Stochastic parameterization: Uncertainties from Convection

Robert S. Plant[1], Richard J. Keane[1], Warren J. Tennant[2] and Michael A. W. Whitall[1]

[1] *Department of Meteorology, University of Reading, PO Box 243, Earley Gate, Reading RG6 6BB, United Kingdom*
[2] *Met Office, Fitzroy Road, Exeter, EX1 3PB, United Kingdom*
r.s.plant@reading.ac.uk, richard.keane@physik.uni-muenchen.de, warren.tennant@metoffice.gov.uk, m.a.w.ball@reading.ac.uk

ABSTRACT

In 2005, the ECMWF held a workshop on stochastic parameterisation, at which the convection was seen as being a key issue. That much is clear from the working group reports and particularly the statement from working group 1 that "it is clear that a stochastic convection scheme is desirable". The present note aims to consider our current status in comparison with some of the issues raised and hopes expressed in that working group report.

1 Introduction

A good indication that some substantial progress have been made since the 2005 ECMWF workshop is that this note will necessarily be a rather partial and somewhat subjective review. It aims to offer some examples rather than attempting to be comprehensive. Six years ago it may have been unnecessary to make that caveat. Fortunately various other contributions to the current workshop also deal with stochastic aspects of convection, and we recommend the reader to consult also the contributions by Majda and by Randall in order to obtain a more complete picture of the current state of the art. In particular, to avoid undue overlap, some useful work from their groups may be neglected, or else mentioned only in passing here.

2 Typical convective parameterization

We begin with a reminder of some basic structural features of the parameterizations that are commonly used in current NWP and climate models. Most current parameterizations belong to the tradition of Arakawa and Schubert (1974) in which convection is characterised by an ensemble of clouds within some area of tolerably uniform forcing (Fig. 2). Each cloud is averaged over its life cycle and its vertical structure is modelled as a "plume", which is described in terms of the mass flux. The plumes interact with their environment but in all other respects are assumed to be independent and non-interacting. Given that the usual plume equations are (almost) linear in mass flux then a common simplification is to make a summation over plumes, thereby representing the entire ensemble with an equivalent "bulk" plume. The great strengths of this bulk, mass-flux approach are that it avoids any separate, explicit consideration of each type of cloud that occurs in the ensemble and also of the cloud area and updraft velocity within each type. Even assuming that we can accept the simplified picture in Fig. 2, these are clearly very strong simplifications of that picture. It is not surprising then that the great weaknesses of the approach are exactly the same points. For example, a very crude treatment of cumulus cloud

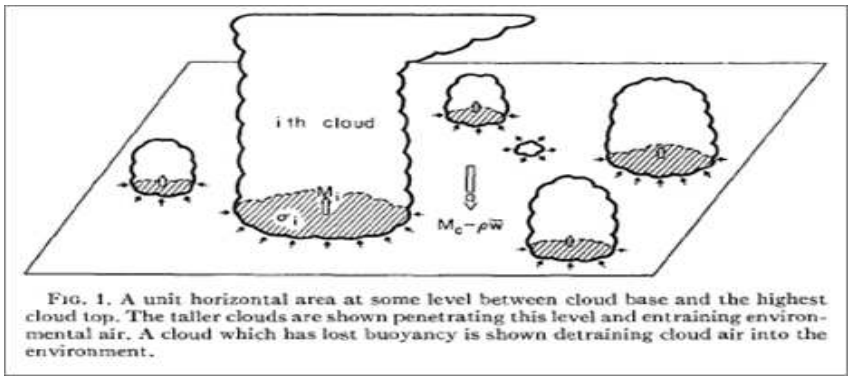

FIG. 1. A unit horizontal area at some level between cloud base and the highest cloud top. The taller clouds are shown penetrating this level and entraining environmental air. A cloud which has lost buoyancy is shown detraining cloud air into the environment.

Figure 1: Schematic of the idealization of convection used in many current parameterizations: an ensemble of cumulus clouds is contained within a grid box of the parent model. Taken from Arakawa and Schubert (1974).

microphysics (which is not linear and which does distinnguish between area and vertical velocity) is required for consistency.

Given a description of the vertical structure, it then remains to provide an overall amplitude for the strength of convective activity: in effect, to determine how many clouds are present within the grid area. This can be determined by assuming "convective quasi-equilibrium," the notion that the tendency of the convective ensemble to stabilize the atmosphere is close to being in balance with the tendency for destabilization (i.e., the convective "forcing") arising from large-scale processes, such as radiative or advective cooling.

The uncertainties associated with parameterization are often considered in three categories:

1. Structural errors. Clearly there are some fundamental issues with and strong assumptions in the methodology outlined above.

2. Parameter uncertainties. The results of massive multi-parameter experiments have highlighted the entrainment rate of a bulk mass-flux scheme as being the largest source of parameter uncertainty for climate projections (e.g. Knight et al., 2007).

3. Inherent process uncertainties. Given a particular state of the parent NWP or climate model, it may not be possible, even in principle, to determine properties of the unresolved state sufficiently well for the feedback to the resolved state to be unambiguous. An extreme example would be where convective initiation is very marginal and sensitive to subtle, inhomogeneous details of boundary layer structure (e.g. Hanley et al., 2011). More generally, various coarse-graining studies of CRM simulations (e.g. Shutts and Palmer, 2007) have demonstrated that a given large-scale state is consistent with many sub-grid states.

In the remainder of this note, we will discuss some aspects of category 3. Before doing so, it may be worth remarking that the distinction between categories 1 and 2 is not clear. The bulk entrainment rate (and any dependencies associated with it in some particular scheme) may itself be regarded as a parmeterization embedded within the convective parameterization. Entrainment parameterizes cloud-environment interactions, and in a bulk scheme it also implicitly encodes assumptions about the relative occurence of different cloud types. Indeed, uncertainties from all three categories might reasonably be considered to also apply to the treatment of entrainment.

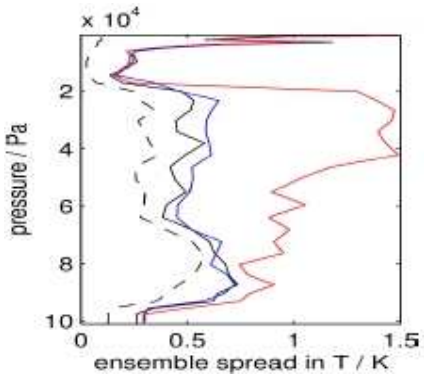

Figure 2: Time-averaged profiles of ensemble spread in temperature for: the deterministic MetUM with small initial-condition perturbations (dashed); default multiplicative noise SPPT method, with random numbers chosen for stochastic multipliers applied to the total parameterized tendencies of T and q (black); multiplicative noise with separate uncorrelated random numbers chosen for the stochastic multipliers of each parameterized process (blue); and, multiplicative noise with separate uncorrelated random numbers chosen for the stochastic multipliers of $\partial T/\partial t$ and $\partial q/\partial t$ (red).

3 The physics of fluctuations

There is now good evidence from various studies that introducing some stochastic component to an ensemble weather forecasting model may be helpful, in that the spread-error relationship of the ensemble can be improved without any undue damage to skill measures. Since such outcomes have been achieved for various forms of the stochastic component, one might therefore be tempted to adopt a purely practical approach towards specifying that component. However, it is worth stressing that one can never become completely agnostic about the relevant physics. In other words, introducing a stochastic component to the model automatically carries with it assumptions about the physics that require some justification.

As a trivial example, let us consider a possible additive noise contribution, ε, to the potential temperature equation:

$$\frac{\partial \theta}{\partial t} + \underline{u}.\underline{\nabla}\theta = P_\theta(X,\alpha) + \varepsilon \tag{1}$$

where P_θ represents deterministic parameterizations that depend upon a set of parameters α and on the resolved-scale state X. Suppose now that we decide to reformulate our model in terms of the transformed variable $\eta = e^\theta$. This evolves according to

$$\frac{\partial \eta}{\partial t} + \underline{u}.\underline{\nabla}\eta = P_\eta(X,\alpha) + \varepsilon\eta \tag{2}$$

so that additive noise has become multiplicative noise. Of course, such phrases are quite meaningless in themselves: it is necessary to specify for what variable the noise is to be considered additive or multiplicative. And, such a choice of variable cannot be made without consideration of the physics.

To give a specific example, let us consider some variations on the SPPT method as used at ECMWF. In that method, multiplicative noise is applied to the total parameterized tendencies of $\partial T/\partial t$ and $\partial q/\partial t$. Ball and Plant (2008) studied the method, amongst others, in a single-column model experiment with the Met Office Unified Model (MetUM) for the TOGA-COARE test case. As an extension to that study, Fig. 3 shows the ensemble spread in temperature for the SPPT method and some simple modifications of it. There is little impact from treating each parameterization separately but a strong impact from treating the temperature and moisture tendencies separately. Indeed, the spread produced by decorrelating the

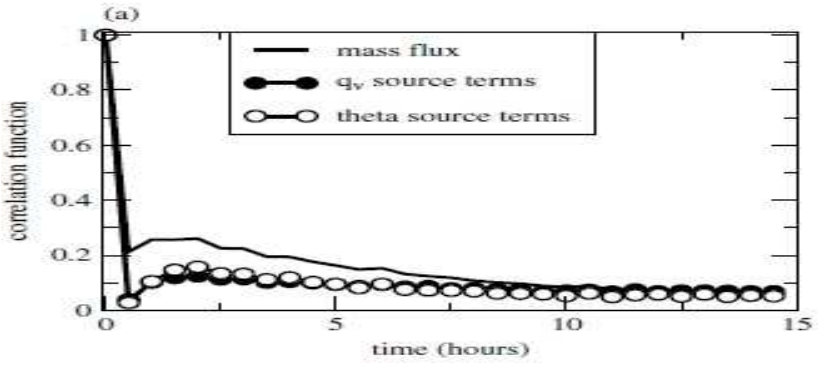

Figure 3: Auto-correlation function for parameterized mass flux, and the convection-scheme tendencies for moisture and potential temperature. Results taken from a simulation with the MetUM at N48L38 resolution. Taken from Stiller (2009).

noise in the two tendencies is larger than that produced by quenched[1] random noise (not shown). The reason for such artificially large spread is that the parameterized increments to T and q are strongly negatively correlated. Condensation and evaporation are key parameterized processes, and a scatter plot of the total parmeterization tendencies reveals that many of the total increments lie on or very close to the line $C_p\Delta T = -L\Delta q$ which would indicate a pure phase change. Decorrelating the noise applied to the two increments obscures that fundamental physical relationship, and so damages the fidelity of the simulations.

Let us now return to the working group report 1 from the earlier ECMWF workshop (Craig et al., 2005) and reconsider issues highlighted there.

3.1 Physical and artificial noise

When a mass-flux convective parameterization is used in an NWP or climate model, it does not behave in a smooth way but rather exhibits strong on/off behaviour with strong timestep-to-timestep variability. This remains the case even in highly-idealized single-column model experiments performed by the authors in which the forcing for convection is specified and constrained to be time invariant. This small-scale, high-frequency variability has not been well characterized or studied, and there is rather little in the journal literature that explicitly addresses the issue. One exception is Stiller (2009) who highlights the problem that it produces for data assimilation. Figure 3.1 shows that there is a very rapid fall off of the auto-correlation function for parameterized convective tendencies, on a timescale that appears to be set by the model timestep rather than anything physical. There is almost no correlation in the tendencies at neighbouring timesteps.

Such artificial on/off noise is present in many NWP and climate models and may be having upscale effects. It would seem to be worthy of dedicated study, not least because artificial noise in the output from one physics scheme implies that artificial noise is present in the input to other physics schemes. That may potentially lead to systematic errors in respresenting (say) cloud-radiation effects. For the purposes of the present discussion, however, we will content ourselves with noting an important consequence. If one wishes to develop an explicit representation for some physical source of inherent convective fluctuations then a naive application of that representation will not necessarily produce fluctuations in the parent model with the desired characteristics. Certainly it will be essential to make careful checks that

[1] i.e., using the default method and randomly choosing tendency multipliers at the first timestep only which are then left fixed throughout the model integration

those characteristics are reproduced in practice, and possibly it may be necessary to take steps to remove the artificial noise.

3.2 Scale-dependence of parameterization

The purpose of a parameterization scheme such as that for convection is to represent the effects of processes that take place below the parent model's filter scale. It immediately follows that any change to the filter may require a change in the appropriate representation of what is sub-filter. Most parameterizations in operational use contain no explicit recognition of the length and timescales on which they operate. That may well be an acceptable approximation over a wide range of the filter length and timescales, at least for the mean response of the unresolved processes. It is however less plausible as an acceptable approximation for the stochastic aspect of a parameterization, as the simple considerations below indicate. The consequence is that ideally a parameterization (but particularly a stochastic parameterization) should adapt automatically to the grid size of the parent model. This is important in the sense that one test for a good representation of the fluctuations is an ability to capture their variation with lengthscale. It may also be important for practical reasons, certainly if parameterization is to be handled in a satisfactory way in conjunction with some of the new dynamical cores that are being developed, which have adaptive grids.

Convective instability is released in a discrete fashion, a finite number of clouds appearing in a finite area in response to some large-scale destabilization mechanism. For a typical GCM grid box of size (say) $(100\text{km})^2$ the number of deep convective clouds that are typically found in the box is on the order of a few, or maybe less, and certainly far from being large enough to produce a steady response to a steady forcing (Plant and Craig, 2008; Shutts and Palmer, 2007). The fluctuations in mass flux about a mean response can be determined theoretically from a statistical mechanics approach, subject to certain assumptions (Craig and Cohen, 2006). Fortunately the key assumptions required are very familiar ones from the perspective of a traditional deep convective parameterization (Sec. 2): specifically, that there is an equilibrium between the large-scale forcing and the ensemble-mean convective response and that the clouds can be assumed to be non-interacting. Moreover, the predictions are in good agreement with CRM simulations, even in organized and time-varying cases where one might expect the main assumptions to have broken down (Cohen and Craig, 2006; Davoudi et al., 2010).

Notice that the key point of this analysis is the need for a clear distinction to be made between ensemble and spatial averaging. Only for a strong enough forcing and a large enough grid box do the two coincide, but more generally, the spatially-averaged convective state is a sample from the full cloud ensemble that is the basis of a mass-flux convective parameterization.

The Craig and Cohen (2006) theory has been translated into a practical parameterization by Plant and Craig (2008). Their scheme uses the mass-flux formalism, and operates as follows.

1. An average in the horizontal and over time is performed to determine the large-scale state.
2. Properties of equilibrium statistics are determined dependent upon the large-scale state: the ensemble-mean cloud-base mass flux is determined from the scheme's CAPE-based closure, and the mean mass flux of a single cloud must also be determined. This latter quantity is important in setting a scale for the fluctuations. In principle, it may be a function of the large-scale state (Davies, 2011, personal communication) but available CRM data seem to suggest that at cloud base it is a weak function, and so a constant value is used in the parameterization.
3. Given the above quantities, the theoretical pdf's are then fully specified for the number and properties of the clouds within a single grid box. Those pdf's are sampled randomly.
4. Output tendencies are computed for the sampled set of clouds.

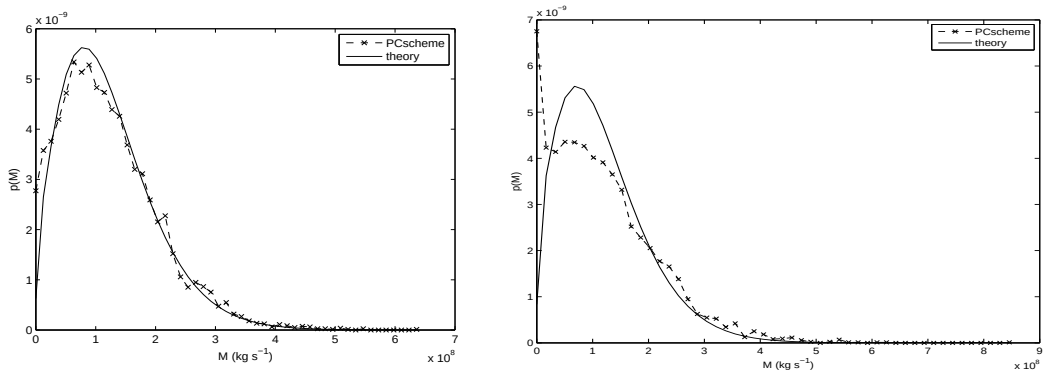

Figure 4: PDFs of total parameterized mass flux over a horizontal area of $(64\,\mathrm{km})^2$ *in RCE simulations with* 32*km grid length with the MetUM. Left: a simulation in which a large-scale state for input to the Plant and Craig (2008) scheme has been computed by averaging over* $\sim (160)km^2$ *and for* $\sim$ 1*hr. Right: with no such averaging. Crosses show simulation data and the solid line is the theoretical prediction.*

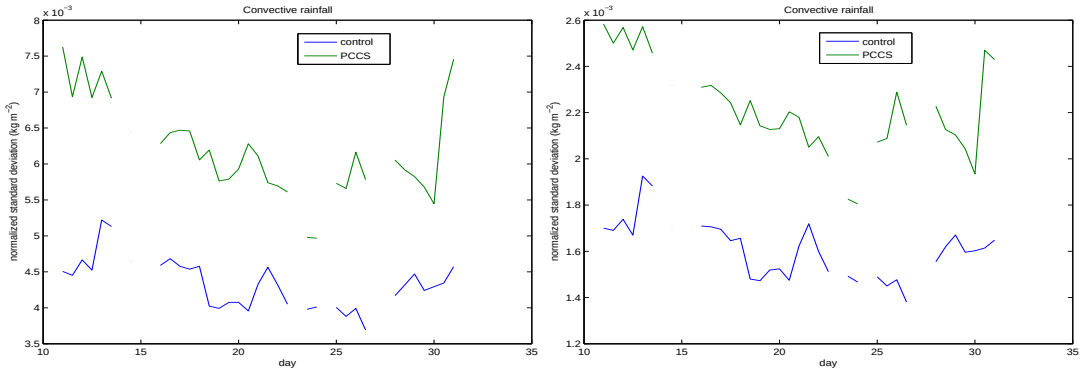

Figure 5: Normalized standard deviation in a trial of the Plant and Craig (2008) parameterization in MOGREPS. Results (green) and compared to those from the MetUM standard deterministic parameterization (blue). The convcetive rainfall was averaged over $(48km)^2$ *(left) and* $(120km)^2$ *(right).*

Notice that the parameterization distinguishes between the grid-scale state and the large-scale state, with the pdfs being a function of the latter. In order to demonstrate the practical importance of the distinction Keane and Plant (2011) performed idealized radiative-convective equilibrium experiments in a three-dimensional domain with parameterized convection. This is a controlled situation that matches well with the picture of convection in Fig. 2, so it is an important test of any parameterization that it should be able to describe the situation well (i.e., in good agreement with equivalent CRM experiments). The underpinning theoretically-predicted pdf for mass flux over a finite area can be successfully reproduced (Fig. 3.2) regardless of the grid length used. That is not the case, however, if grid-scale input is used. In essence, fluctuations (whether physically-based or otherwise) in the input state can damage the closure calculations for determining the mass flux that is required to balance the imposed forcing.

The scheme is currently being trialled in the Met Office MOGREPS ensemble system, and some preliminary results are shown in Fig. 3.2 . This indicates that enhanced variability remains on scales longer than the 24km grid length.

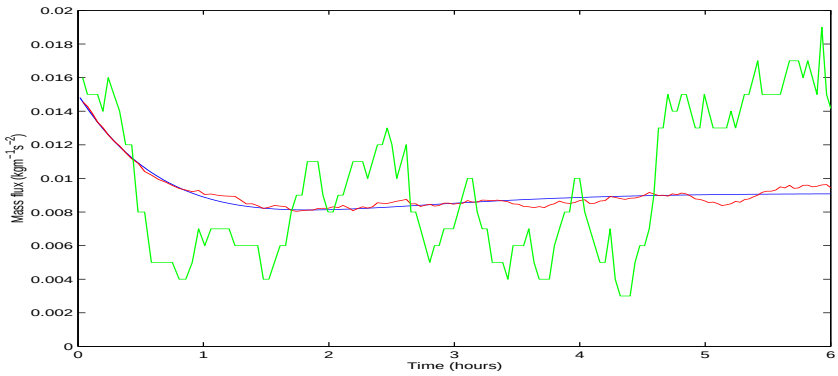

Figure 6: An example solution of the ordinary differential equations of Pan and Randall (1998) (blue) is shown alongside numerical results from an equivalent, individual-level stochastic model. The system size was such that there are an average of 10 clouds present at equilibrium within the simulated system. The green line shows a single realization of the individual-level model and the red line the ensemble mean from 100 realizations.

3.3 Prognostic closures

Non-equilibrium closures for convective parameterizations have been explored on the basis of the convective-energy-cycle equations. The first attempt to do this was by Pan and Randall (1998) who considered the equation set

$$\frac{dA_i}{dt} = F_i - \gamma_{ij} M_j \quad ; \quad \frac{dK_i}{dt} = A_i M_i - \frac{K_i}{\tau_i} \tag{3}$$

supplemented by an ansatz

$$K_i = \alpha M_i^2 \tag{4}$$

Here A is the cloud work function of Arakawa and Schubert (1974) (a generalization of the CAPE), M is the mass flux, K the convective kinetic energy, F the forcing and γ and τ are vertical-structure and dissipation parameters that are treated as constants. The subscripts label cloud types. In the past few years there has been a revival of interest in these and other related equation sets (Davies et al., 2009; Wagner and Graf, 2010; Yano and Plant, 2011).

A natural question to ask is how one might treat stochastic effects from finite cloud number in out-of-equilibrium systems such as these. One possibility is to consider a model formulated at the individual cloud level using simple birth and death probabilities suitably modulated by the evolving cloud work function. Recently Plant (2011) showed that such models can be formulated so that they are completely equivalent to the above ordinary differential equations, in the sense that the ode's are reproduced in the limit of infinite system size. A numerical example is given in Fig. 3.3. Moreover, the birth–death rules used and the associated probabilities are very strongly constrained by making the link to appropriate ode's. The individual level model can also be made consistent with the equilibrium fluctuations in a finite-size system that were predicted by Craig and Cohen (2006).

Note that stochastic birth–death processes have previously been used to describe deep convection (e.g. Majda and Khouider, 2002), albeit in a rather different context and motivated by uncertainties in the triggering process. See also the contribution to this workshop from Majda for details of a further application.

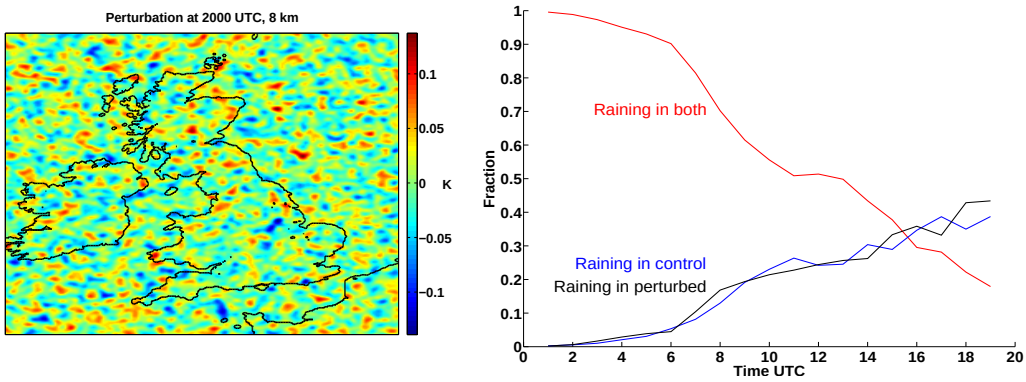

Figure 7: On the left is shown an example of a perturbation field applied to the temperature within the boundary-layer in a convective-scale simulation of a case of widespread convection across the UK. On the right are shown the fractions of rainy grid points in a control (without boundary layer perturbations) and a perturbed simulation.

3.4 Effects of sub-grid variability

There have been various demonstrations in the recent literature from simulations at convective-scale resolution (i.e., without convective parameterization) that small boundary layer fluctuations can easily shift the locations of precipitating convective cells . An example is given in Fig. 3.4, taken from the study by Leoncini et al. (2010). Such fluctuations provide a source of ensemble spread for NWP forecasts at convective-scale resolution. It may be needless to remark that this provides a simple example of how the absence of a convective parameterization certainly does not mean that we remove the uncertainties associated with convection.

In terms of accounting for boundary layer fluctuations when using a convective parameterization, we refer the reader to earlier work by Majda and Khouider (2002) and Bright and Mullen (2002), the former study having been mentioned earlier and the latter applying stochastic perturbations to the triggering function in the Kain-Fritsch parameterization. Development of the Majda and Khouider (2002) approach has been reported by Majda et al. (2008) but we are not aware of any other recent studies explicitly dealing with this issue.

It may be worth noting, however, that there seems to be increasing interest in mass flux closures related to the quantity $\exp(-\mathrm{CIN}/\mathrm{TKE})$. See for example the poster contribution to this workshop from Hohenegger. Such closures are strongly motivated by ideas about boundary layer fluctuations, but so far at least do not seem to have been considered in a stochastic sense.

More generally, this topic clearly raises important issues about the coupling between the boundary-layer and the convective parameterizations, which remain to be addressed.

3.5 Propagation

There are longstanding and well documented issues in NWP and climate models regarding propagation and organization of convection. At least in part, this may be due to the lack of communication between grid cells. At the 2005 workshop, there were two possibilities raised. One was that a cellular-automata approach might provide a suitable mechanism for inter-cell communication. Some work along these lines has recently been conducted by Bengtsson-Sedlar et al. (2011) and is further described in her contribution to this workshop.

The other possibility was that communication could be achieved by allowing for cold pool propagation

between grid boxes. A recent study to mention in this context is Grandpeix and Lafore (2010) who proposed a density current parameterization for cold pools, coupled to a convective parameterization. The authors suggest that it might be extended to provide a mechanism for horizontal propagation. Their approach is not stochastic, although a stochastic aspect would appear natural in any such extension, given that the cold pool propagation out of a grid box must to some extent depend on where within the grid box the downdraft source is assumed to occur.

4 Summary

Some issues in the stochastic parameterization of convection have been discussed. It is clear that there has been considerable progress made since the 2005 ECMWF workshop. However, some important issues raised then remain important issues now, while other issues identified then have still to be addressed in any concerted way.

References

Arakawa, A. and W. H. Schubert (1974). Interaction of a cumulus cloud ensemble with the large-scale environment. Part I. *J. Atmos. Sci. 31*, 674–701.

Ball, M. A. and R. S. Plant (2008). Comparison of stochastic parameterization approaches in a single-column model. *Phil. Trans. Roy. Soc. A 366*, 2605–2623.

Bengtsson-Sedlar, L., H. Körnich, E. Källén, and G. Svensson (2011). Large-scale dynamical response to sub-grid scale organization provided by cellular automata. *To appear in: J. Atmos. Sci.*.

Bright, D. R. and S. L. Mullen (2002). Short-range ensemble forecasts of precipitation during the southwest monsoon. *Weather and Forecasting 17*, 1080–1100.

Cohen, B. G. and G. C. Craig (2006). Fluctuations in an equilibrium convective ensemble. Part II: Numerical experiments. *J. Atmos. Sci. 63*, 2005–2015.

Craig, G., K. Fraedrich, J. Lin, M. Moncrieff, R. Plant, D. Thomson, and A. Tompkins (2005). Working group 1 report: Issues in convection. In *ECMWF Workshop on Representation of Sub-grid Processes using Stochastic–Dynamic Models, Reading, UK, 6-8 June 2005*, pp. ix–xii.

Craig, G. C. and B. G. Cohen (2006). Fluctuations in an equilibrium convective ensemble. Part I: Theoretical formulation. *J. Atmos. Sci. 63*, 1996–2004.

Davies, L., R. S. Plant, and S. H. Derbyshire (2009). A simple model of convection with memory. *J. Geophys. Res. 114*, D17202.

Davoudi, J., N. A. McFarlane, and T. Birner (2010). Fluctuation of mass flux in a cloud resolving simulation with interactive radiation. *J. Atmos. Sci. 67*, 400–418.

Grandpeix, J.-V. and J.-P. Lafore (2010). A density current parameterization coupled with Emanuel's convection scheme. Part I: The models. *J. Atmos. Sci. 67*, 881–897.

Hanley, K. E., D. J. Kirshbaum, S. E. Belcher, N. M. Roberts, and G. Leoncini (2011). Ensemble predictability of an isolated mountain thunderstorm in a high-resolution model. *To appear in: Quart. J. R. Meteorol. Soc.*.

Keane, R. J. and R. S. Plant (2011). Length and time scales for stochastic convective parameterization. *Submitted to: Quart. J. R. Meteorol. Soc.*.

Knight, C. G., S. H. E. Knight, N. Massey, T. Aina, C. Christensen, D. J. Frame, J. A. Kettleborough, A. Martin, S. Pascoe, B. Sanderson, D. A. Stainforth, and M. R. Allen (2007). Association of parameter, software, and hardware variation with large-scale behavior across 57,000 climate models. *Proc. Natl. Acad. Sci. 104*, 12259–12264.

Leoncini, G., R. S. Plant, S. L. Gray, and P. A. Clark (2010). Perturbation growth at the convective scale for CSIP IOP18. *Quart. J. R. Meteorol. Soc. 136*, 653–670.

Majda, A. J., C. Franzke, and B. Khouider (2008). An applied mathematics perspective on stochastic modelling for climate. *Phil. Trans. Roy. Soc. A 366*, 2429–2455.

Majda, A. J. and B. Khouider (2002). Stochastic and mesoscopic models for tropical convection. *Proc. Nat. Acad. Sci. 99*, 1123–1128.

Pan, D.-M. and D. A. Randall (1998). A cumulus parameterization with prognostic closure. *Quart. J. R. Meteorol. Soc. 124*, 949–981.

Plant, R. S. (2011). A new modelling framework for statistical cumulus dynamics. *To appear in: Phil. Trans. Roy. Soc. A*.

Plant, R. S. and G. C. Craig (2008). A stochastic parameterization for deep convection based on equilibrium statistics. *J. Atmos. Sci. 65*, 87–105.

Shutts, G. J. and T. N. Palmer (2007). Convective forcing fluctuations in a cloud-resolving model: Relevance to the stochastic parameterization problem. *J. Clim. 20*, 187–202.

Stiller, O. (2009). Efficient moist physics schemes for data assimilation. II: Deep convection. *Quart. J. R. Meteorol. Soc. 135*, 721–738.

Wagner, T. M. and H.-F. Graf (2010). An ensemble cumulus convection parameterization with explicit cloud treatment. *J. Atmos. Sci. 67*, 3854–3869.

Yano, J.-I. and R. S. Plant (2011). Finite departure from convective quasi-equilibrium: Periodic cycle and discharge-recharge mechanism. *Submitted to: Quart. J. R. Meteorol. Soc.*.