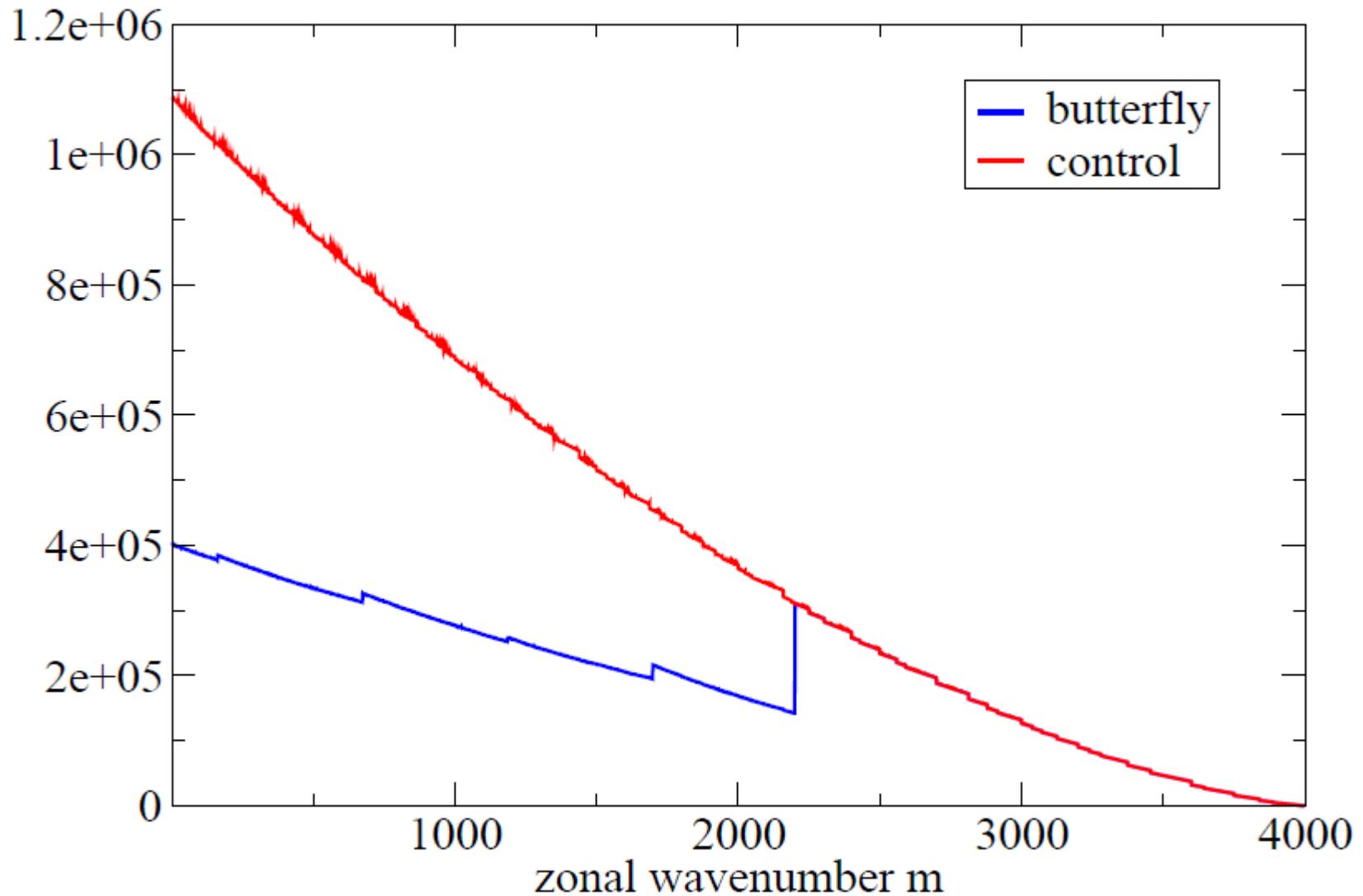
Selected projects to prepare for exascale computing in Meteorology (NWP)

- ***ICOMEX*** – ***ICO*sahedral grid Models for *EX*ascale Earth system simulations (2011-2014)**
- **Gung-Ho** – **Development of the Next Generation Dynamical Core for the UK MetOffice (2 phases, 2011-2013, 2013-2016)**
- **CRESTA** – **Collaborative *R*esearch into *EX*ascale Systemware, *T*ools & *A*pplications (2011-2014)**

T3999 6h forecast - inverse transforms: CPU time vs. wave number

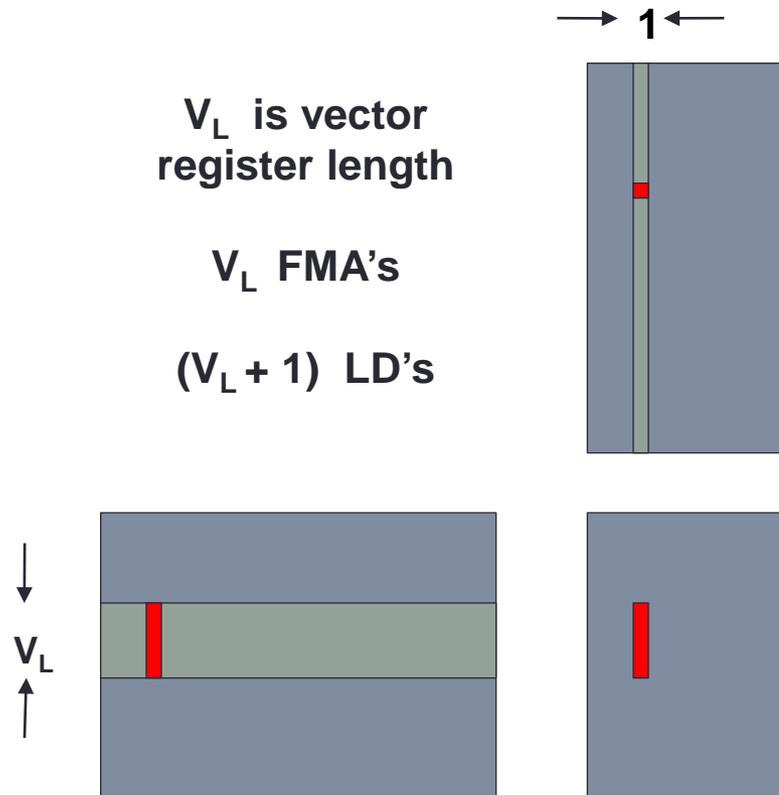


T3999 6h forecast - inverse transforms: Floating point operations vs. wave



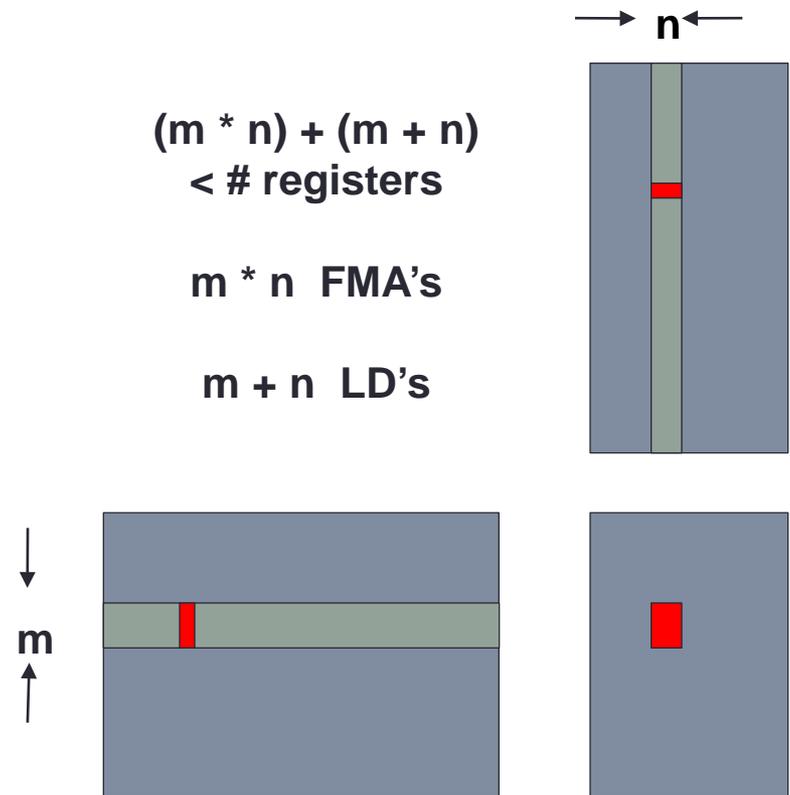
Why is Matrix Multiply (DGEMM) so efficient?

VECTOR



FMA's \approx LD's

SCALAR / CACHE



FMA's \gg LD's