Efficient Preconditioning Techniques Applied to a Parallel Tsunami Simulation Model

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Overview

Tsunami

- Tsunami - Japanese: 'harbour wave'
- Reasons - earthquakes, land slides, volcanic eruptions and meteorite ocean impacts
- motion of the whole water column from surface to bottom
- in deep water ($h = 4000\ m$) tsunami waves have a wave length $\lambda > 200 km$ and an amplitude of a few centimetres
- in coastal regions the wave length decreases and the body of water piles up

Shallow Water Model

- describes 3D flow on the rotating earth by depth-integrated mass and momentum equations in 2 (horizontal) dimensions

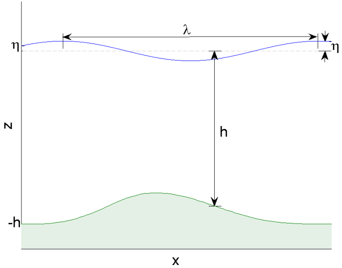
$$\frac{\partial}{\partial t}\eta + (\nabla \cdot \mathbf{u})(\eta + h) = 0, \qquad (1)$$

$$\frac{\partial}{\partial t}\mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + \mathbf{f} \times \mathbf{u} + \frac{\nabla p}{\rho} + \mathbf{g} + \mathbf{F} = 0, \qquad (2)$$

with surface water elevation $\eta(t, x, y)$ and horizontal velocity $\mathbf{u}(t, x, y)$ as unknowns.

Shallow Water Model

- condition - the vertical motion H of the fluid is very small with respect to the horizontal motion L



- $\delta := \frac{H}{L} \ll 1$
- characteristical values: $H = h$, $L = \lambda$

Pressure Term

- Seperately observation of hydrostatic and nonhydrostatic pressure $p = p_h + \hat{q}$
- Hydrostatic pressure $p_h = p_a + \rho g(\eta - z)$
- Here the atmospheric pressure p_a at the sea surface is neglected.

Classical, hydrostatic Shallow Water Equations ($\hat{q} \equiv 0$):

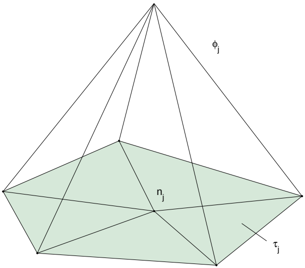
$$\tilde{\eta}_t + \nabla \cdot (\tilde{\mathbf{u}} H) = 0, \tag{3}$$

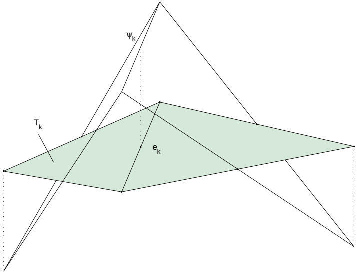
$$\tilde{\mathbf{u}}_t + (\tilde{\mathbf{u}} \cdot \nabla)\tilde{\mathbf{u}} + \mathbf{f} \times \tilde{\mathbf{u}} + g\nabla\tilde{\eta} + \mathbf{F} = 0, \tag{4}$$

with $\tilde{\mathbf{u}} = (\tilde{u}, \tilde{v})$.

TsunAWI - Discretization

- time - Leapfrog time-stepping scheme with Robert-Asselin-Filter
- space - P_1-P_1^{NC} Finite Element Method on unstructured grids





Nonhydrostatic Correction Terms

- Idea: nonhydrostatic model = hydrostatic model + nonhydrostatic correction (R. Walters, 05)
- linearization of depth-integrated $\hat{q} = \frac{1}{2}(q_\eta + q_{-h})$
- boundary condition at the surface: $q_\eta = q(t, x, y, \eta) = 0$
- correction term depends only on nonhydrostatic bottom pressure $q := q_{-h}$

Additional Unknown: Bottom Pressure q

- Inclusion of nonhydrostatic correction equations in the integral continuity equation

$$\int \phi_i(\nabla \cdot \mathbf{u} + \partial_z w)\mathrm{d}V = 0, \tag{5}$$

- partial integration and sorting of the terms depending on q to the left and others to the right

$$\mathbf{Aq} = \mathbf{b}. \tag{6}$$

Additional Unknown: Vertical Velocity w

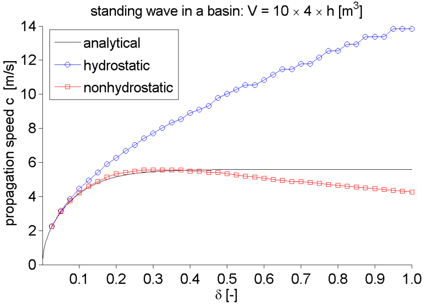
- linearization of the depth-integrated vertical velocity: $w = \frac{1}{2}(w_\eta + w_{-h})$
- kinematic boundary condition: $w_{-h} = -\mathbf{u} \cdot \nabla h$
- momentum equation in z-direction with $q \equiv 0$

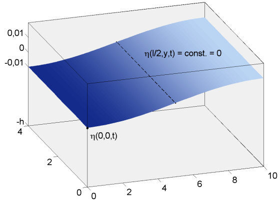
- FEM $\rightarrow$ 2 additional systems of equations
- saving work by *Lumping*: Approximation of mass matrix by diagonal matrix

Nonhydrostatic approach: costs

- 3 additional unknowns: q, w_{-h}, w_{η}
- 1 system of linear equations $\mathbf{Aq} = \mathbf{b}$
  - computation of the components of **A** and **b** in each timestep
  - pattern of **A** remains
- correction of $\tilde{u}$, $\tilde{v}$, $\tilde{w}$

Exampel: Standing Wave In A Basin





$$\delta = \frac{H}{L} = \frac{h}{\lambda}$$

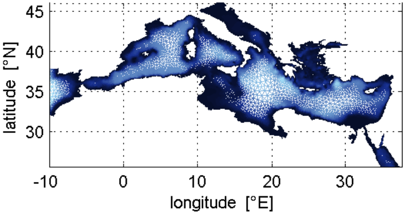
- hydrostatic: good results with $\delta < 0.1$
- nonhydrostatic: good results almost up to $\delta < 0.5$

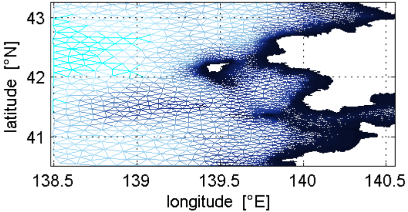
Overview

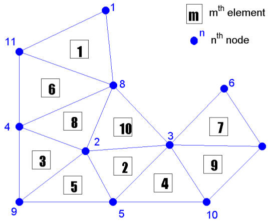
Software Packages

- used software package: METIS (G. Karypis, V. Kumar)
- routine METIS_PartGraphRecursive: using multilevel recursive bisection
  - Graph Type I: Element - Element
  - Graph Type II: Node - Node
  - Graph Type III: Node - Element
- minimization of the number of edgecuts to approximate the communication costs

Mesh Partitioning

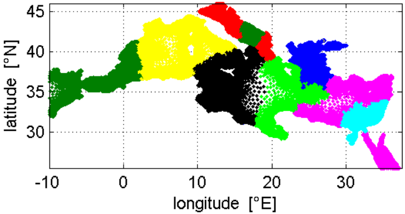


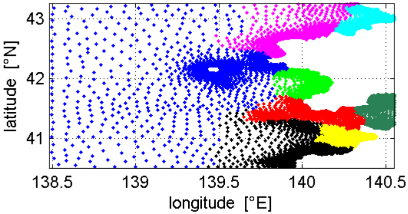


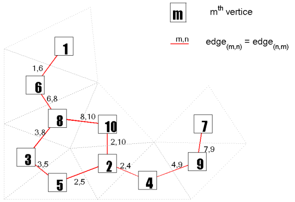


	Nnodes	Nelements
MED	298644	560704
OKU	45028	48330

Graph I : Element - Element

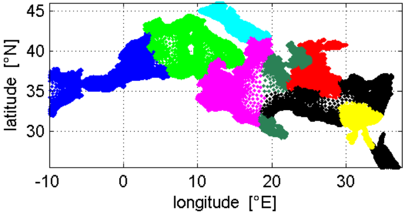


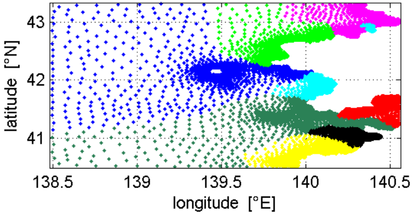


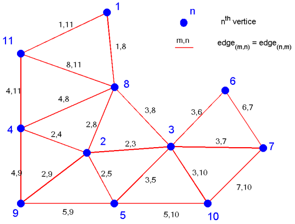


	$nloc/PE$	$\frac{Ninterface}{Nnodes}$
MED	35927 - 38388	0.15%
OKU	5496 - 5708	0.62%

Graph II : Node - Node

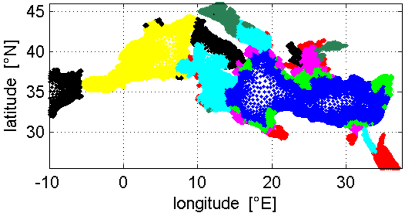


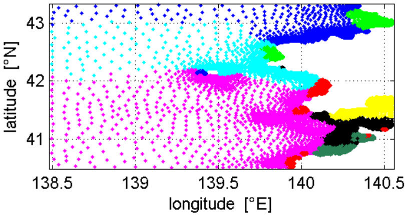


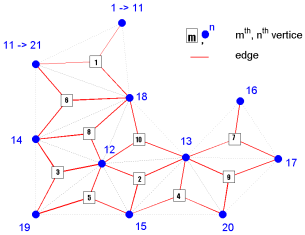


	$nloc / PE$	$\frac{Ninterface}{Nnodes}$
MED	37007 - 37708	0.15%
OKU	5628 - 5629	0.62%

Graph III : Node - Element







	$nloc/PE$	$\frac{Ninterface}{Nnodes}$
MED	35927 - 38388	1.06%
OKU	5557 - 5730	1.45%

Overview

Tsunami Simulation Model

Domain Decomposition Techniques

- Graph Partitioning
- From Mesh to Graph

Solvers and Preconditioners

- Solvers and Preconditioners
- Computer
- Results

Solvers

- PETSc - Portable, Extensible Toolkit for Scientific Computation
- Krylov Subspace Methods
  - GMRES(30)
  - BiCGStab

Preconditioners

- PETSc
  - Block Jacobi
  - restricted Additive Schwarz
- pARMS - parallel Algebraic Recursive Multilevel Solver
  - Schur Complement Preconditioner with local Incomplete LU-Factorization

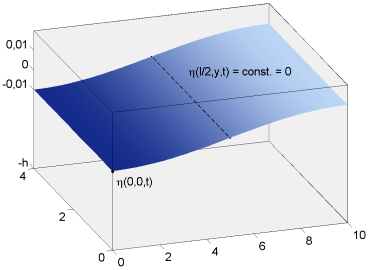
Computer

IBM BladeCenter

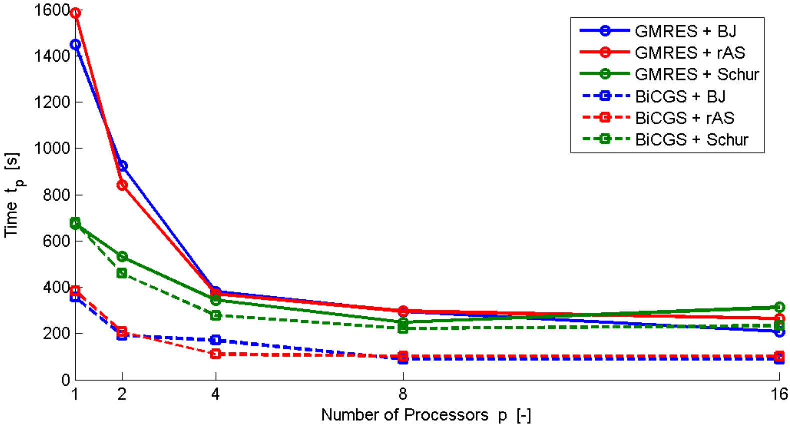
- 14 blades
- 4 Processor cores per blade
- Power 6 processors (4.0 GHz)
- 12 blades with 16 GB memory
- 2 blades with 32 GB memory
- 7.3 TB disk space

Test case : Standing Wave in a Basin

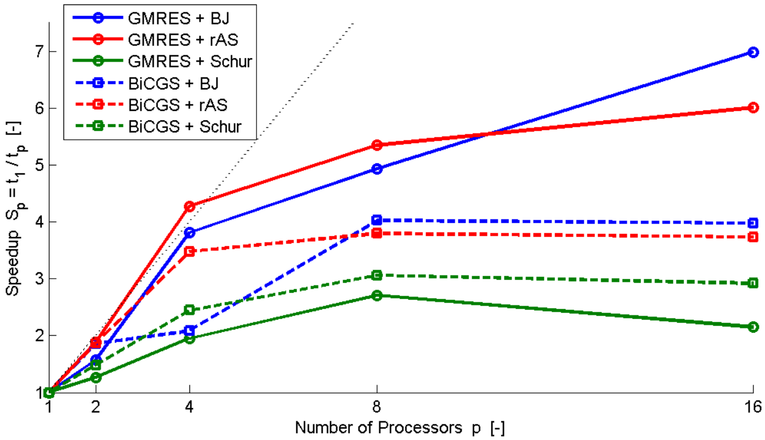
- Nnodes = 40313
- Nelements = 79851
- $\Delta t = 0.001$
- Number of timesteps: 200



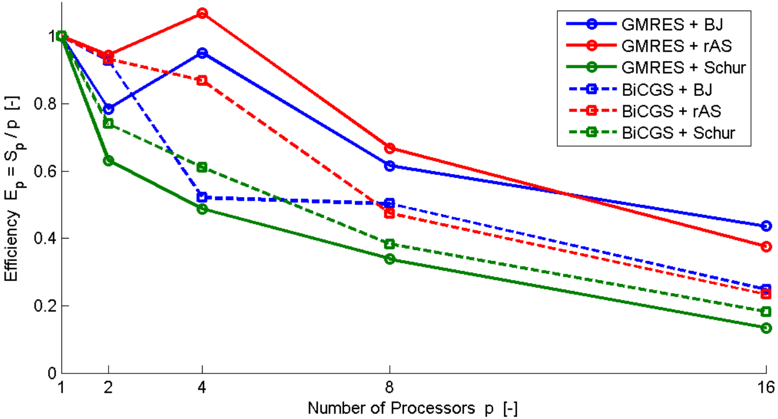
Results: Time



Results: Speedup



Results: Efficiency



Future Plans

Next steps:

- investigation of these techniques applied to a more complex tsunami szenario
- run both TsunAWI + Nonhydrostatic Correction in a parallel way

Aim:

- computation of the nonhydrostatic tsunami model in a reasonable time span