Modeling Atmospheric Circulations with Soundproof Equations

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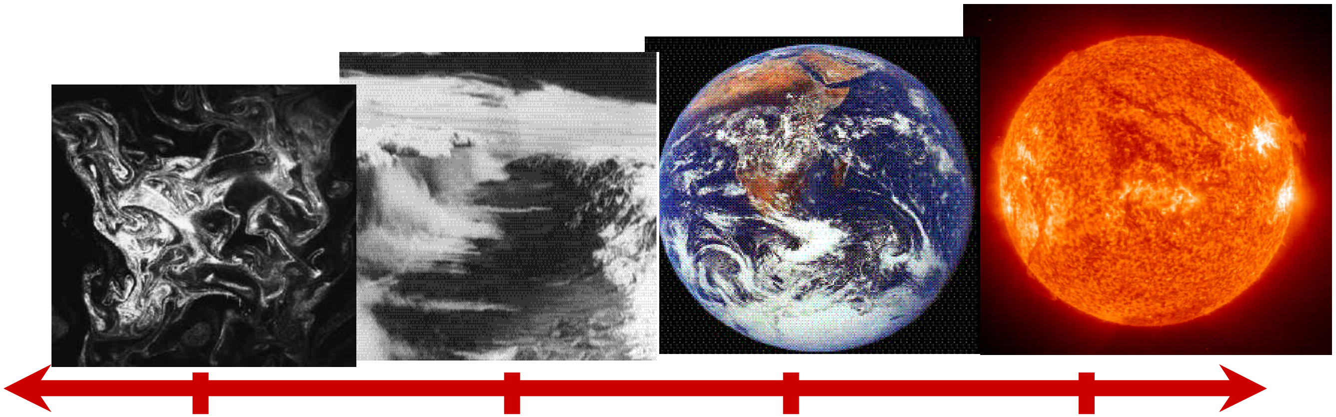
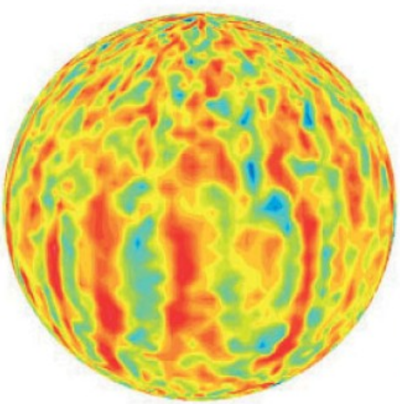
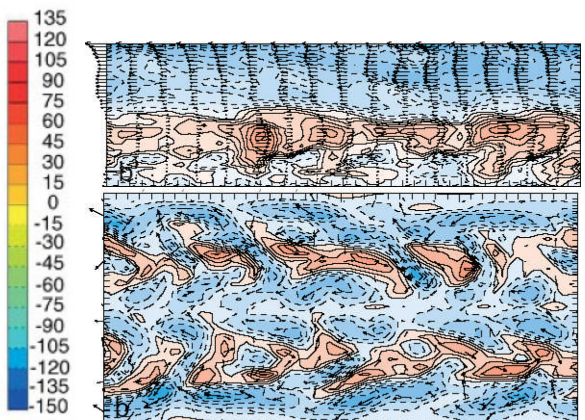


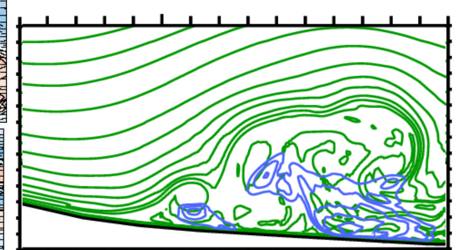
ΔX $O(10^{-2})$ m $O(10^{2})$ m $O(10^{4})$ m $O(10^{7})$ m



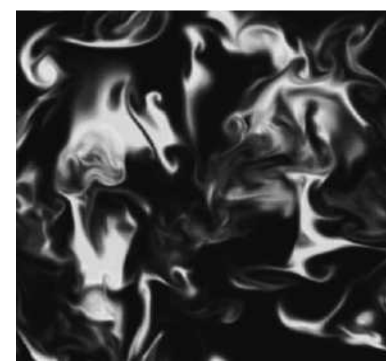
Solar convection



Global flows



Gravity waves



Cloud turbulence

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The applicability of soundproof equations to prediction of weather and climate is questioned.

All leading nonhydrostatic NWP codes are based on the compressible Euler equations.

Yet, there is no set of equations uniformly adopted throughout the NWP community (*J. Comput. Phys.*, 2008, vol. **227**).

However, soundproof models progress, expand their predictive capabilities, and keep attracting interests of the community.

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Davies et al, 2003, *QJR*, **129**: quantifying the departures of normal modes of atmospheric soundproof PDEs from that of fully compressible Euler equations

Prusa & Smolar. and Wedi & Smolar., 2003-2004, *JCP*, **190** & **193**: time dependent geometry of soundproof models ➔ flexible boundaries and model couplers

Durran, 2008, *JFM*, **601**: generalized pseudo-incompressible system with an arbitrary reference state

Abiodun, Prusa & Gutowski, 2008, *Clim. Dyn*., 31: comparison of CAM3 dynamics cores in aqua-planet simulations, including the anelastic nonhydrostatic model EULAG ❶

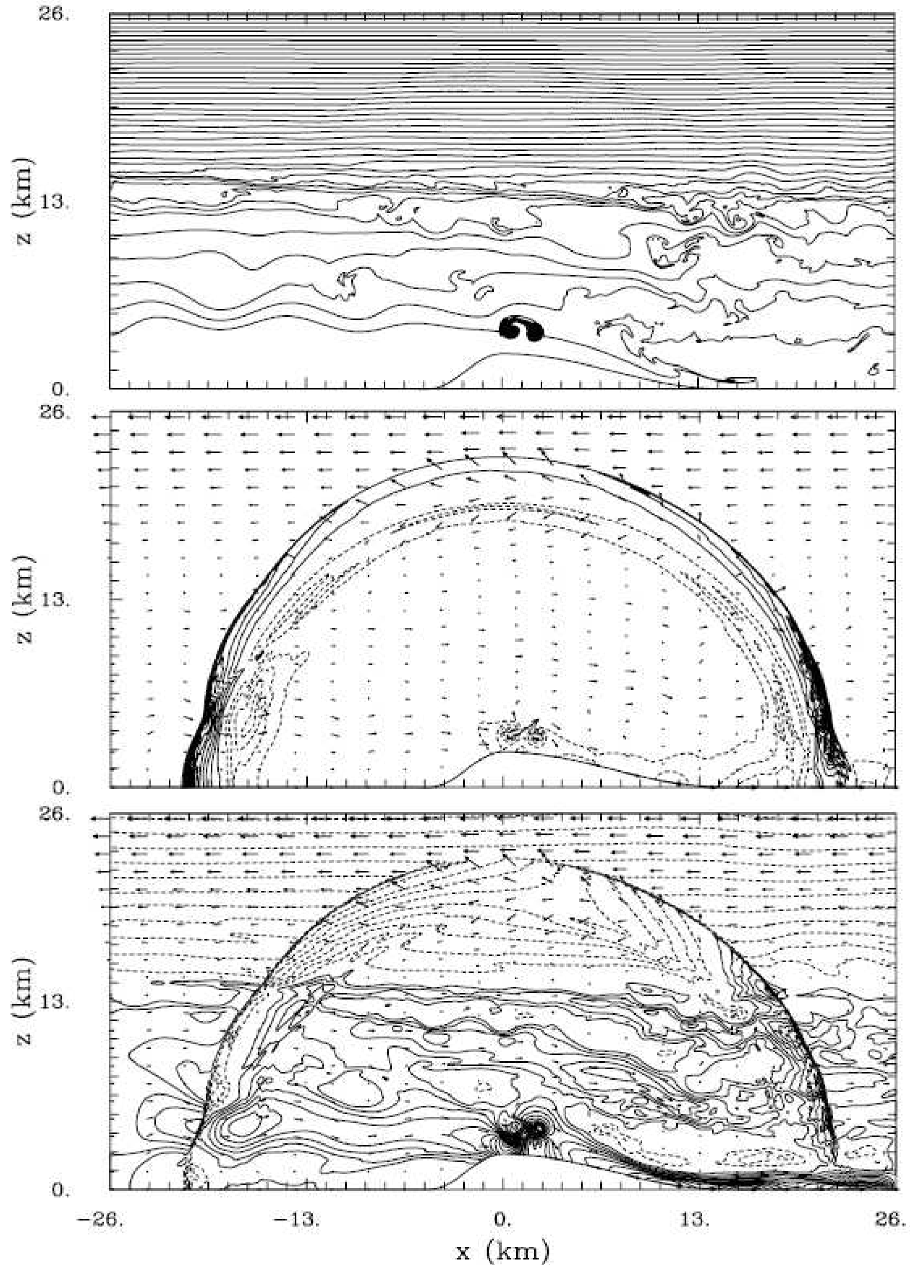
Arakawa & Konor, 2009, *MWR*, **137:** a hybride of nonhydrostatic soundproof and hydrostatic primitive PDEs

Szmelter & Smolar., 2009-2010, JCP, 228 & 229: common structured/unstructured numerical environment for compressible/soundproof systems on differential manifolds ❷

Klein & coauthors, 2010, *JAS*, **67**: extended validity regimes of soundproof models

EULAG's key features (www.eulag.org)

- A suit of governing PDEs; numerical laboratory
- Conservative, nonoscillatory, forward in time (NFT) semi-implicit numerics
- Robust elliptic solver; exact projection
- Static /dynamic grid stretching with 2nd order accuracy



extreme event

Aqua-Planet Simulation ❶

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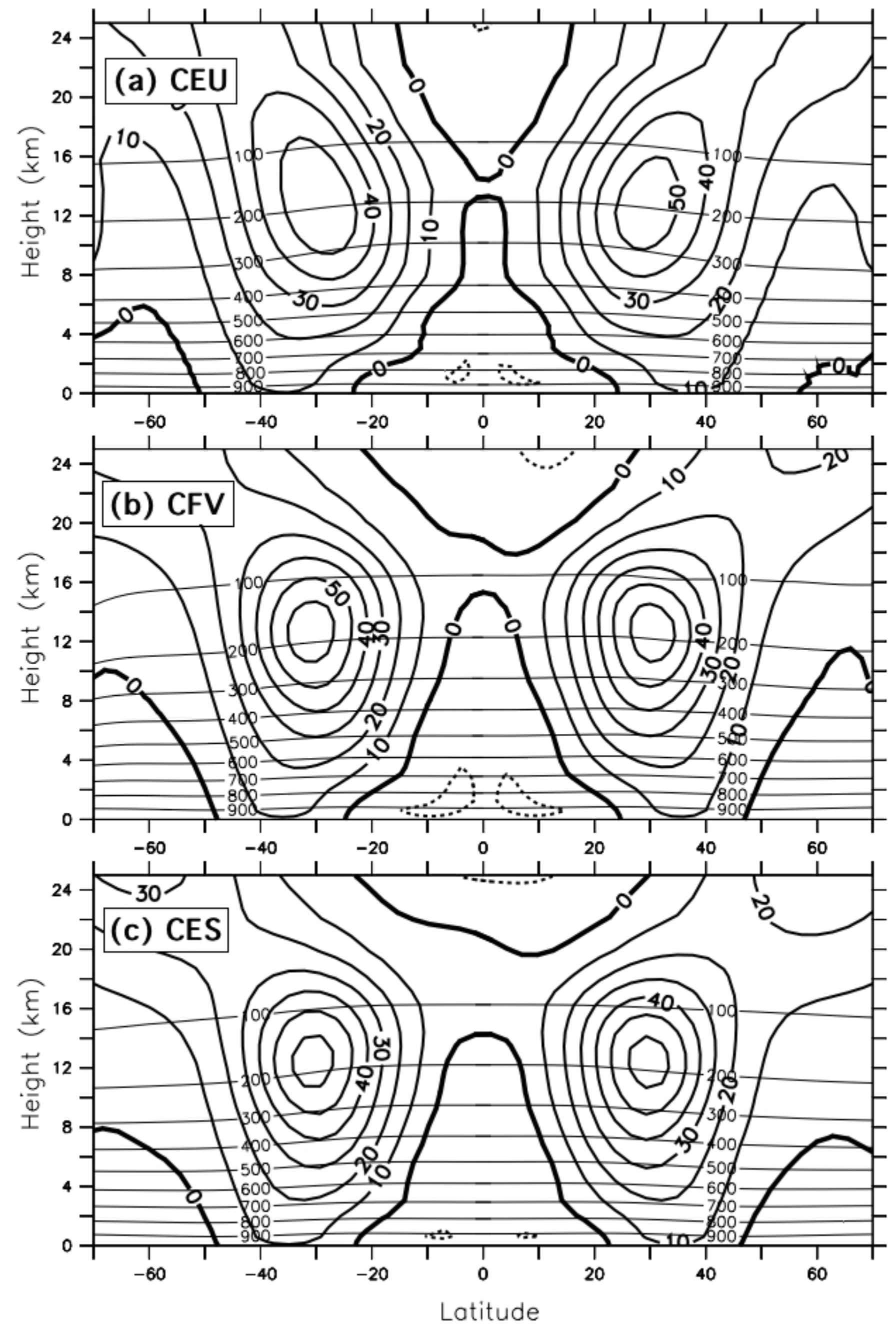
Abiodun, Prusa & Gutowski, 2008, *Clim. Dyn.*, 31

- CAM3 Cores: EULAG, FV and ESP
- Experiment: Aqua-planet; time = 18-6 months
- Forcing: Idealized, zonally symmetric SST
- Horizontal resolutions: 2˚x2.5˚ [EULAG, FV],T42 [ESP]
- Vertical grid: 26 levels
- Time step: 600s (EULAG), 900s (FV and ESP)
- Initialization: Eulag started from rest, FV and ESP from their standard initial conditions

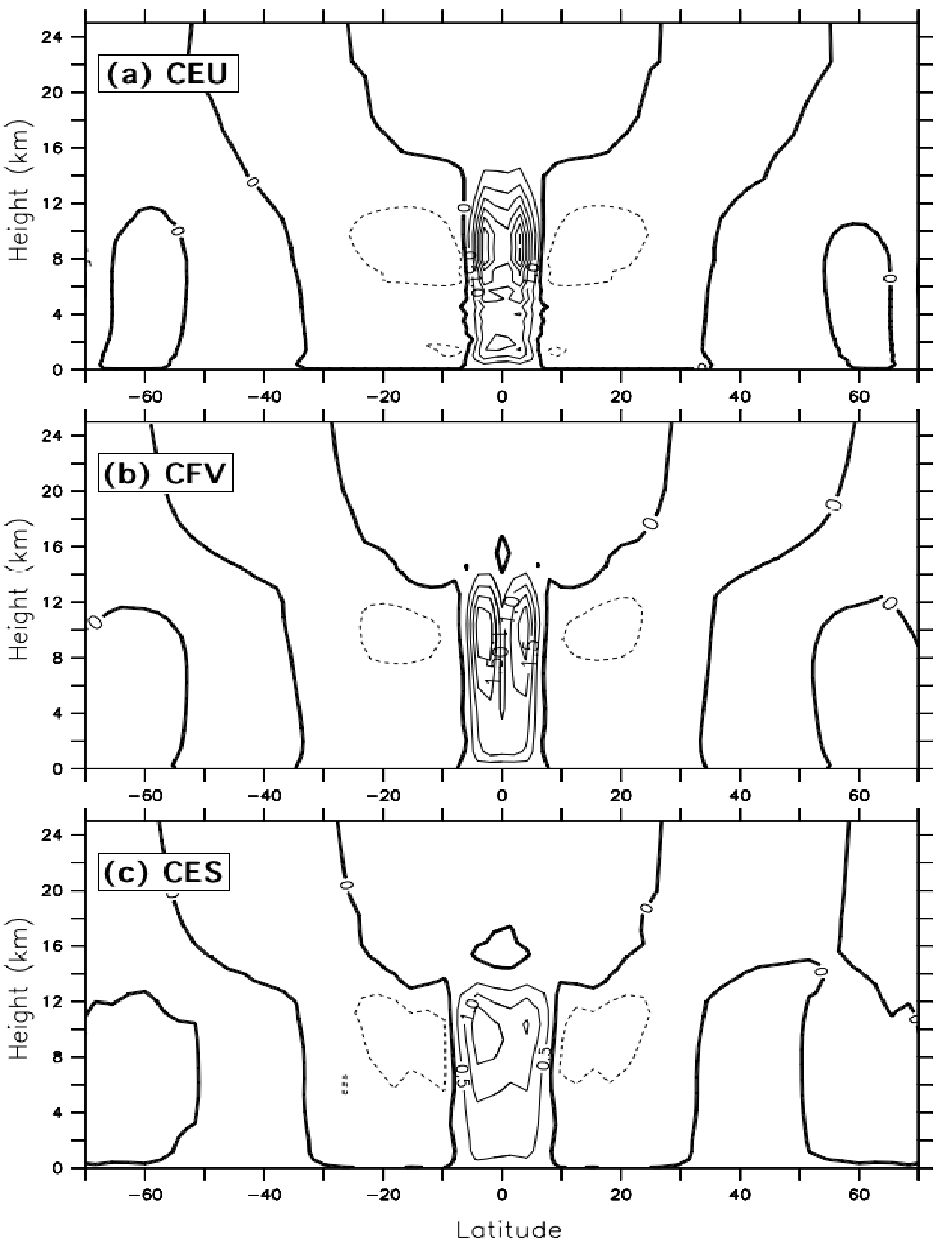
Zonally Averaged Zonal Wind

- Westerly Jet cores:
  - EULAG (55 m/s)
  - FV (65 m/s)
  - ESP (60 m/s)
- Easterly peaks:
  - EULAG (10 m/s)
  - FV (10 m/s)
  - ESP (10 m/s)



Zonally Averaged Vertical Wind

- Maximum updrafts:
  - EULAG (4.0 cm/s)
  - FV (2.2 cm/s)
  - ESP (1.8 cm/s)
- Updraft locations:
  - ~ + 3˚ off equator



Precipitation

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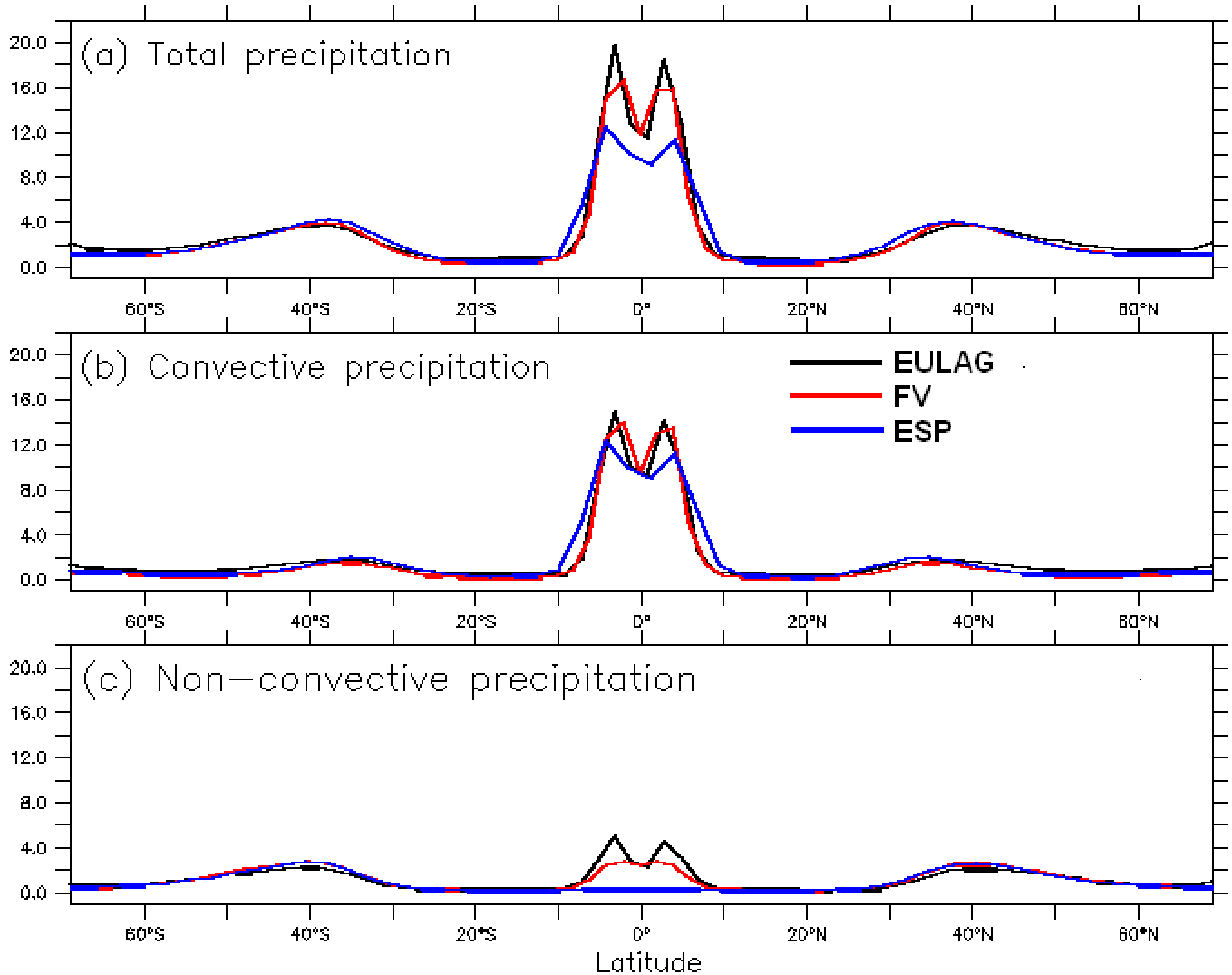

Precipitation (mm/day)

Power Spectra: Kinetic Energy

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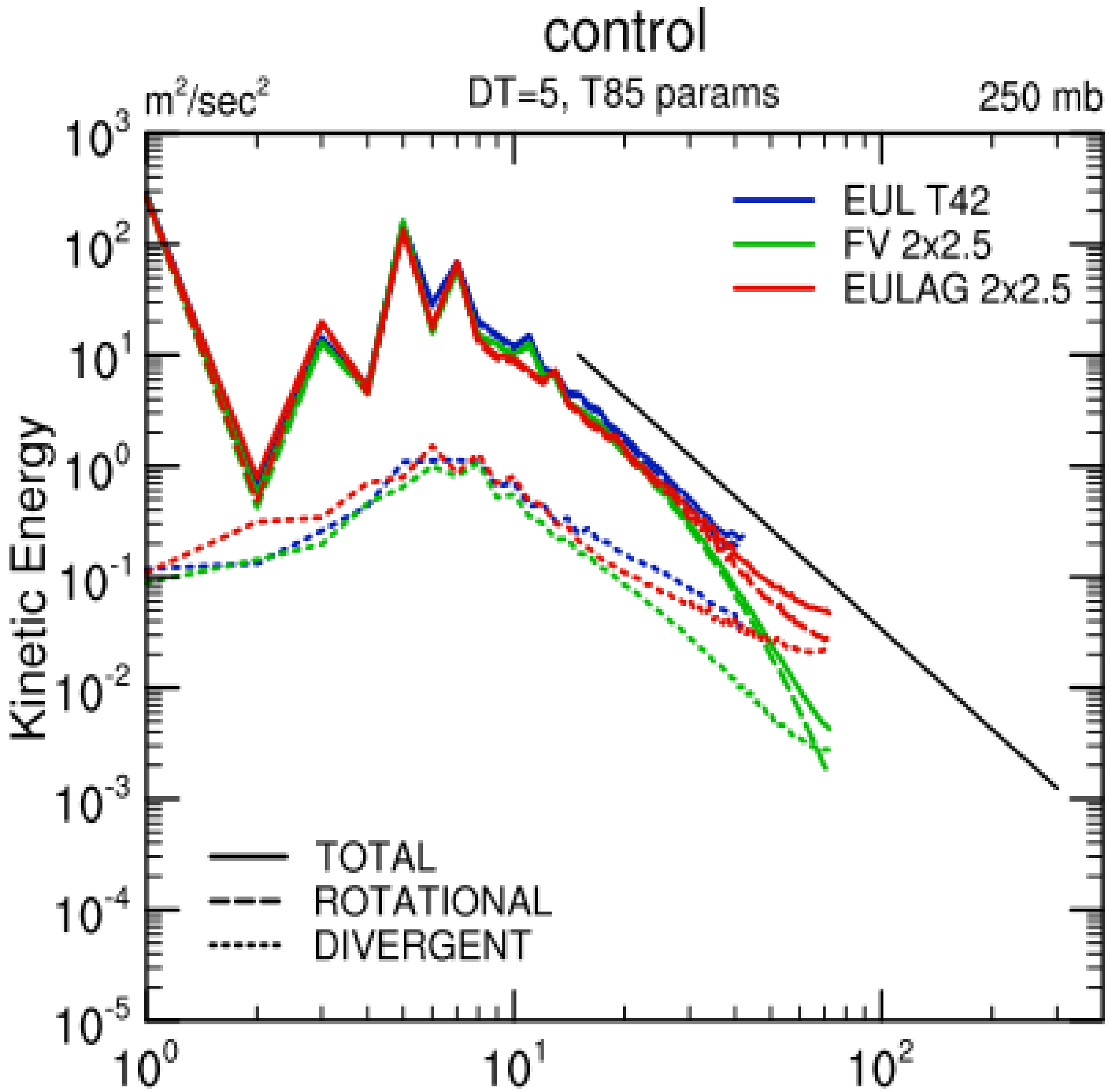
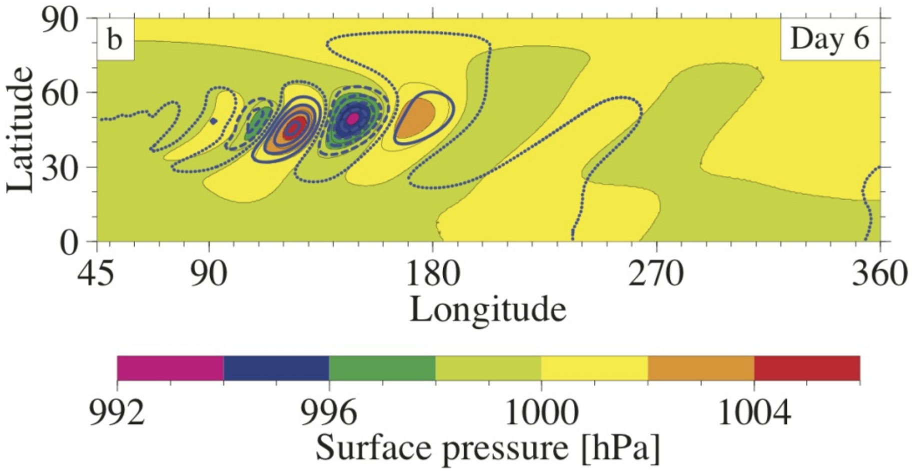
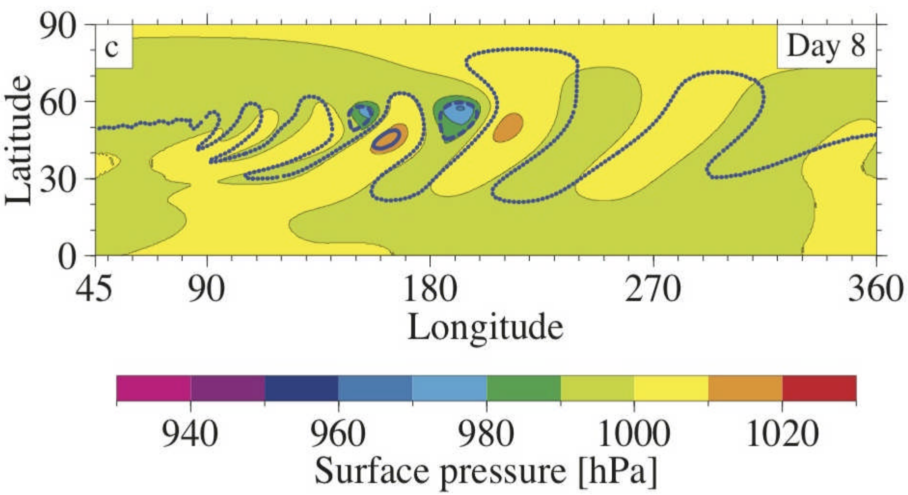


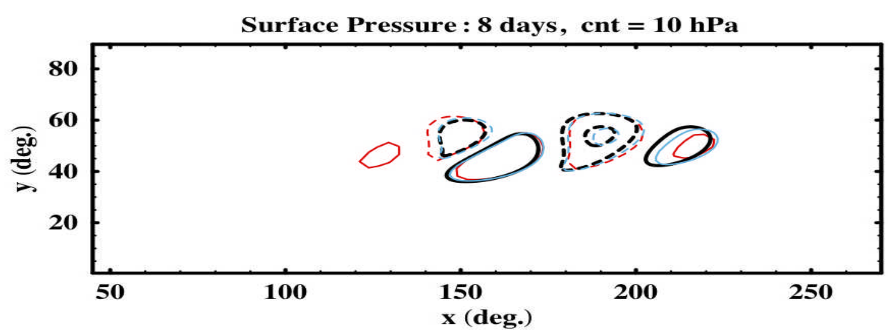
(D. Williamson, NCAR, 2007)

CAM-EULAG aqua-planet simulation agrees well with CAM3. Similar conclusion applies to baroclinic instability:



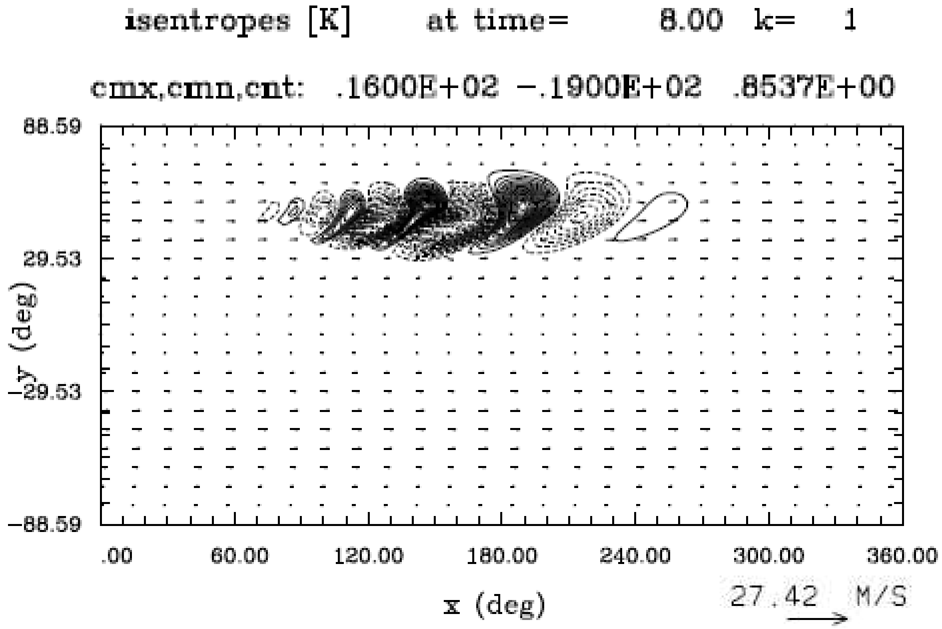
Lines & color: EULAG , 1.4˚, & Jablonowski & Williamson (2006), 0.7˚



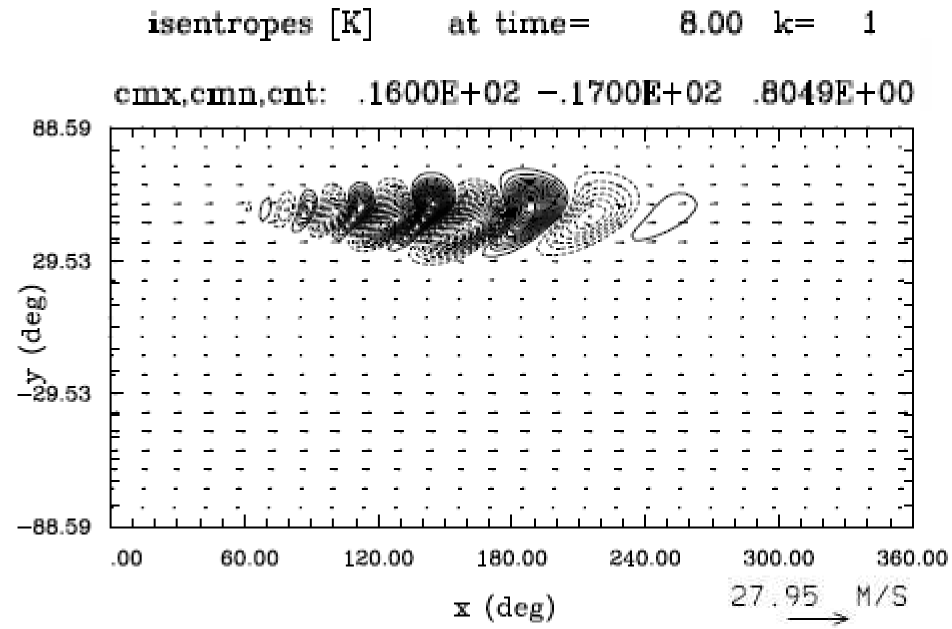


As resolution increases (2.8˚, 1.4˚, 0.7˚) wave ``tightens up'' in the longitudinal, thus improving comparison with JW

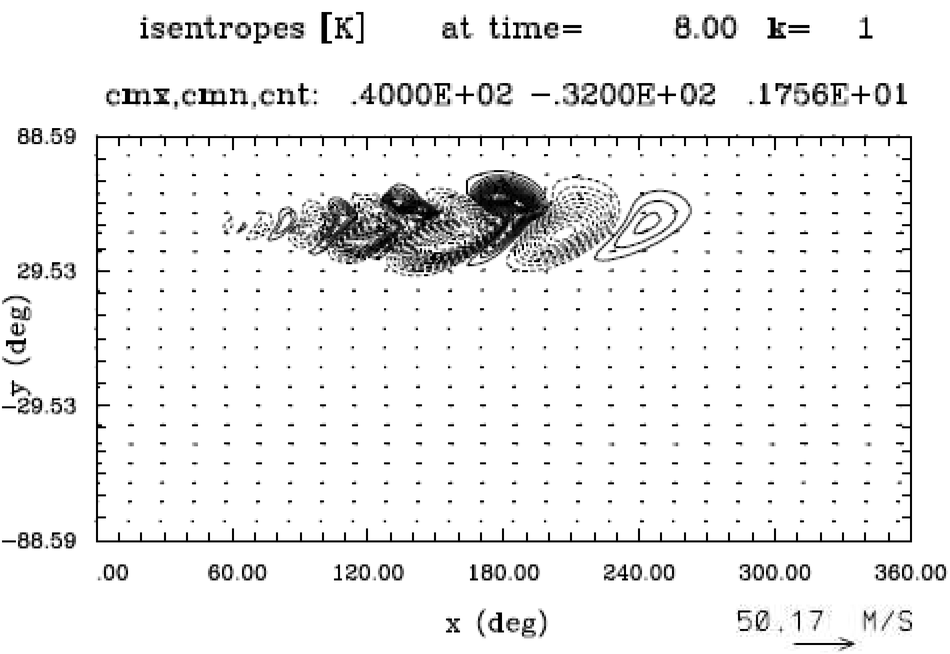
But →



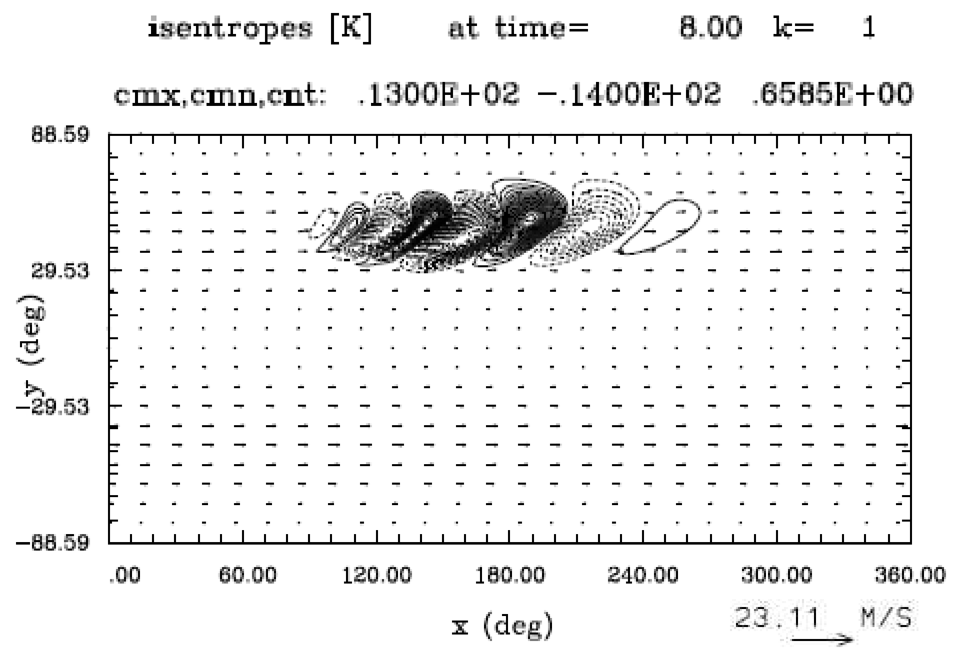
LH, implicit (dt=300s)



LH, explicit (dt=15s) ; ~20 x implicit CPU



PSI, implicit (dt=300s)



LH, implicit (dt=300s), SL2

Unstructured-mesh framework for atmospheric flows ❷

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Smolarkiewicz ↻ Szmelter, pubs in JCP, IJNMF, 2005-2010

- Differential manifolds formulation $\frac{\partial G\mathbf{\Phi}}{\partial t} + \nabla \cdot (\mathrm{V}\mathbf{\Phi}) = G\boldsymbol{\mathcal{R}}\ , \mathrm{V}(\mathrm{x}, t) := G\dot{\mathrm{x}}$

- Finite-volume NFT numerics with a fully unstructured spatial discretization, heritage of EULAG and its predecessors

$$\mathbf{\Phi}_i^{n+1} = \mathcal{A}_i(\mathbf{\Phi}^n + 0.5\delta t\,\boldsymbol{\mathcal{R}}^n,\ \mathrm{V}^{n+1/2}, G) + 0.5\delta t\,\boldsymbol{\mathcal{R}}_i^{n+1}$$

- Focus (sofar) on wave phenomena across a range of scales and Mach, Froude & Rossby numbers

- Sustained accuracy of structured grid discretization

- Static and dynamic mesh adaptivity

The edge-based discretisation

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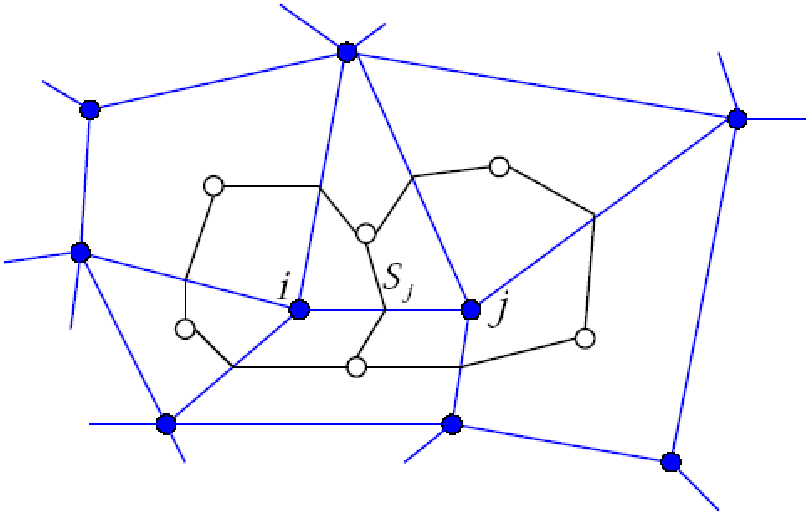
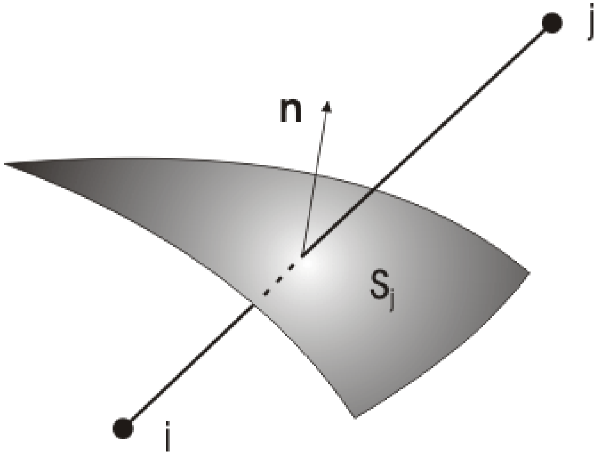
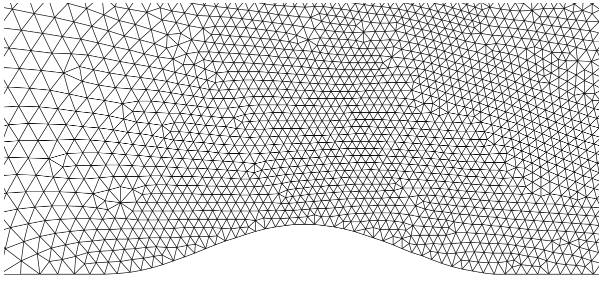
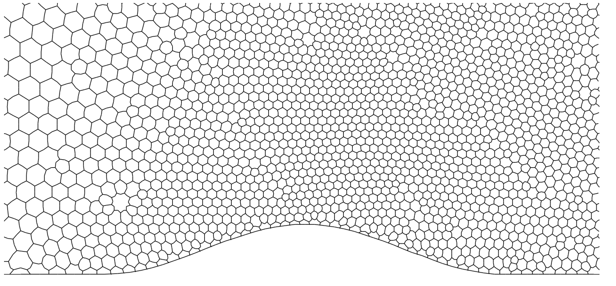






Edges



Dual mesh, finite volumes

Nonhydrostatic Boussinesq mountain wave

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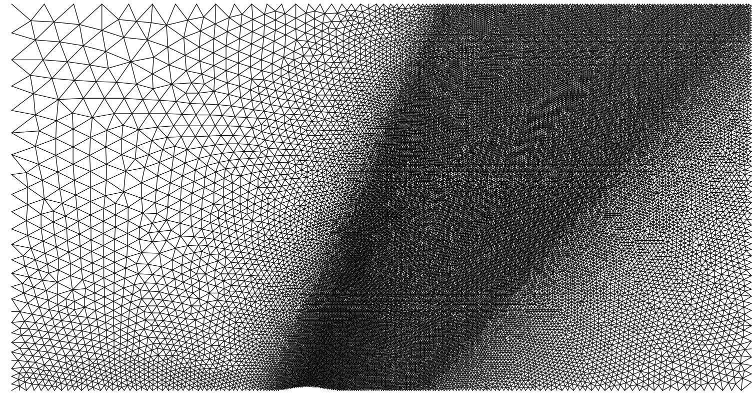
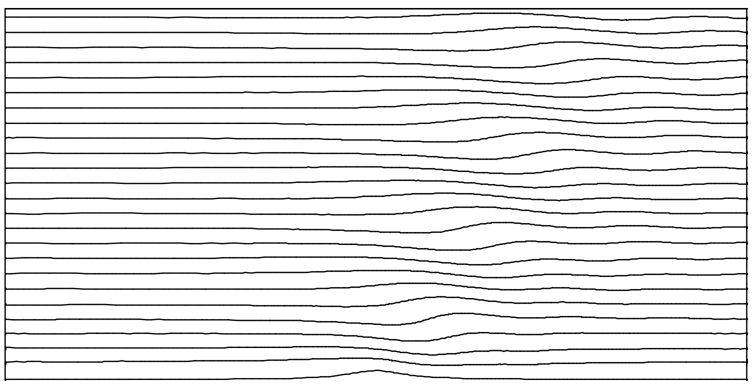
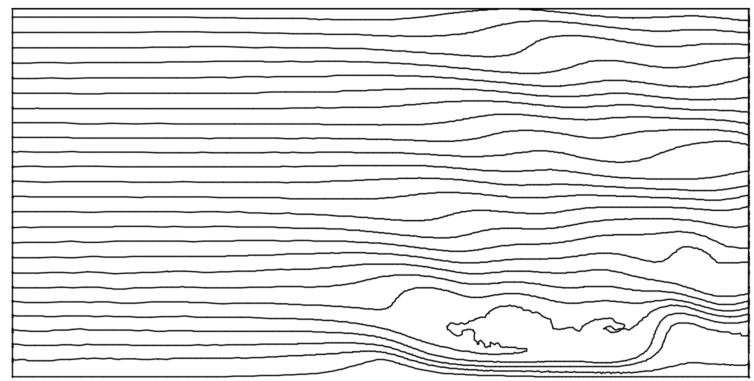
$$\nabla \bullet (\mathbf{V}\rho_o) = 0 \ ,$$

$$\frac{\partial \rho_o V^I}{\partial t} + \nabla \bullet (\mathbf{V}\rho_o V^I) = -\rho_o \frac{\partial \tilde{p}}{\partial x^I} + g\rho_o \frac{\theta'}{\theta_o}\delta_{I2}$$

$$\frac{\partial \rho_o \theta}{\partial t} + \nabla \bullet (\mathbf{V}\rho_o \theta) = 0 \ .$$







$Fr \lesssim 2$ $\quad$ $NL/U_o = 2.4$ $\quad$ $Fr \lesssim 1$

Comparison with the EULAG's results and the linear theories (Smith 1979, Durran 2003):
3% in wavelength; 8% in propagation angle; wave amplitude loss 7% over 7 wavelenghts

Non-Boussinesq amplification and breaking of vertically propagating gravity wave

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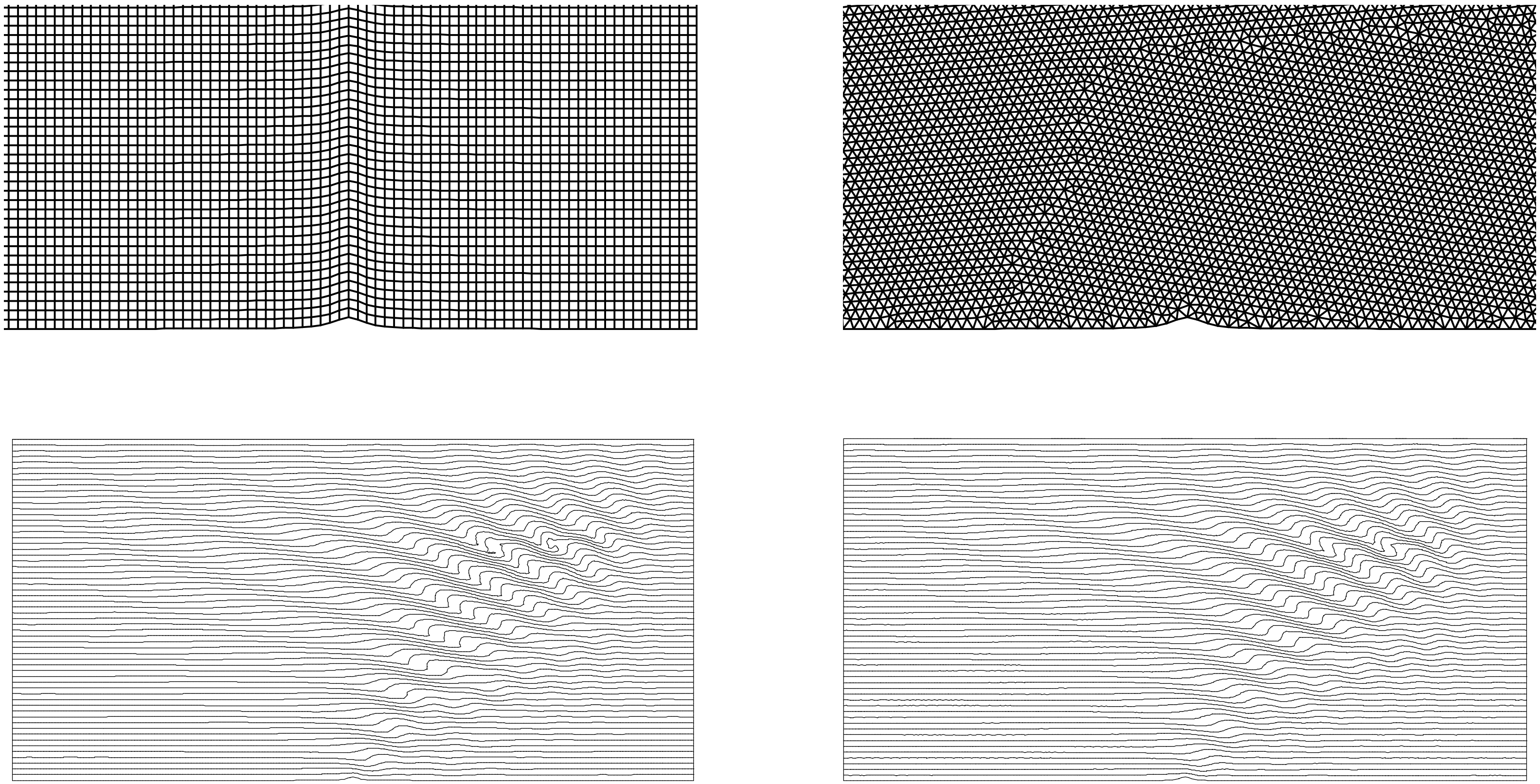
anelastic reference profiles: Bacmeister and Schoeberl, *JAS*, 1989



$NL/U_o \approx 1$, $Fr \approx 1.6$; $\lambda_o = 2\pi$ km □ $H_\rho \Rightarrow A(H/2) = 10h_o = \lambda_o$

Smolarkiewicz & Margolin, *Atmos. Ocean*, 1997; Klein, *Ann. Rev. Fluid Dyn.*, 2010

A global hydrostatic sound-proof model

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$$\frac{\partial G\mathcal{D}}{\partial t} + \nabla \cdot (G\mathbf{v}^*\mathcal{D}) = 0 \,,$$

$$\frac{\partial GQ_x}{\partial t} + \nabla \cdot (G\mathbf{v}^*Q_x) = G\left(-\frac{1}{h_x}\mathcal{D}\frac{\partial M}{\partial x} + fQ_y - \frac{1}{G\mathcal{D}}\frac{\partial h_x}{\partial y}Q_xQ_y\right) \,,$$

$$\frac{\partial GQ_y}{\partial t} + \nabla \cdot (G\mathbf{v}^*Q_y) = G\left(-\frac{1}{h_y}\mathcal{D}\frac{\partial M}{\partial y} - fQ_x + \frac{1}{G\mathcal{D}}\frac{\partial h_x}{\partial y}Q_x^2\right) \,,$$

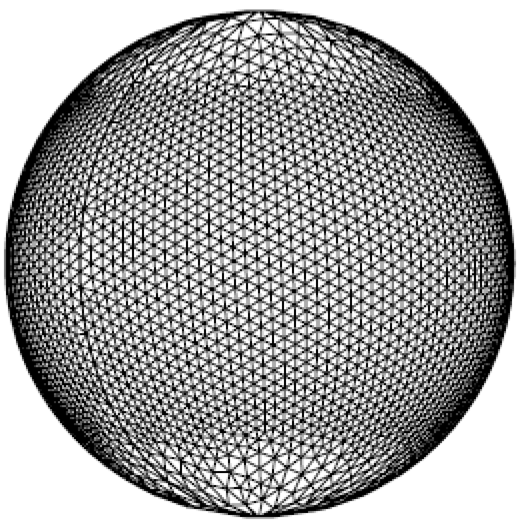
$$\frac{\partial M}{\partial \zeta} = \Pi \,.$$

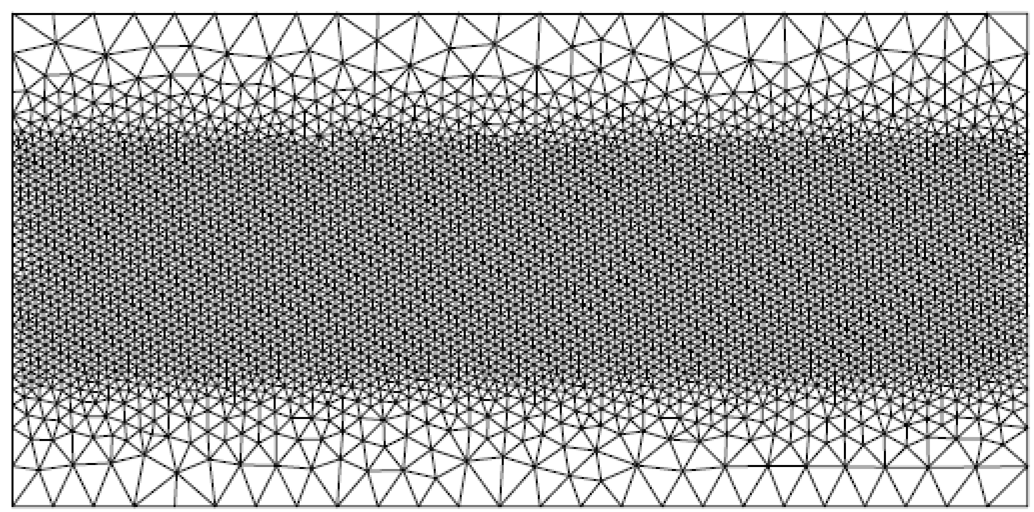
$$\mathcal{D} \equiv \partial p / \partial \zeta$$

$$M \equiv gh + \zeta$$

Isosteric/isopycninc model: $\zeta = \rho^{-1}$, =p

Isentropic model: $\zeta = \theta$, $\Pi = c_p (p/p_o)^{R_d/c_p}$



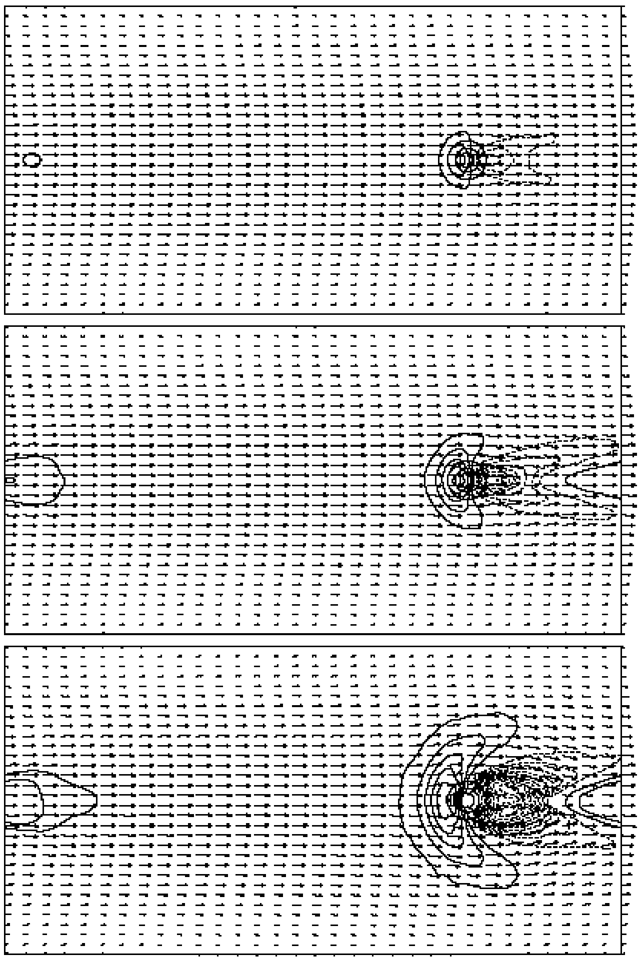


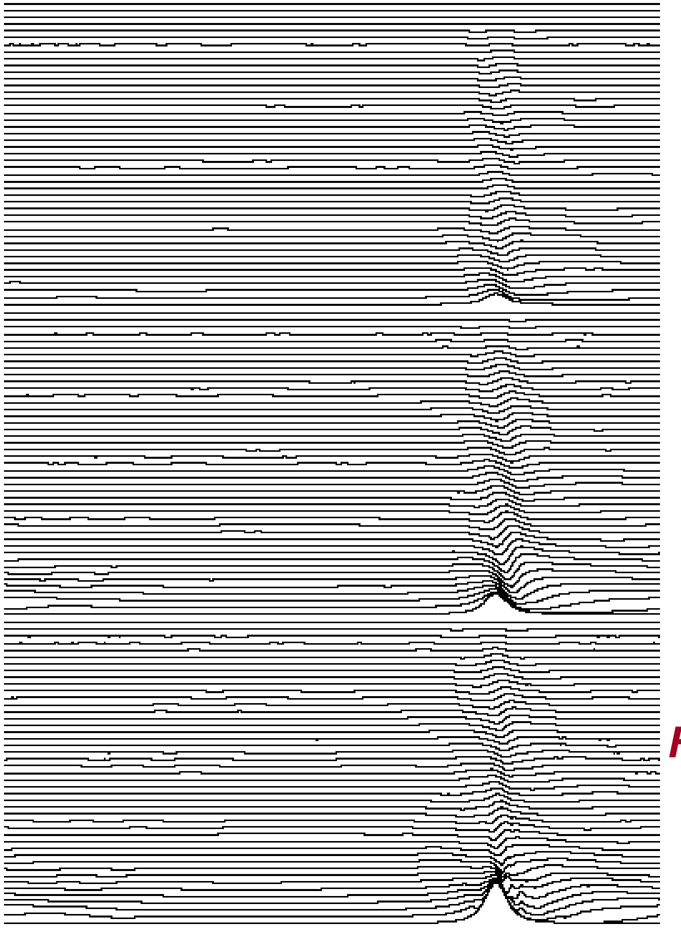
(Szmelter & Smolarkiewicz, J. Comput. Phys. 2010)

Stratified (mesoscale) flow past an isolated hill on a reduced planet

4 hours

$Fr = U_0/Nh$





Hunt & Snyder J. Fluid Mech. 1980; Smolar. & Rotunno, J. Atmos. Sci. 1989 ; Wedi & Smolar., QJR 2009

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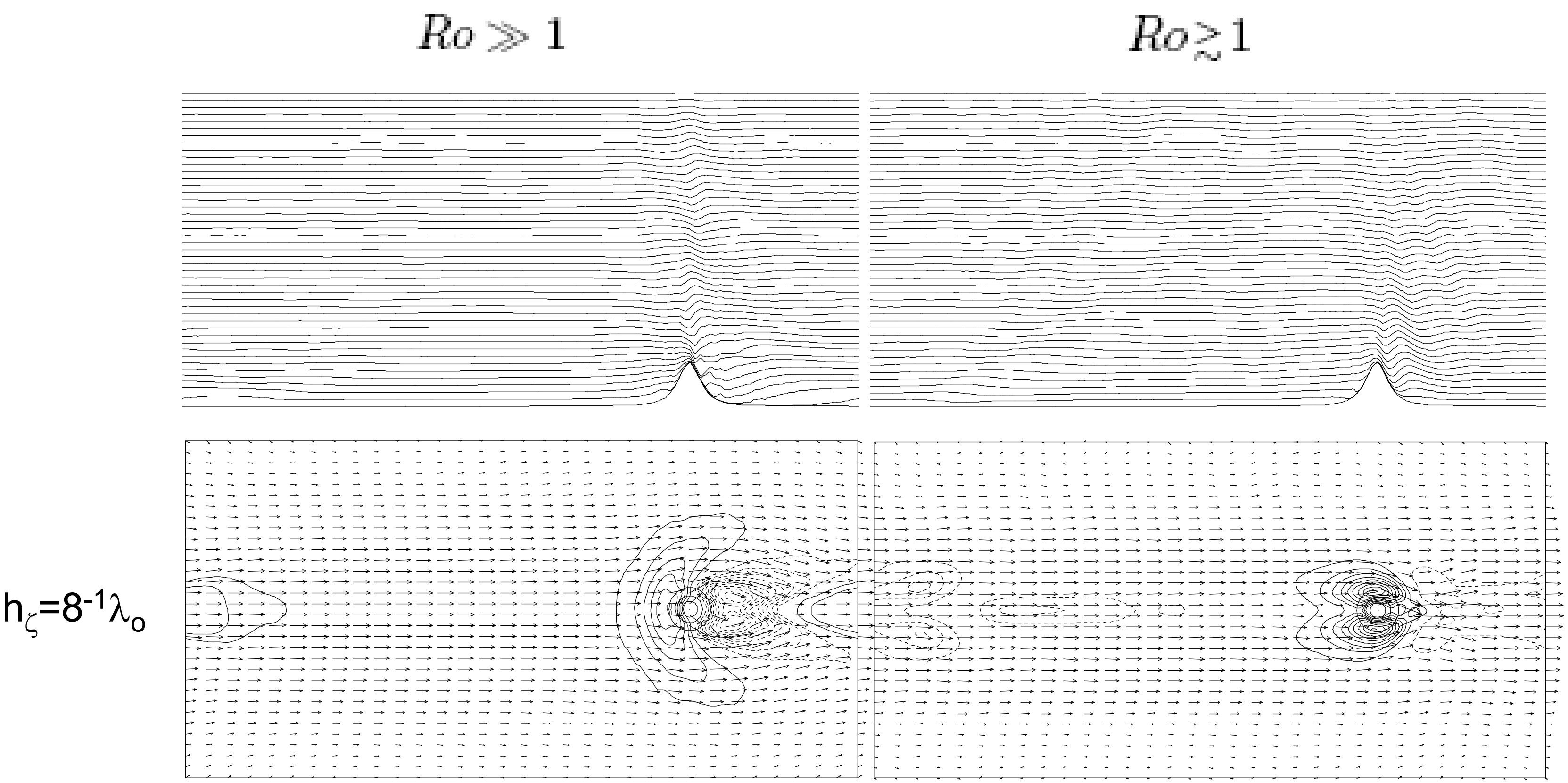
Fr=0.5



Smith, Advances in Geophys 1979; Hunt, Olafsson & Bougeault, QJR 2001

Conclusions:

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While some soundproof models may be better than others, it is difficult to find an example relevant to NWP and climate studies to show conclusively a failure of the soundproof approximation.

While some unstructured-mesh numerical techniques may be superior to others, there is sufficient evidence of the potential and merits of finite-volume numerics with a fully unstructured spatial discretisation for modelling atmospheric circulations of all scales.

It seems feasible that future atmospheric models will blend various equations and numerical methods