Diagnostics of model numerical cores

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Introduction

- ***Reductionism*: Assume that essential flow characteristics remain unchanged when the domain, the forcings or the equations are changed. Typically, a complete physical similarity is precluded, but an approximate dynamic similarity can be of substantial practical importance *(Buckingham, 1914)*.**
- **Often a hierarchy of numerical model setups with gradually increased complexity is used.**
- **Diagnostics include the comparison with analytic solutions, with large-eddy or global-scale simulation benchmarks, or with geophysical laboratory experiments and direct numerical simulations (DNS) thereof.**

Hierarchy of reduced models

- Single-column model (1-D, model physics)
- X-Z slice model (2-D, multi-layer, physics and/or dynamics)
- Shallow water model (2-D, single layer, model dynamics)
- The full numerical model (3-D, multi-layer, physics and/or dynamics)
  - Limited-area domain simulations
  - Reduced-size planet simulations
  - Held-Suarez “model climate” simulations
  - Aquaplanet “model climate” simulations
  - DNS/LES of a laboratory experiment

What is the model dynamical core ?

- Historically, the “dynamical” (dry) part of the model without any diabatic forcings (i.e. physics).
- However, in NWP with increased resolution O(0.1-10km) more and more of the physics (e.g. gravity waves, moist processes, convection) are resolved and it becomes increasingly ambiguous if a parametrized (subgrid-scale) term should be computed at all and if it should be computed in the “physics” interface or the “dynamics”.

The shallow water model

$$\frac{\partial \mathbf{v}_h}{\partial t} + \mathbf{v}_h \cdot \nabla \mathbf{v}_h + f\mathbf{k} \times \mathbf{v}_h + g\nabla h = 0$$

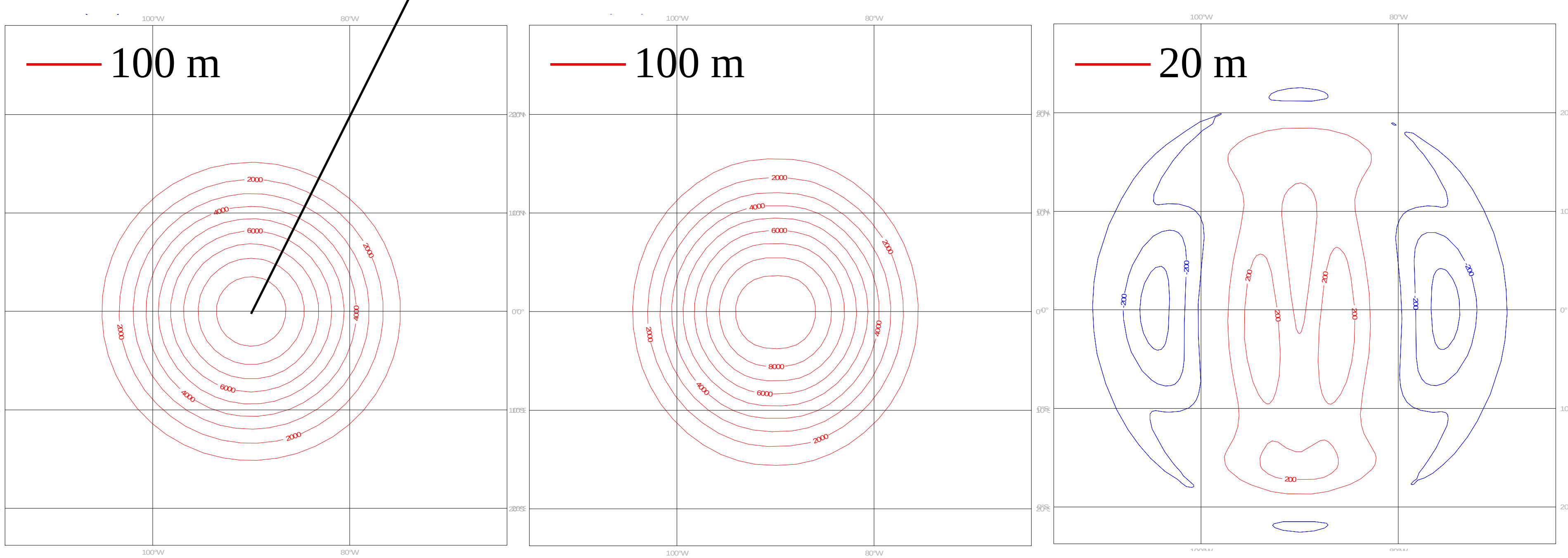
$$\frac{\partial h}{\partial t} + \mathbf{v}_h \cdot \nabla h + h\nabla \cdot \mathbf{v}_h = 0$$

- **Justification as a reduced model (Taylor, 1936; Matsuno, 1966)**
- **Published test cases** *Williamson et. al., JCP 102, p. 211-224 (1992)*
- **Example 1: Advection of cos-bell hill over the pole**
- **Example 2: The Rossby-Haurwitz wave and the influence of the lunar gravitational potential**

Example 1: Advection of Cosine bell over the pole (alpha=90 degrees)

IFS-SHW

$h_0 = 1000m$



Initial

After 12 days == 1 rotation

Difference

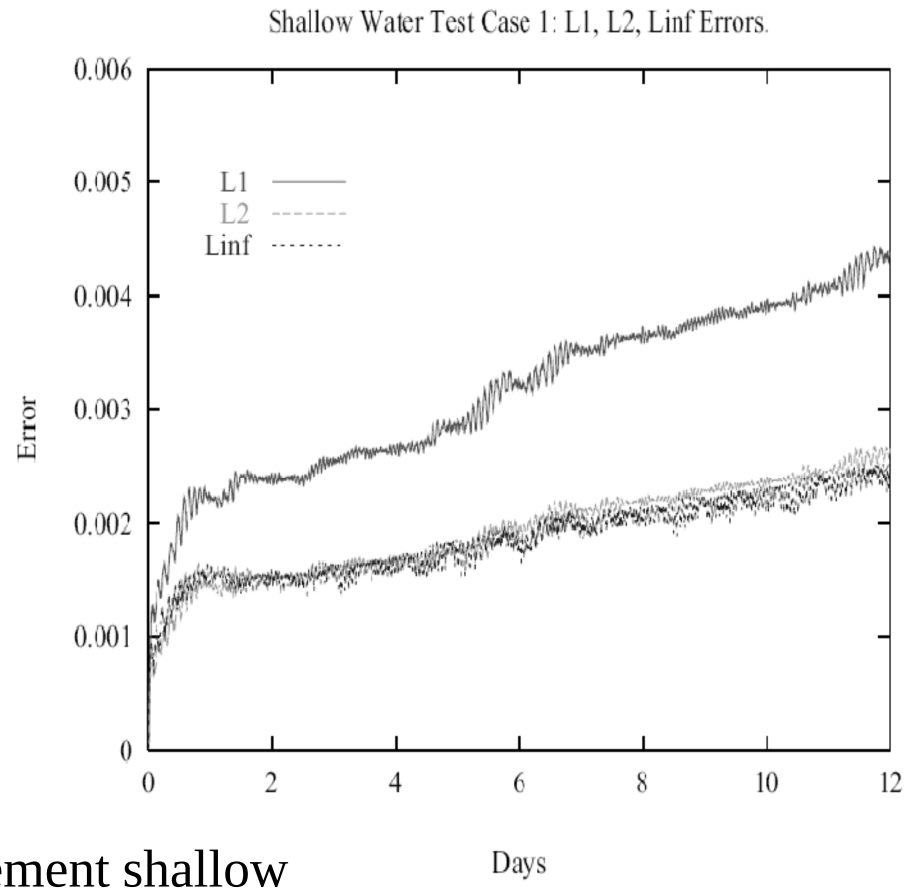
Example 1: Advection of Cosine bell over the pole (alpha=90 degrees)

- **Error measures graphed as a function of time or as a function of resolution**

$$l_1(h) = \frac{I[|h(\lambda,\theta) - h_T(\lambda,\theta)|]}{I[|h_T(\lambda,\theta)|]}$$

$$l_2(h) = \frac{\sqrt{I[(h(\lambda,\theta) - h_T(\lambda,\theta))^2]}}{\sqrt{I[h_T(\lambda,\theta)^2]}}$$

$$l_\infty(h) = \frac{\max_{all\,\lambda,\theta} |h(\lambda,\theta) - h_T(\lambda,\theta)|}{\max_{all\,\lambda,\theta} |h_T(\lambda,\theta)|}$$



Example: Semi-implicit spectral element shallow water model on the cubed sphere
(Thomas and Loft, J. Sci. Comp., 17, p.339-350 2002)

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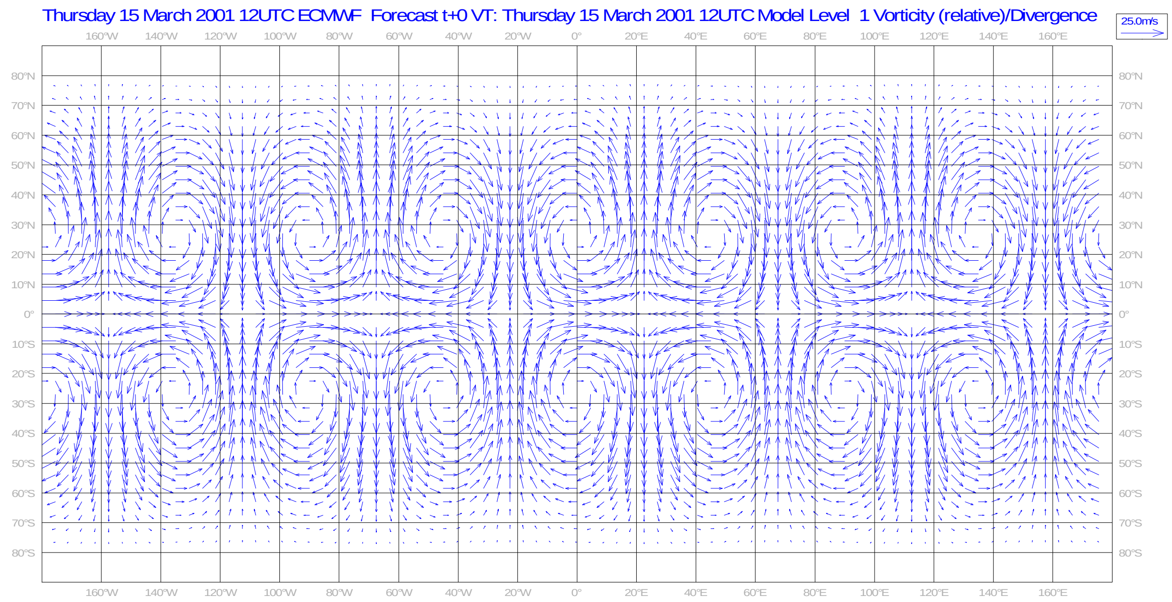
Example 2: The Rossby-Haurwitz wave ... and the influence of the Moon

- **Rossby-Haurwitz waves (here wavenumber 4) are steadily propagating solutions of the fully nonlinear nondivergent barotropic vorticity equation on the sphere (Haurwitz, 1940)**



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Example 2: The Rossby-Haurwitz wave … and the influence of the Moon

- **The analytic Rossby-Haurwitz wave initial condition is assumed to evolve similarly steady with the shallow water equations.**
- **The Rossby-Haurwitz wave is found to be unstable for wavenumbers >=4** *(Lorenz 1972, Hoskins, 1973, Thuburn and Li, 2000)*
- **Is the gravitational pull of the Moon sufficient to trigger such an instability of the Rossby-Haurwitz wave ?**

$$\frac{du}{dt} = \ldots - \frac{1}{a\cos\theta}\frac{\partial}{\partial\lambda}\Omega_{tide}$$

$$\frac{dv}{dt} = \ldots - \frac{1}{a}\frac{\partial}{\partial\theta}\Omega_{tide}$$

→ even for wavenumber 3 !

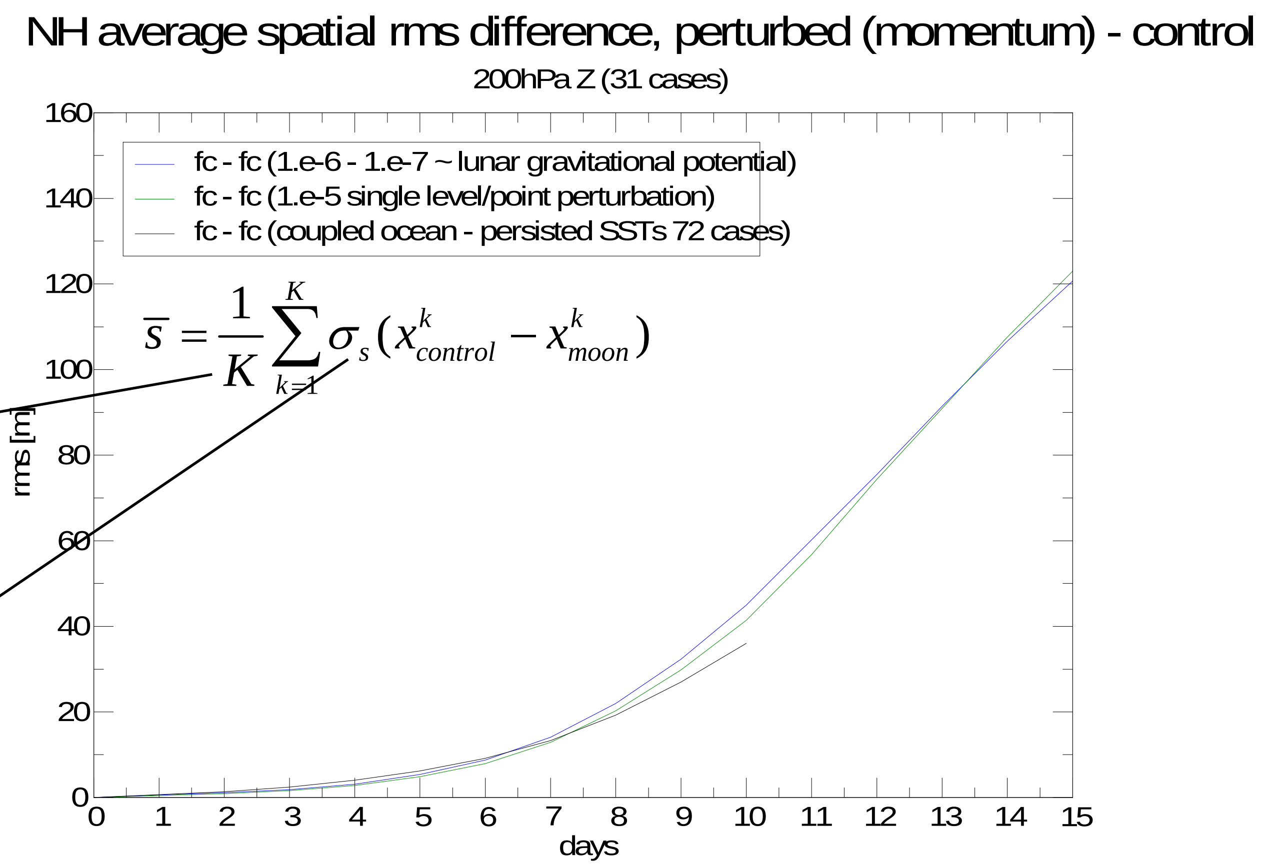
(Skiba, 1993 suggests >=2)

(e.g. Chapman and Lindzen 1970)

Animation:

Example 2: The Rossby-Haurwitz wave … and the influence of the Moon

- **Does the lunar gravitational potential matter for medium-range weather forecast ?**



$$\overline{s} = \frac{1}{K}\sum_{k=1}^{K} \sigma_s (x_{control}^k - x_{moon}^k)$$

number of forecast cases

spatial standard deviation of the forecast difference *(see Jung and Vitart, MWR, Vol.106, 2006 for details)*

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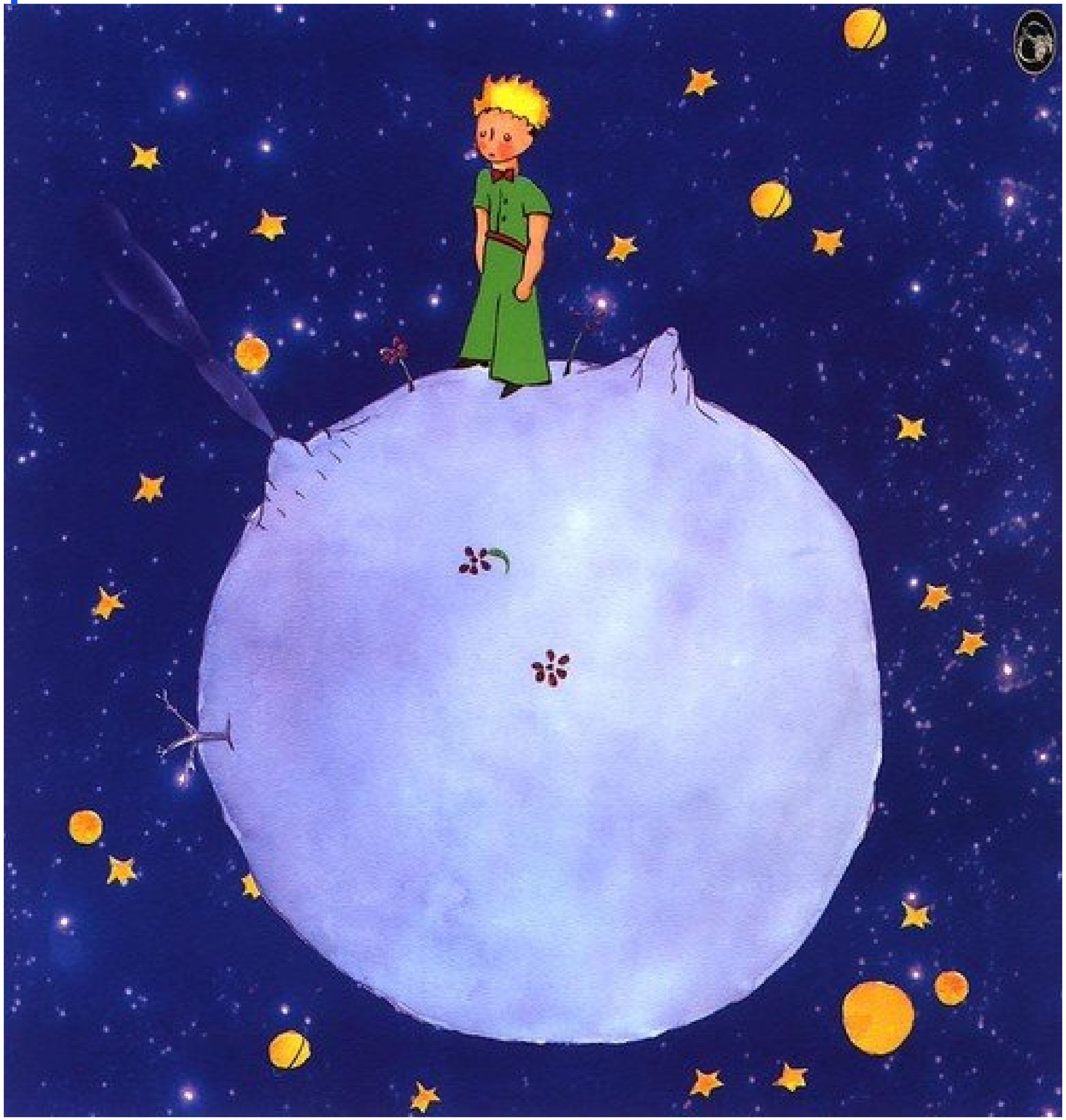
Local- and synoptic-scale simulations on the sphere ...

The size of the computational domain is reduced without changing the depth or the vertical structure of the atmosphere by changing the radius ($a < a_{Earth}$)



Wedi and Smolarkiewicz, Q. J. R. Meteorol. Soc. 135: 469-484 (2009)

Scale analysis for NH local-scale problems

$\delta = 0, \Gamma = 1$

shallow atmosphere approximation

$$\frac{du}{dt} = 2\Omega(v\sin\phi - \delta w\cos\phi) + \frac{uv}{\Gamma a}\tan\phi - \delta\frac{uw}{\Gamma a} - \frac{1}{\rho\Gamma a\cos\phi}\frac{\partial p}{\partial\lambda}$$

$$\frac{dv}{dt} = -2\Omega u\sin\phi - \frac{u^2}{\Gamma a}\tan\phi - \delta\frac{vw}{\Gamma a} - \frac{1}{\rho\Gamma a}\frac{\partial p}{\partial\phi}$$

U^2/L	f_0U	f_0W	U^2/a	UW/a	$\Delta p/\rho L$
10^{-1}	10^{-3}	10^{-4}	10^{-3}	10^{-4}	10^{-1}

a~100km

$$\frac{dw}{dt} = \delta 2\Omega u\cos\phi + \delta\frac{u^2+v^2}{\Gamma a} - \frac{1}{\rho}\frac{\partial p'}{\partial r} - g\frac{\rho'}{\rho}$$

UW/L	f_0U	U^2/a	$dP'/\rho H$	N^2H
10^{-2}	10^{-3}	10^{-3}	10^{-2}	10^{-2}

a~100km

Test-bed for NH effects

- 3D global simulations, without the prohibitive cost, when resolving non-hydrostatic effects.
- Study the influence of the model formulation and/or various numerical choices on selected wave-types in three dimensions.
- Use of the established vertical discretization and/or physical parameterization packages.
- Use of the existing optimized 3D code framework.

Description of experiments

- **All IFS experiments conducted in reduced-grid T_L159 or T_L255 configuration (320 or 512 gridpoints along equator).**
- **Comparison with analytic solutions and with the all-scale research code EULAG on the sphere.**

Hydrostatic approximation

$$\frac{dw}{dt} \square -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \quad \longrightarrow \quad \boxed{\frac{1}{\rho}\frac{\partial p}{\partial z} = -g}$$

⇨ Filters vertically propagating acoustic waves

w is obtained *diagnostically* from the continuity equation.

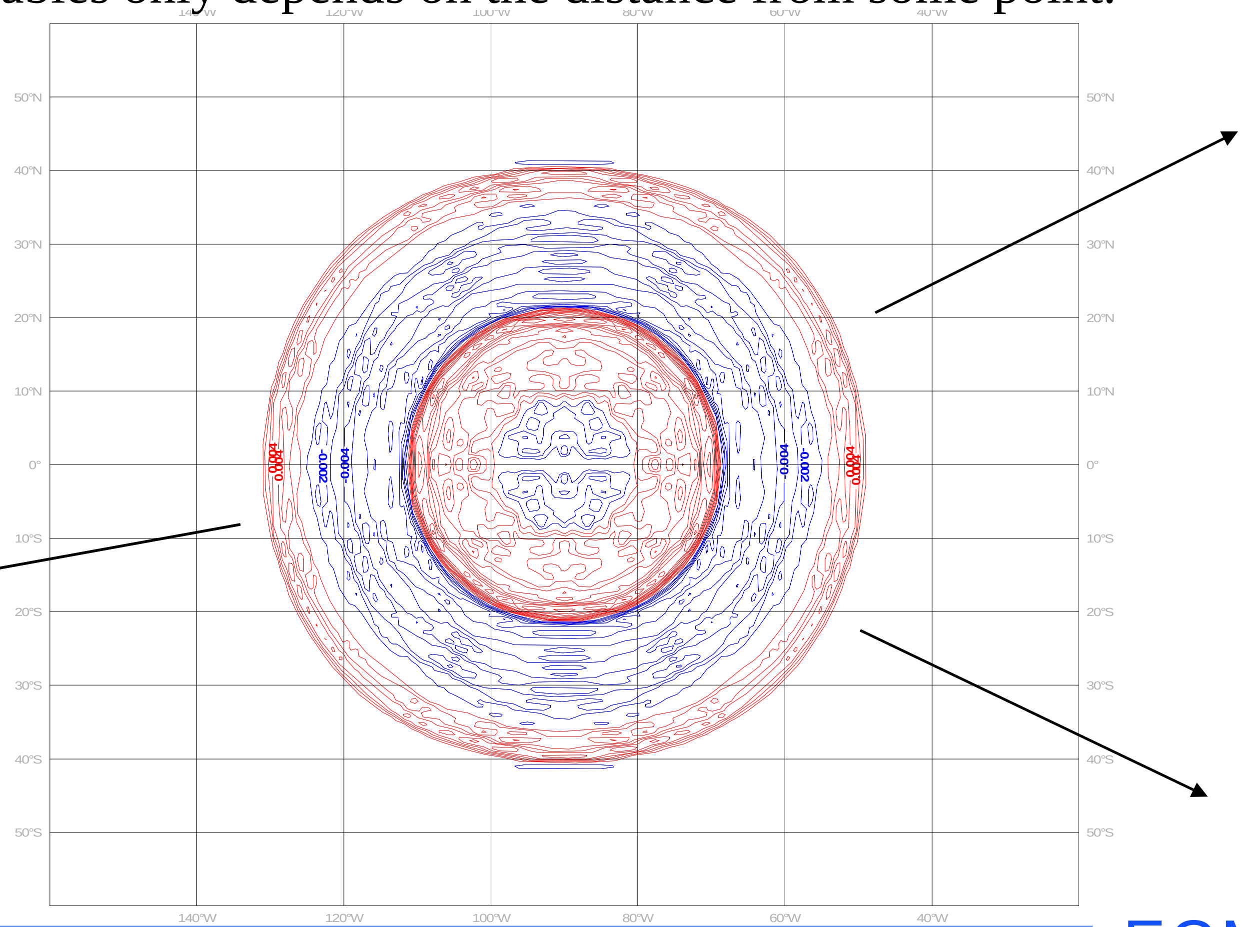
Acoustic waves

- The nonhydrostatic, fully elastic model admits the propagation of sound waves
- Comparison against analytic solution
- Impact of the semi-implicit formulation on the propagation

A spherical acoustic wave in a stratified (isothermal) atmosphere

analytical solution from (Landau and Lifshitz, Fluid Mechanics, §70.)

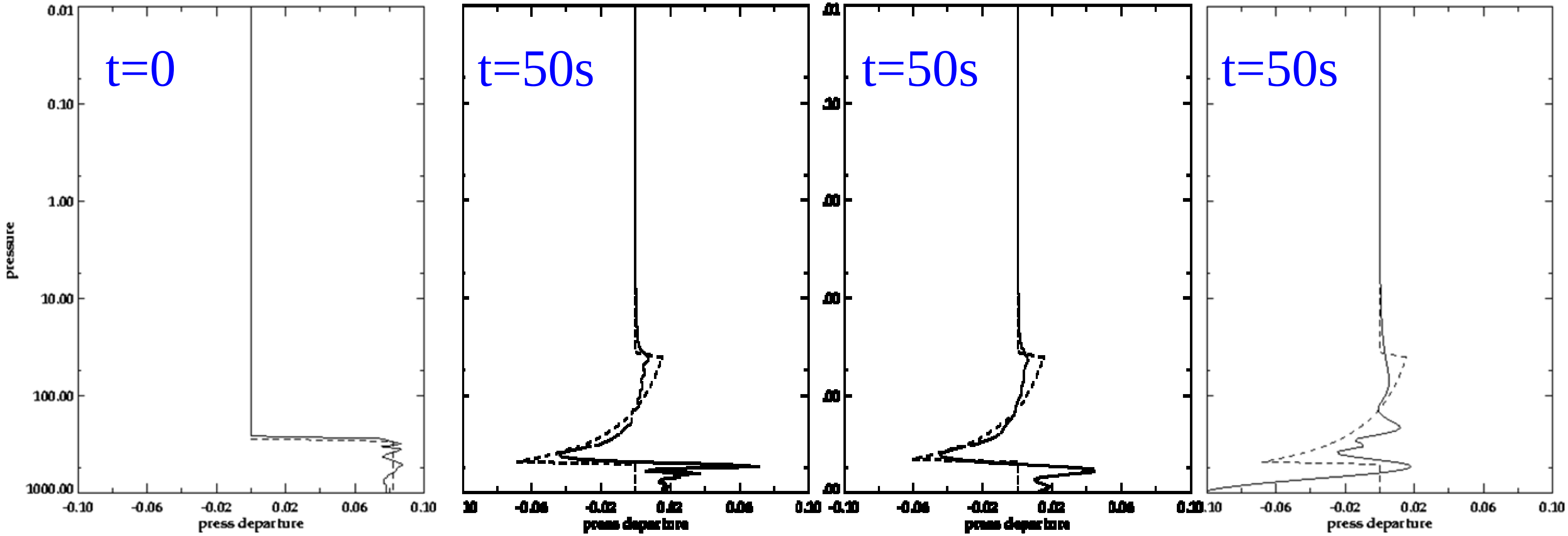
A sound wave in which the distribution of density, velocity and other flow variables only depends on the distance from some point.



C_0=340m/s

A spherical acoustic wave in a stratified (isothermal) atmosphere

analytical solution in dashed line



explicit, dt=0.01s | explicit, dt=0.01s | Semi-implicit, dt=1s | Semi-implicit, dt=10s

NH-IFS T_L159L91

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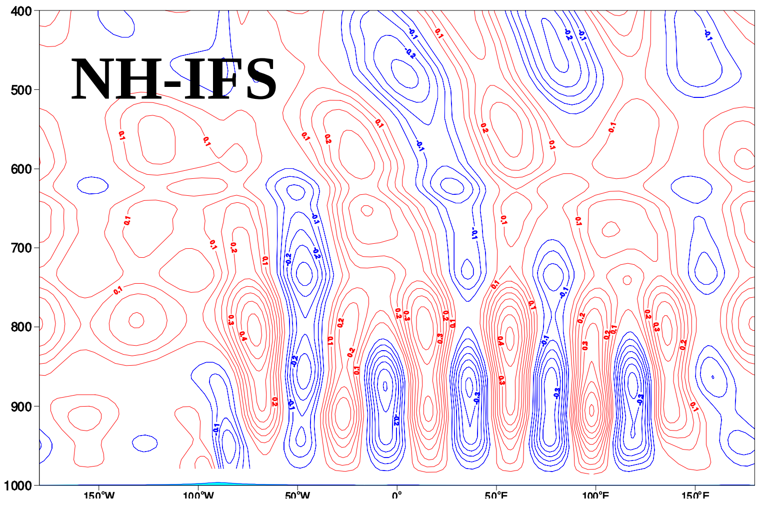
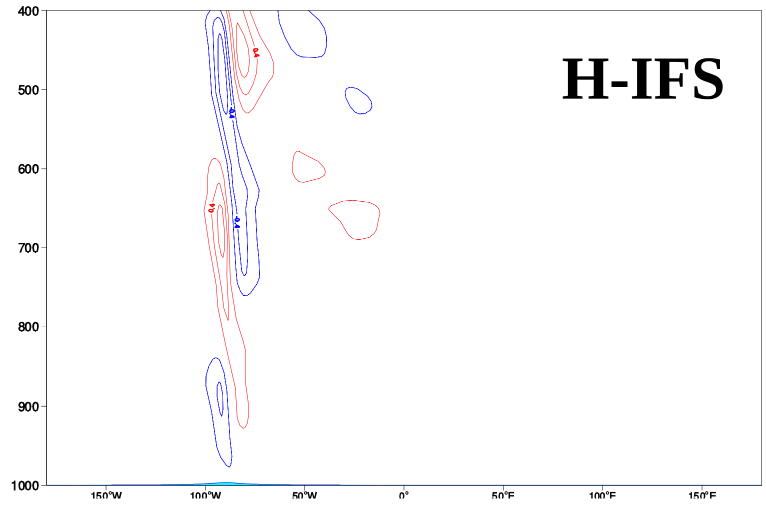
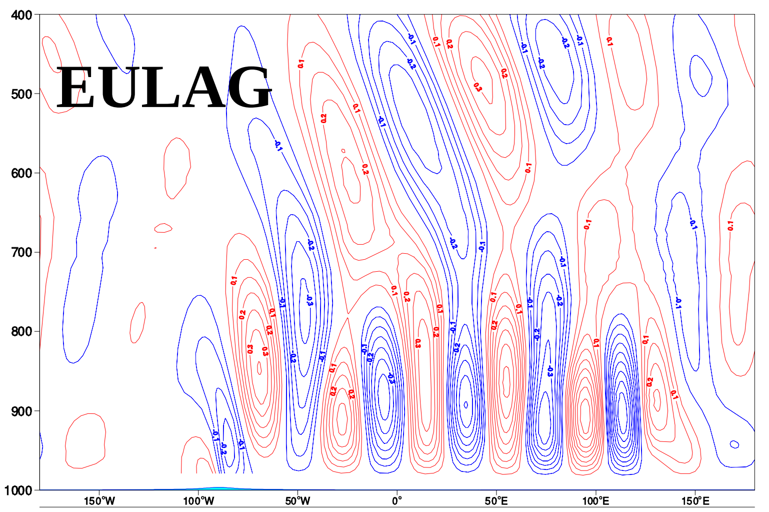
Gravity waves

- **Propagation**
- **Drag**

Quasi two-dimensional orographic flow with linear vertical shear





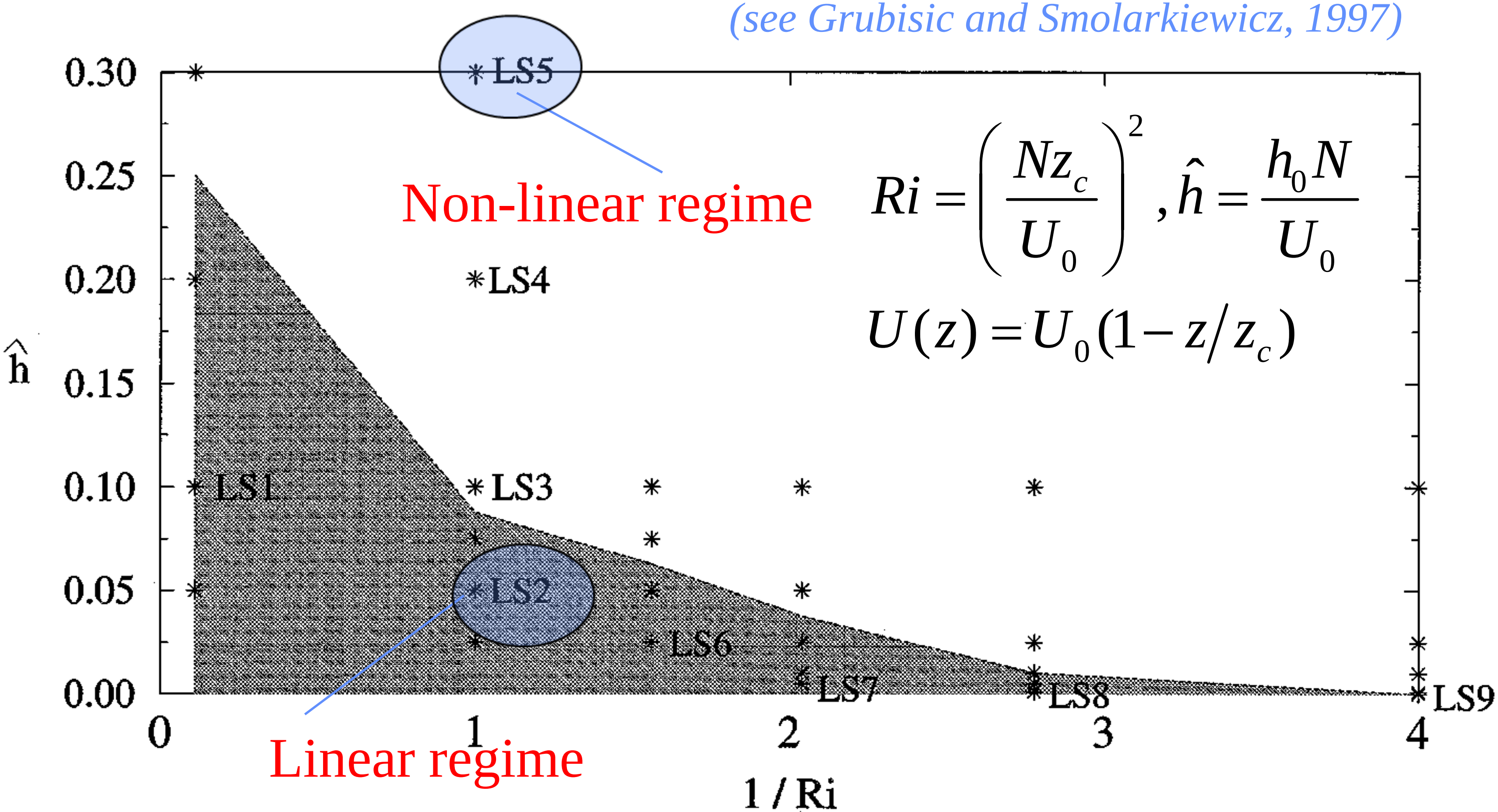


The figures illustrate the correct horizontal (NH) and the (incorrect) vertical (H) propagation of gravity waves in this case (Keller, 1994). Shown is vertical velocity.

The critical level effect on linear and non-linear flow past a three-dimensional hill

$Ri = 1$

(see Grubisic and Smolarkiewicz, 1997)



$$Ri = \left(\frac{Nz_c}{U_0}\right)^2, \hat{h} = \frac{h_0 N}{U_0}$$

$$U(z) = U_0(1 - z/z_c)$$

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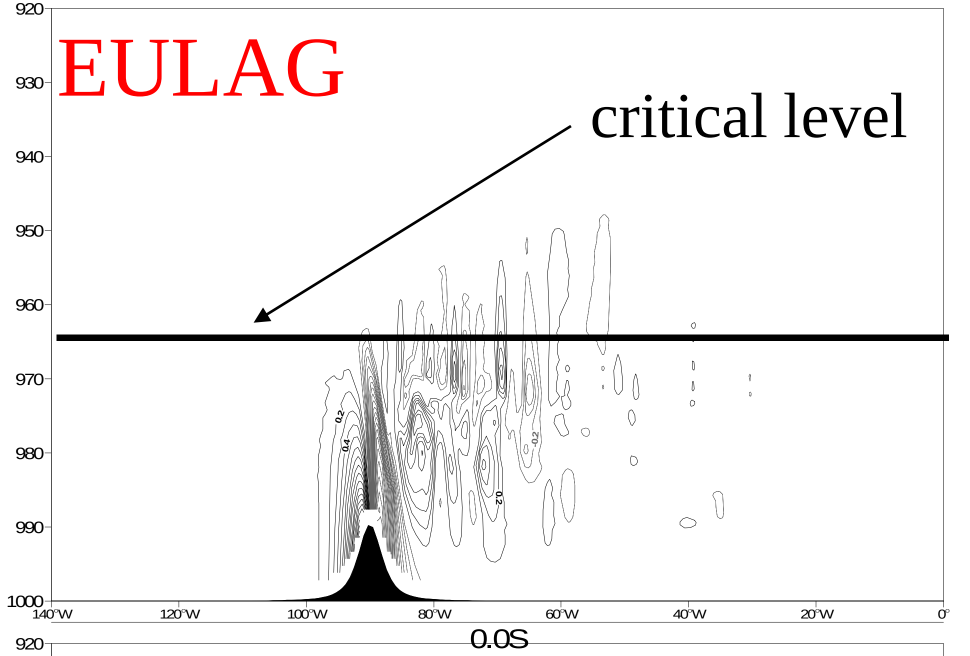
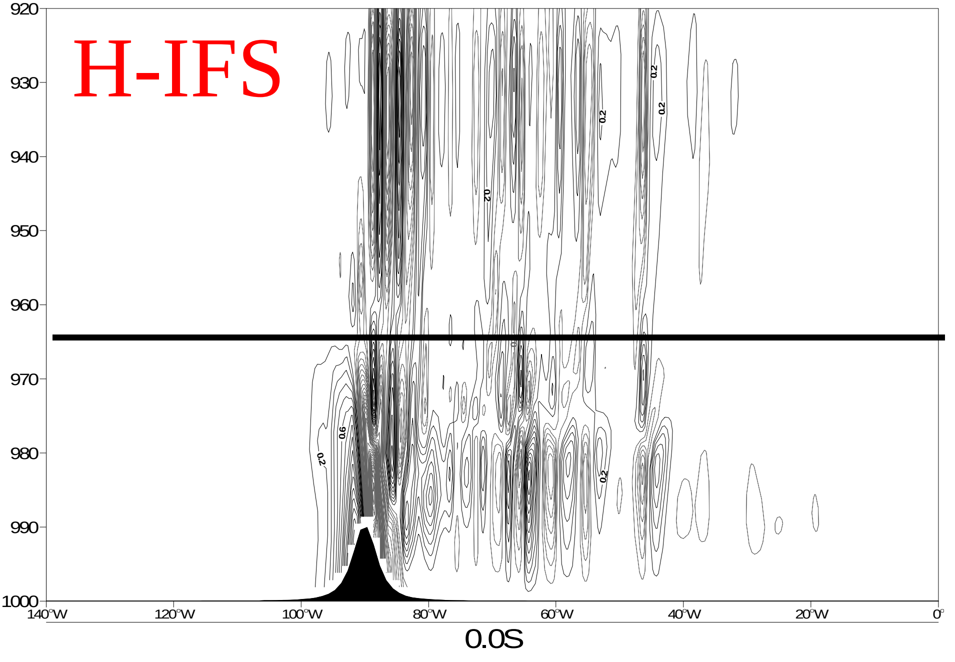
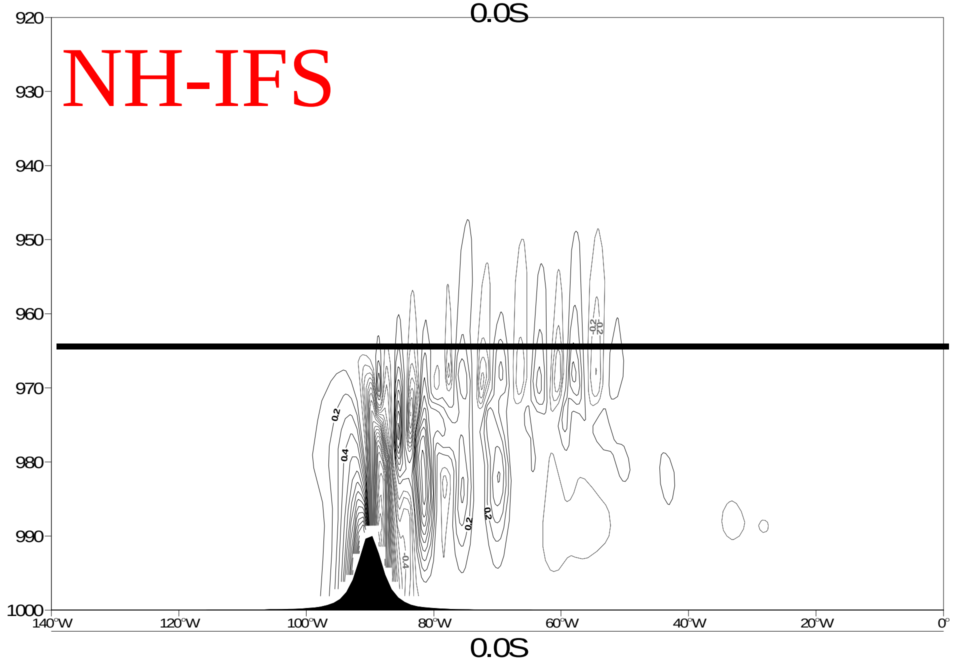
The critical level effect on linear and non-linear flow past a three-dimensional hill

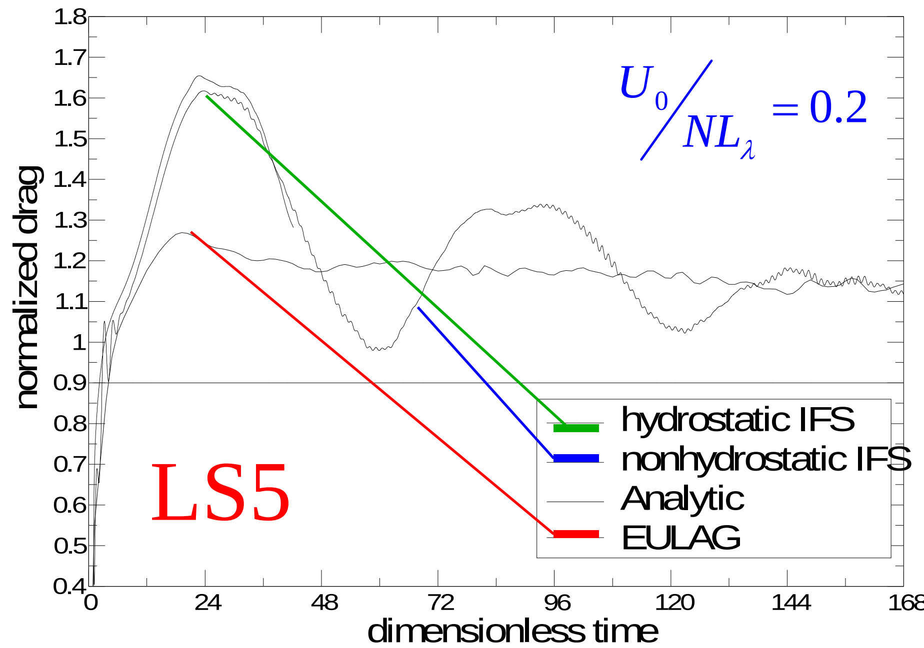
LS5 $U_0 / NL_\lambda = 1$

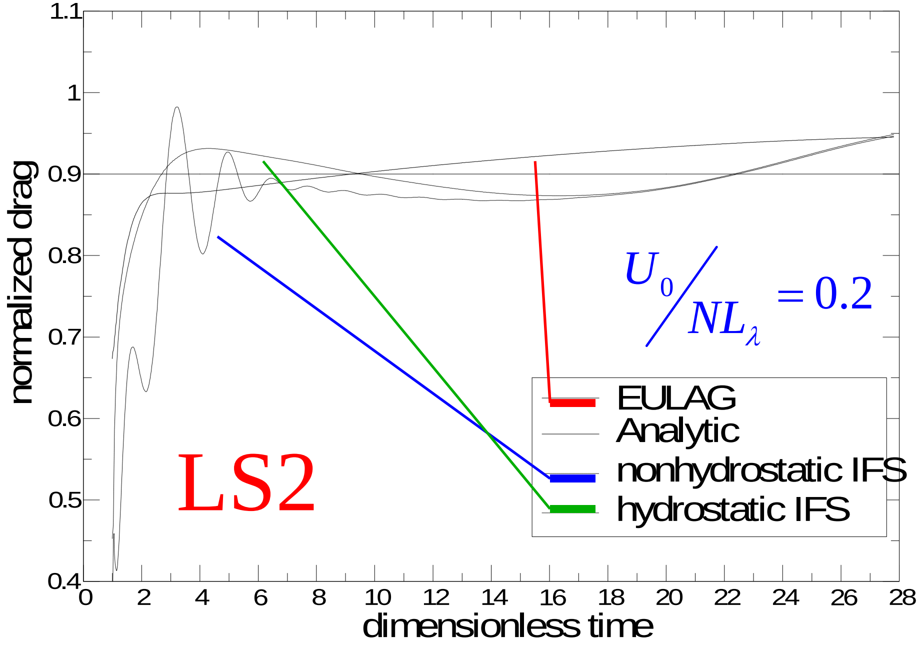


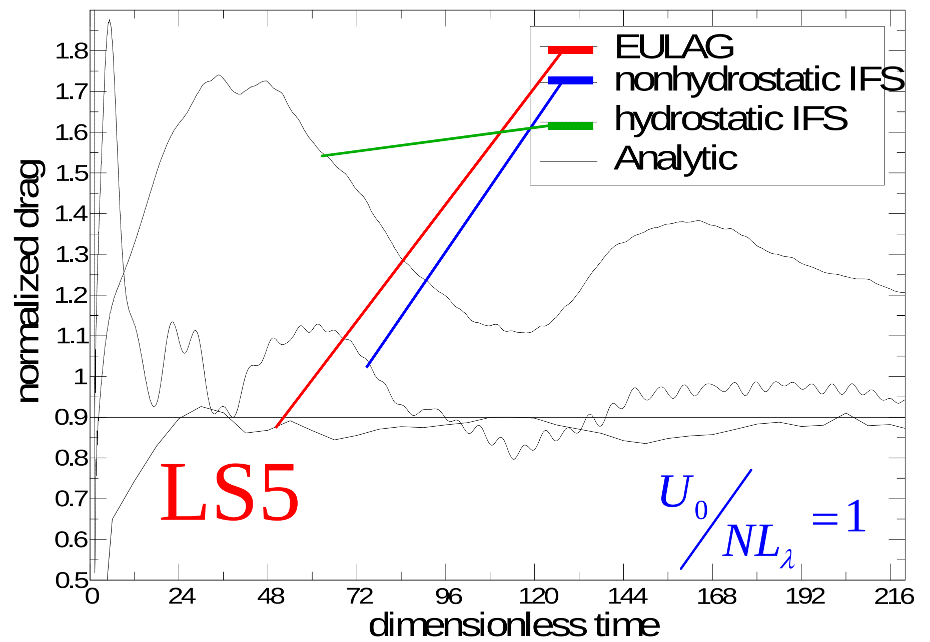




Mountain drag







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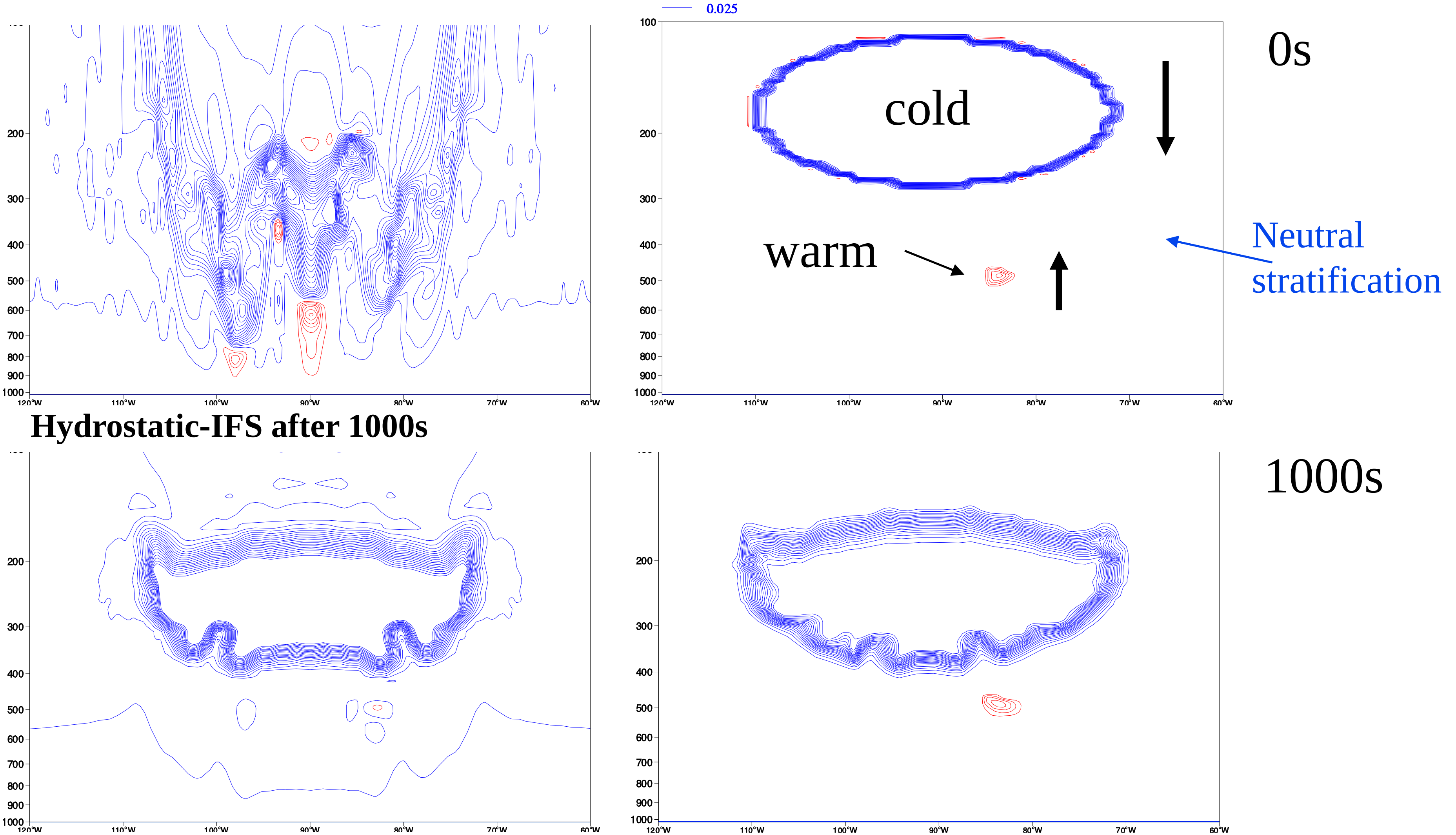
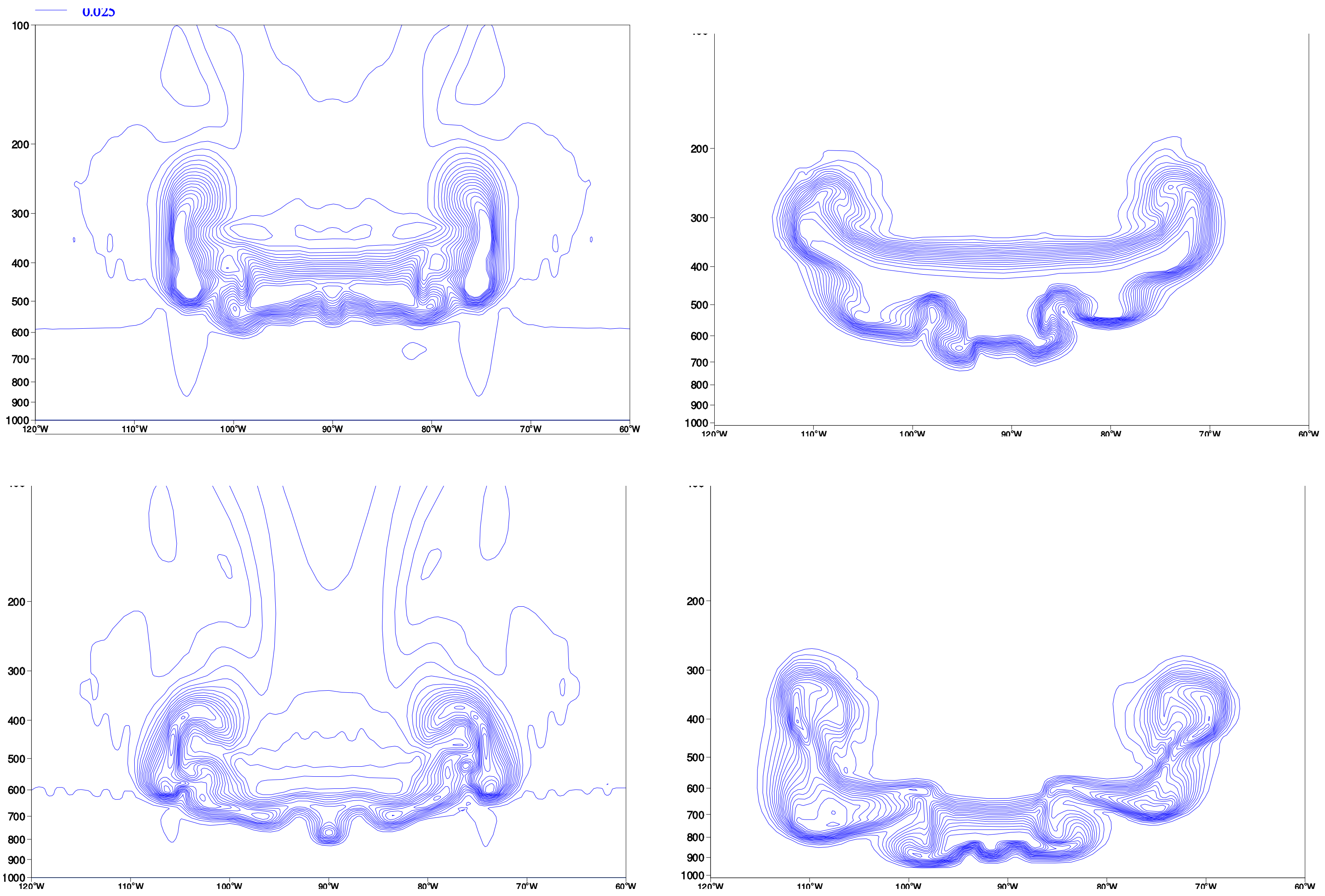
Convective motion (3D bubble test)



NH-IFS EULAG

Convective motion (3D bubble test)



1800s

2400s

NH-IFS

EULAG

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Held-Suarez “model climate” simulations

Held and Suarez (BAMS, 1994)

$$\frac{dv}{dt} = \ldots - k_v v$$

Augment frictional/heating terms

$$\frac{dT}{dt} = \ldots - k_T (T - T_{eq})$$

$$T_{eq} = \max\left\{200K, [315K - (\Delta T)_y \sin^2\phi - (\Delta\theta)_z \log\left(\frac{p}{p_0}\right)\cos^2\phi]\left(\frac{p}{p_0}\right)^{\kappa}\right\}$$

meridional temperature gradient pole - equator

vertical temperature gradient at equator

Held-Suarez simulations / Rossby waves

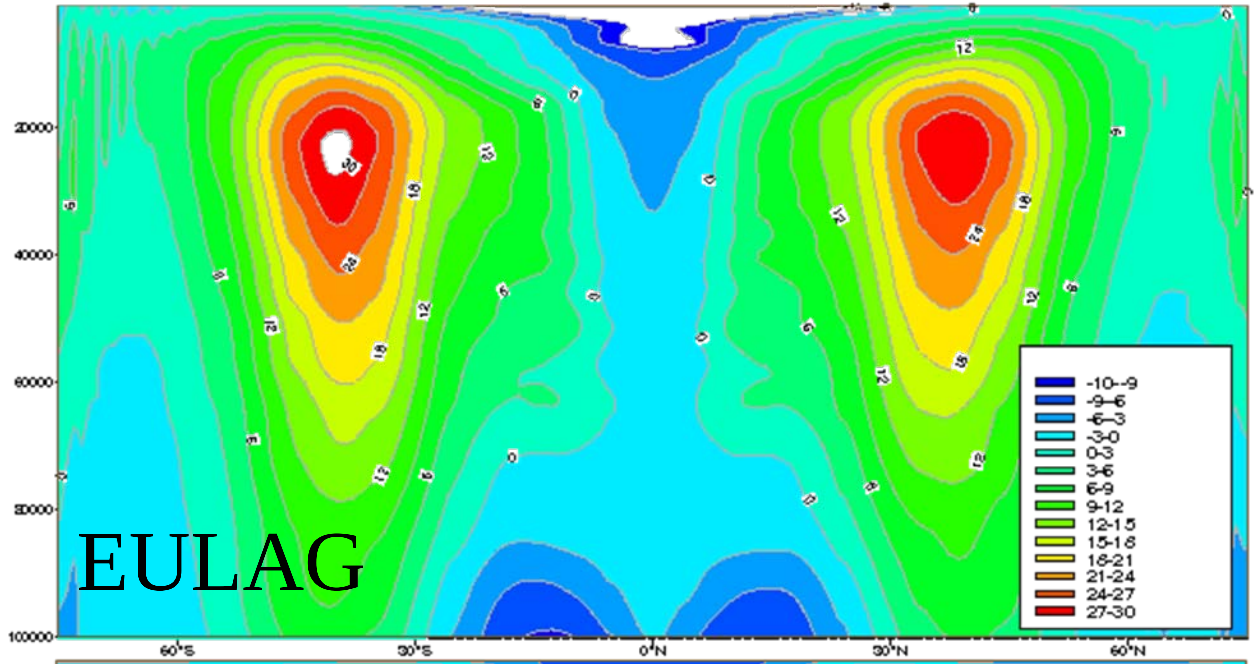
- **Zonal mean diagnostics**
- **Temporal anomalies**
- **Kinetic energy spectra**
- **Wavenumber frequency spectra**

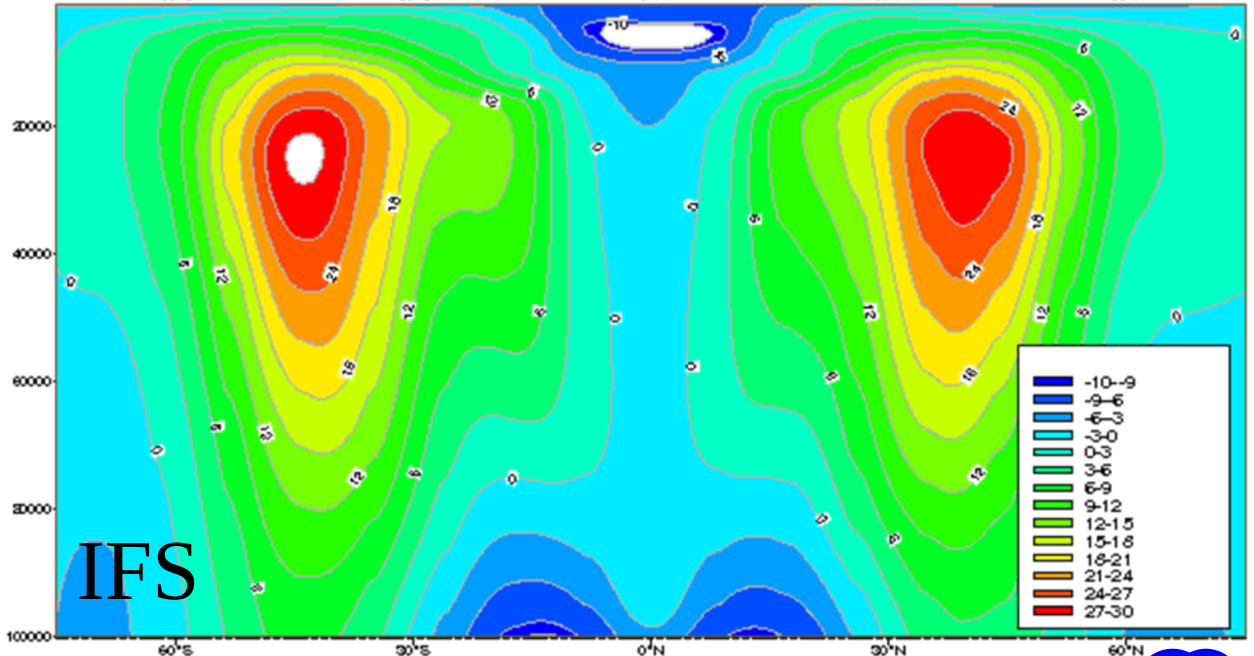
Held-Suarez climate

$a = a_{Earth}/10$

$\Omega = 10 \times \Omega_{Earth}$

zonal-mean zonal wind U





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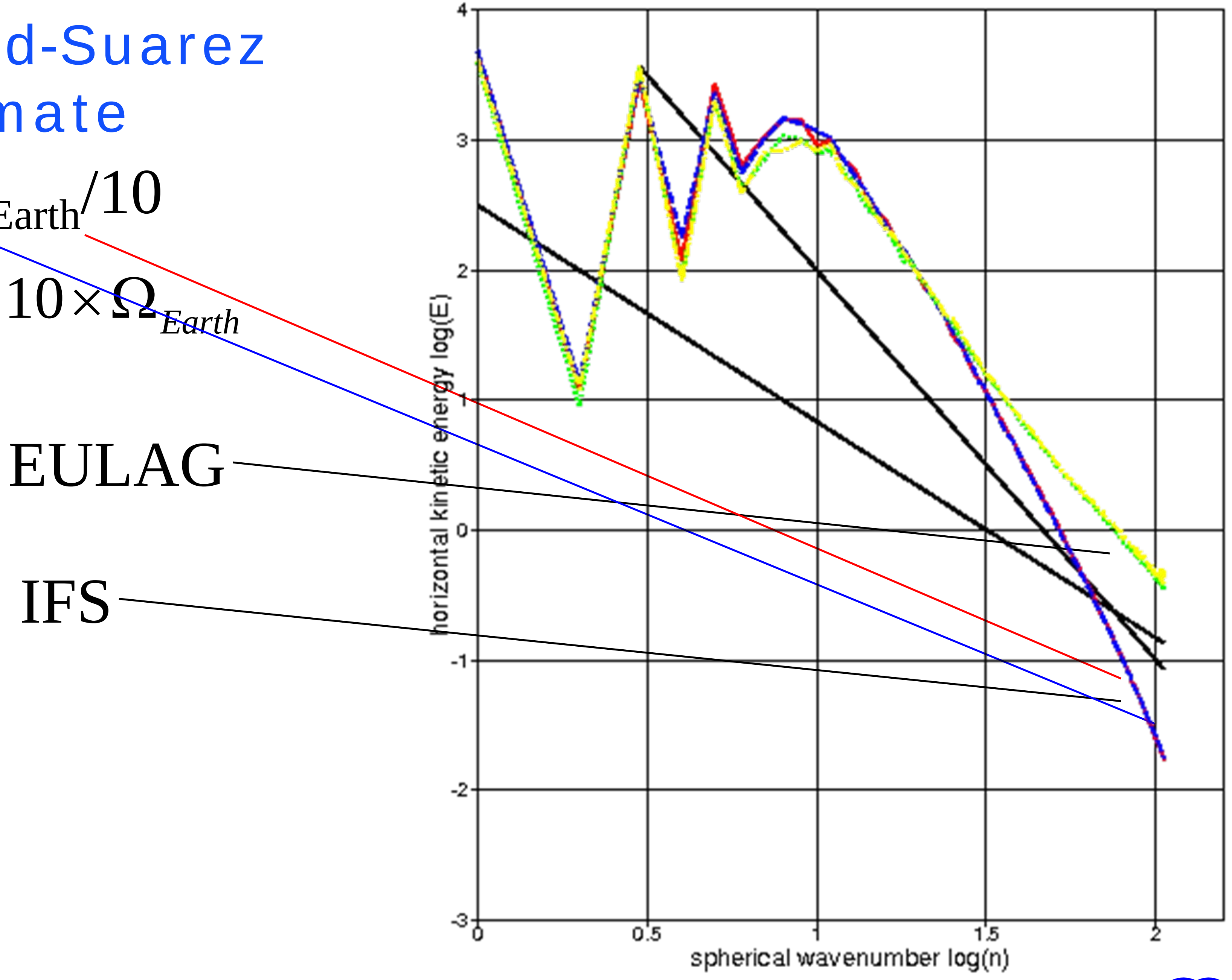
Held-Suarez climate

$a = a_{Earth}/10$

$$\Omega = 10 \times \Omega_{Earth}$$



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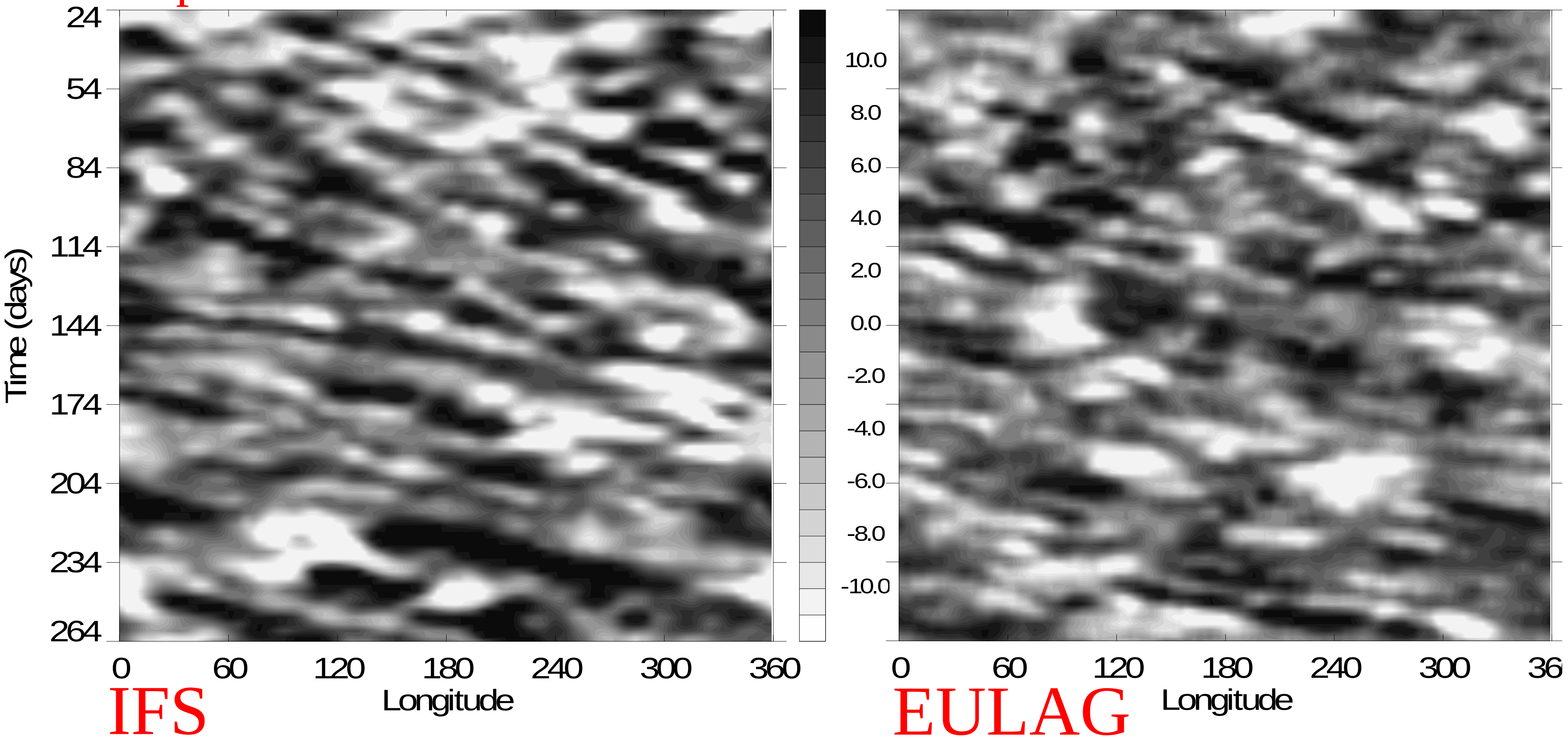
Held-Suarez climate $a=a_{Earth}/10$

Averaged 30ºN-50ºN

Temporal anomalies of zonal wind



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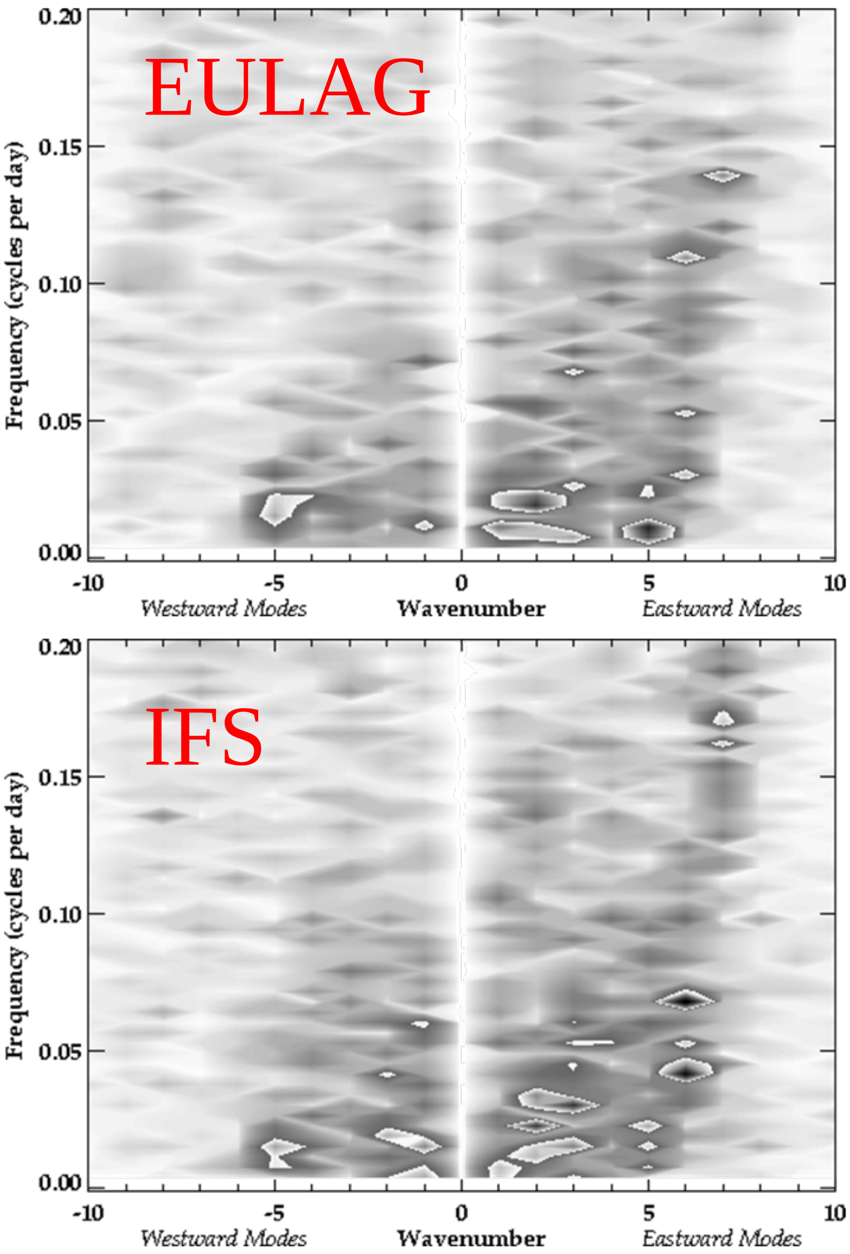
Held-Suarez climate

wavenumber-frequency diagrams of zonal wind

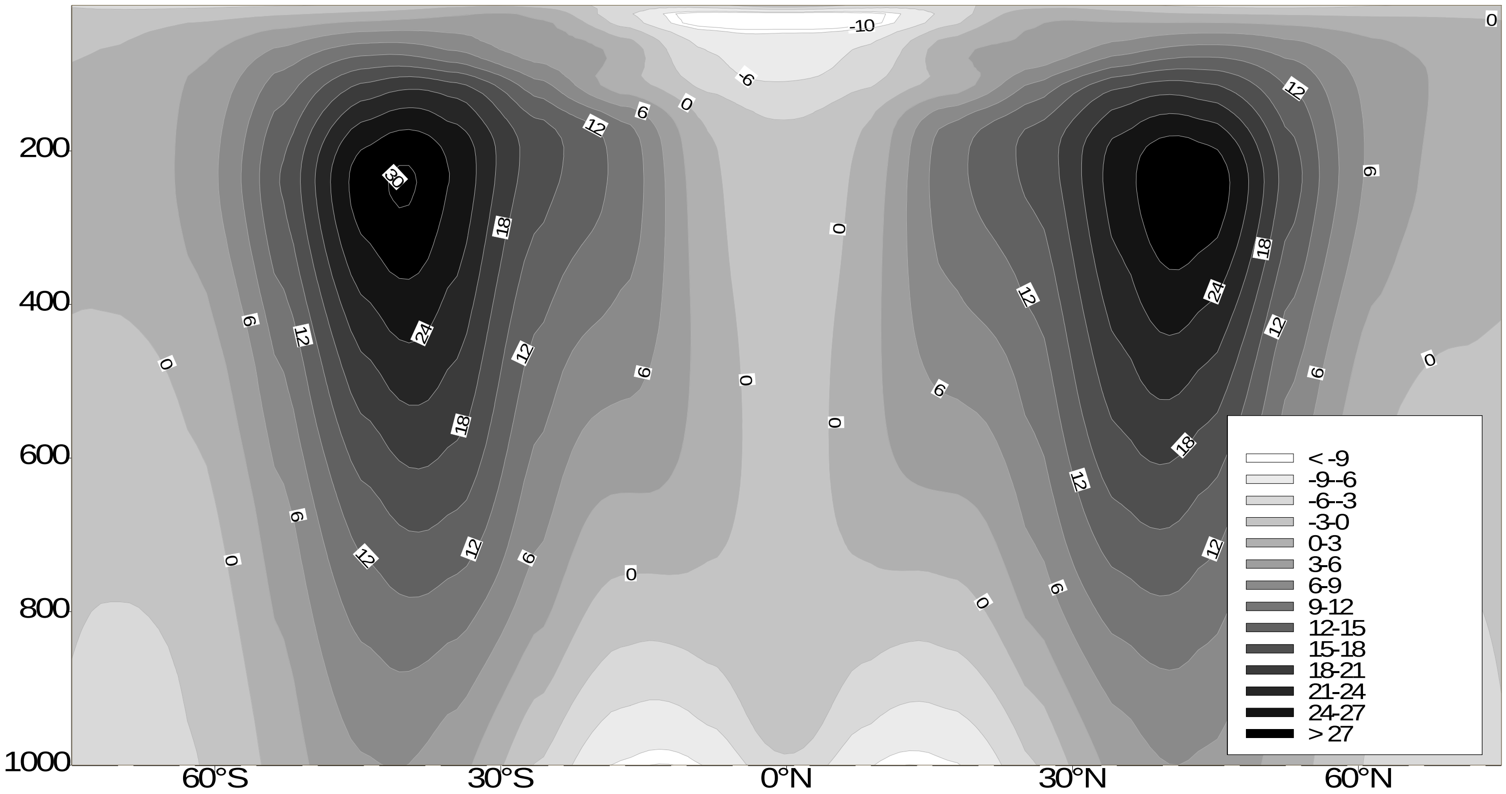


e.g. Rossby wave dispersion:

$$\omega_R \approx \frac{2\Omega N^2 m}{n(n+1)N^2 + f^2 a^2(\delta_A N^2 / c_s^2 + k_z^2 + \Gamma_A^2)}$$

The influence of the shallow vs deep atmosphere model formulation

$$\Omega = 20 \times \Omega_{Earth} \quad a = a_{Earth}/20$$



IFS Held-Suarez simulation with a shallow hydrostatic model

Simmons and Burridge 1981

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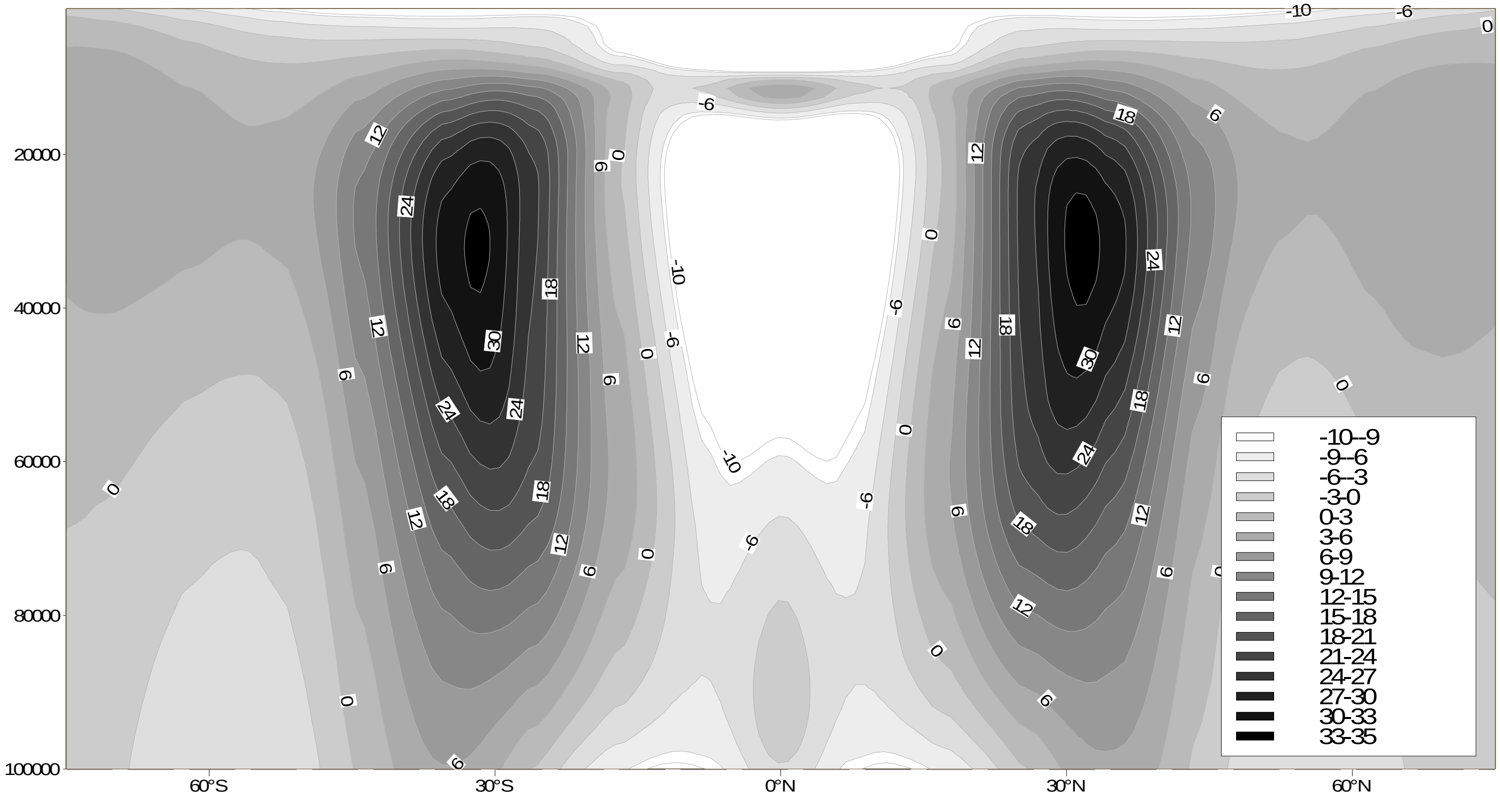
The influence of the shallow vs deep atmosphere model formulation

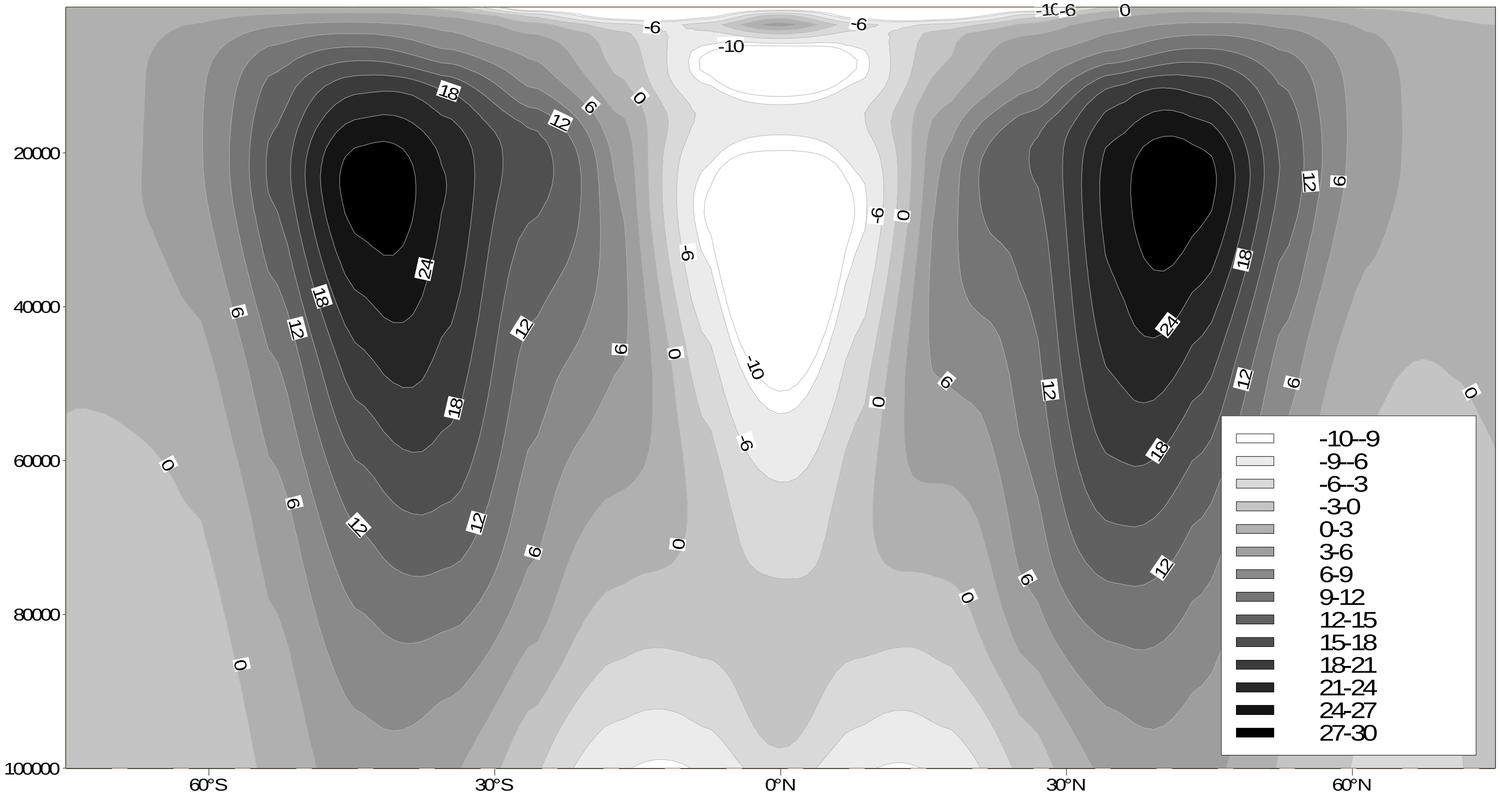
$$\Omega = 20 \times \Omega_{Earth} \quad a = a_{Earth}/20$$



IFS Held-Suarez simulation with a deep hydrostatic model following *White and Bromley, 1995*

The influence of the shallow vs deep atmosphere model formulation

$$\Omega = 20 \times \Omega_{Earth} \quad a = a_{Earth}/20$$



IFS Held-Suarez simulation with a deep non-hydrostatic elastic model following *Wood and Staniforth 2003*

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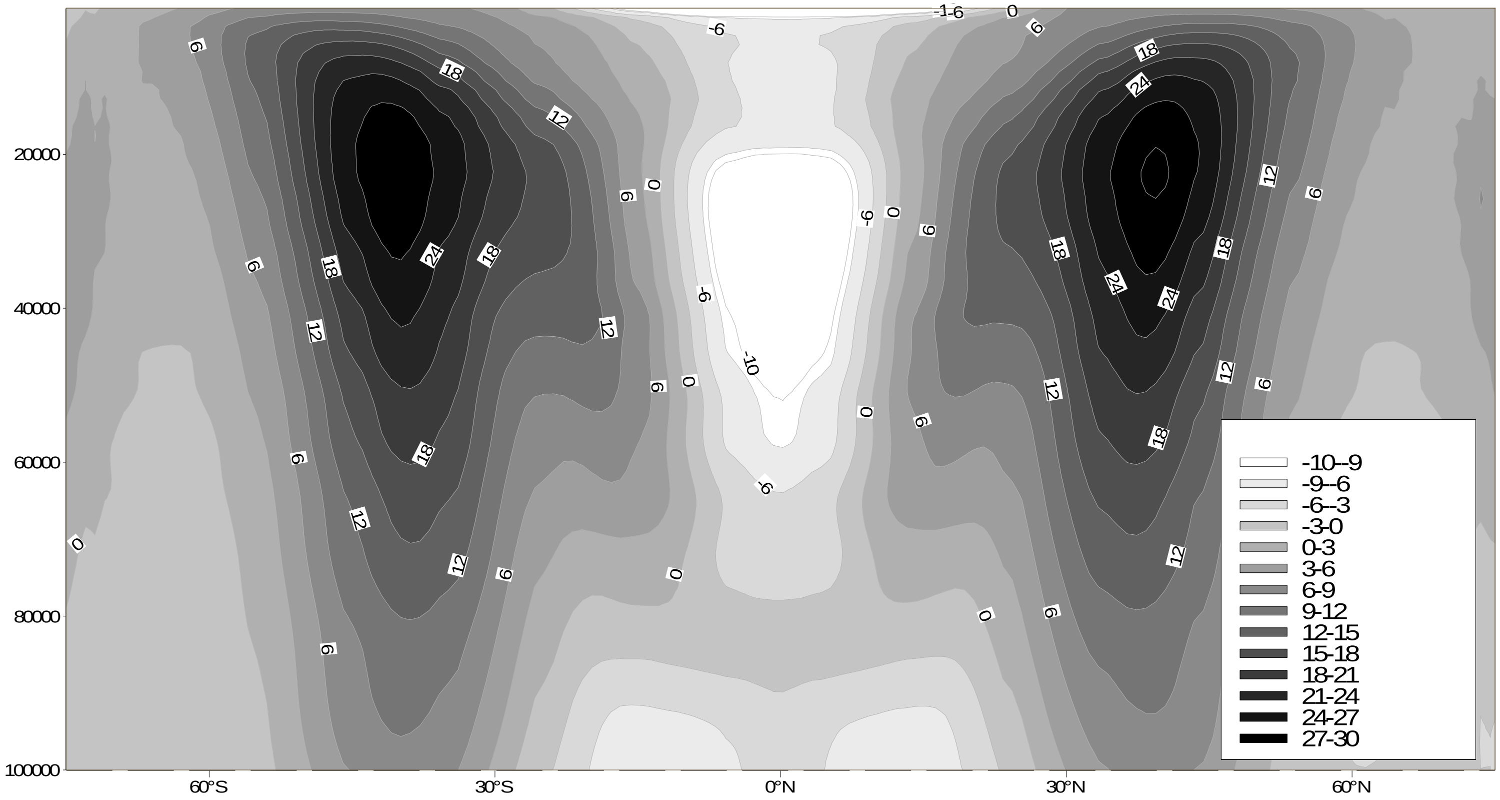
The influence of the shallow vs deep atmosphere model formulation

$$\Omega = 20 \times \Omega_{Earth} \quad a = a_{Earth}/20$$



EULAG Held-Suarez simulation with a deep non-hydrostatic anelastic model, Wedi and Smolarkiewicz, 2009

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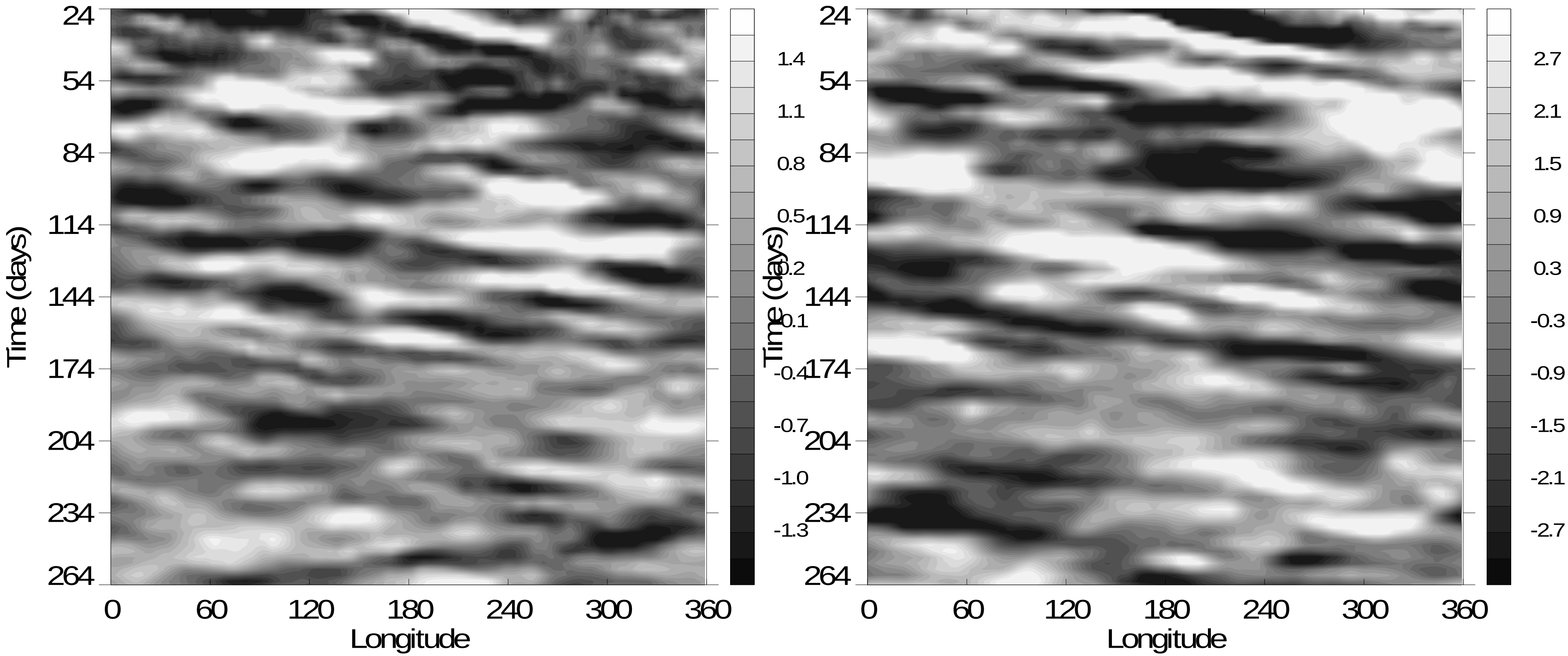
Held-Suarez Tropics

IFS - Temporal anomalies of velocity potential

600hPa 200hPa



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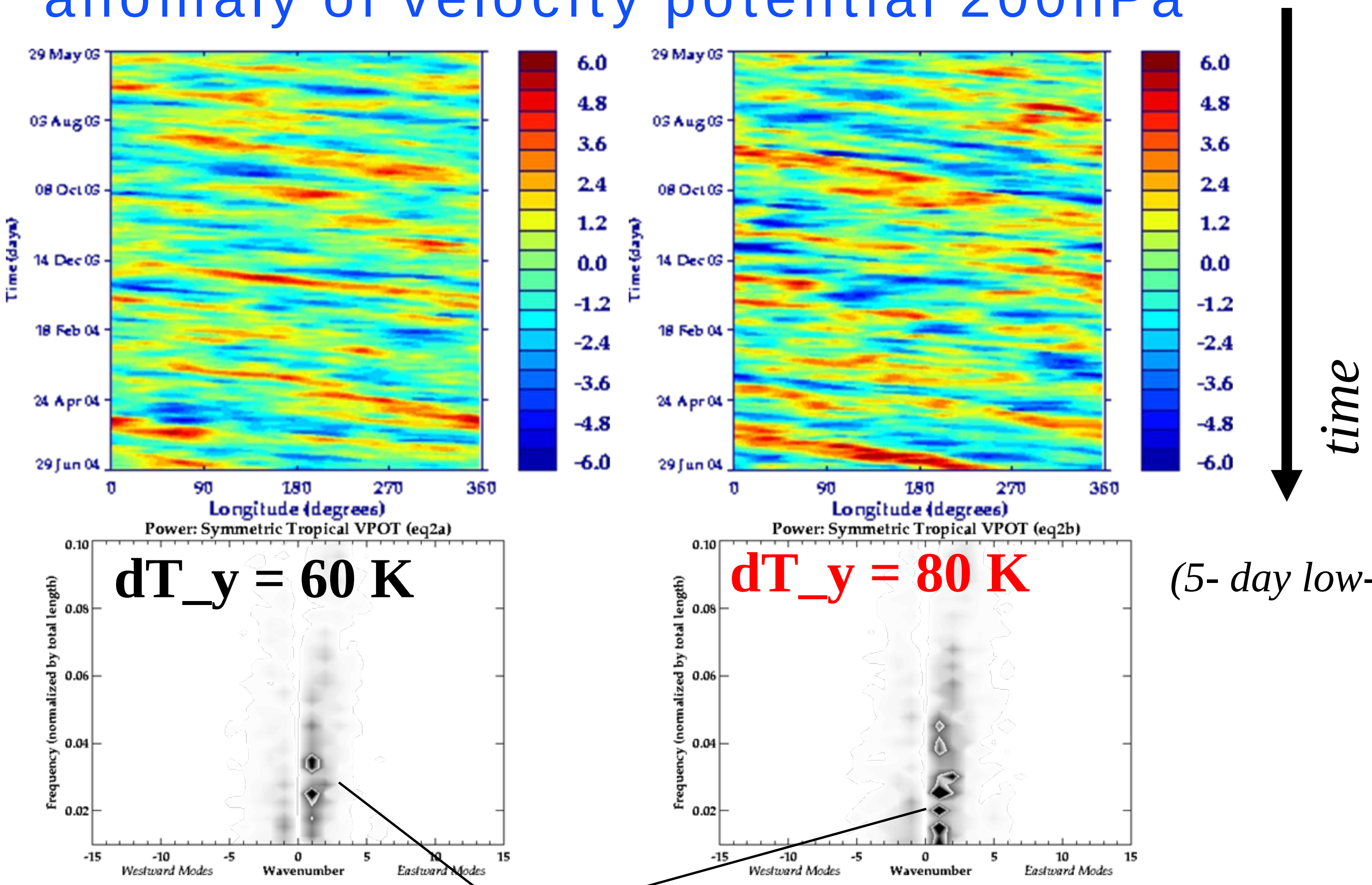
5-year Held-Suarez (dry) simulations: (temporal) anomaly of velocity potential 200hPa



(5- day low-pass filtered)

Eastward propagation

with periods of 39-66 days !

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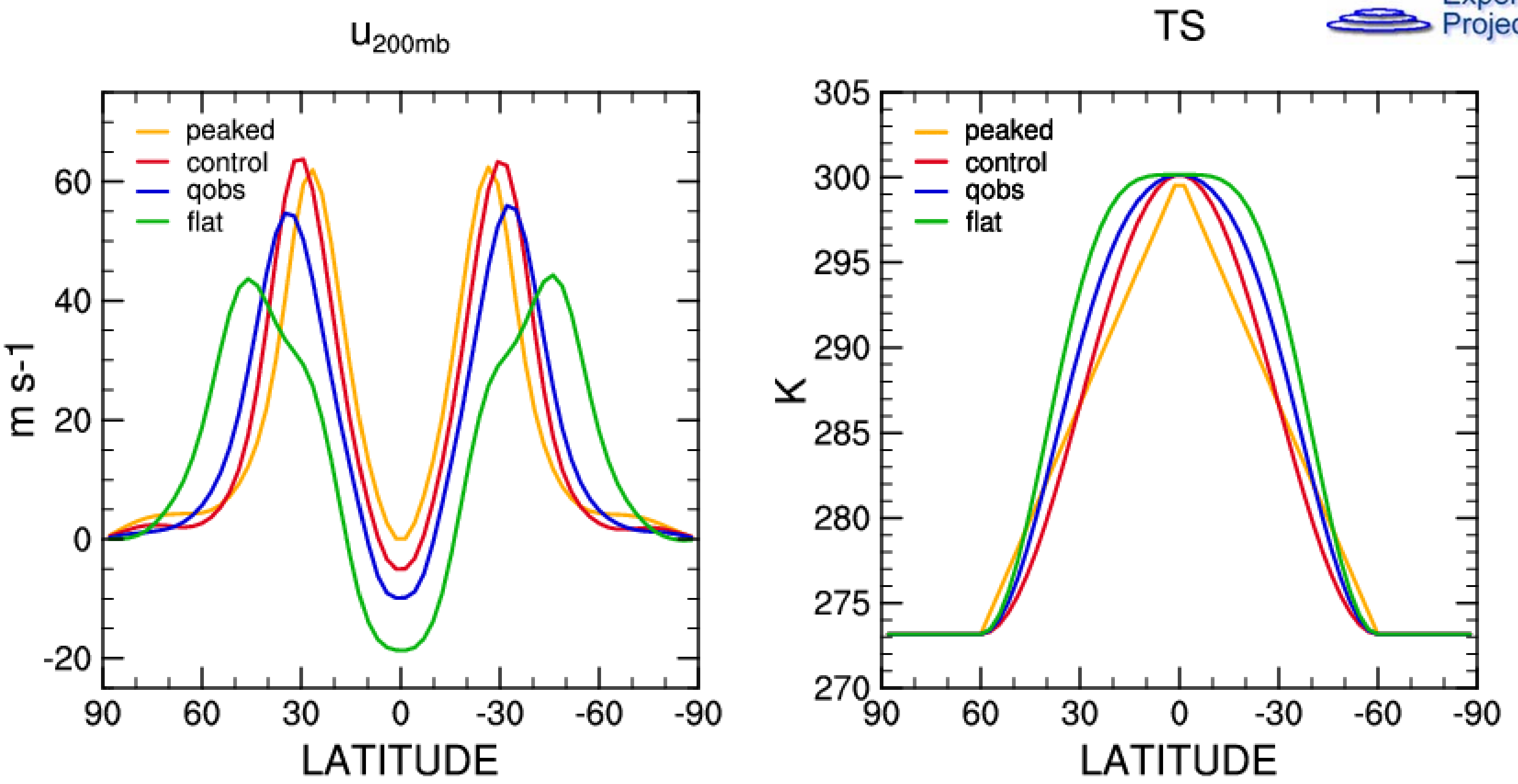
Aquaplanets

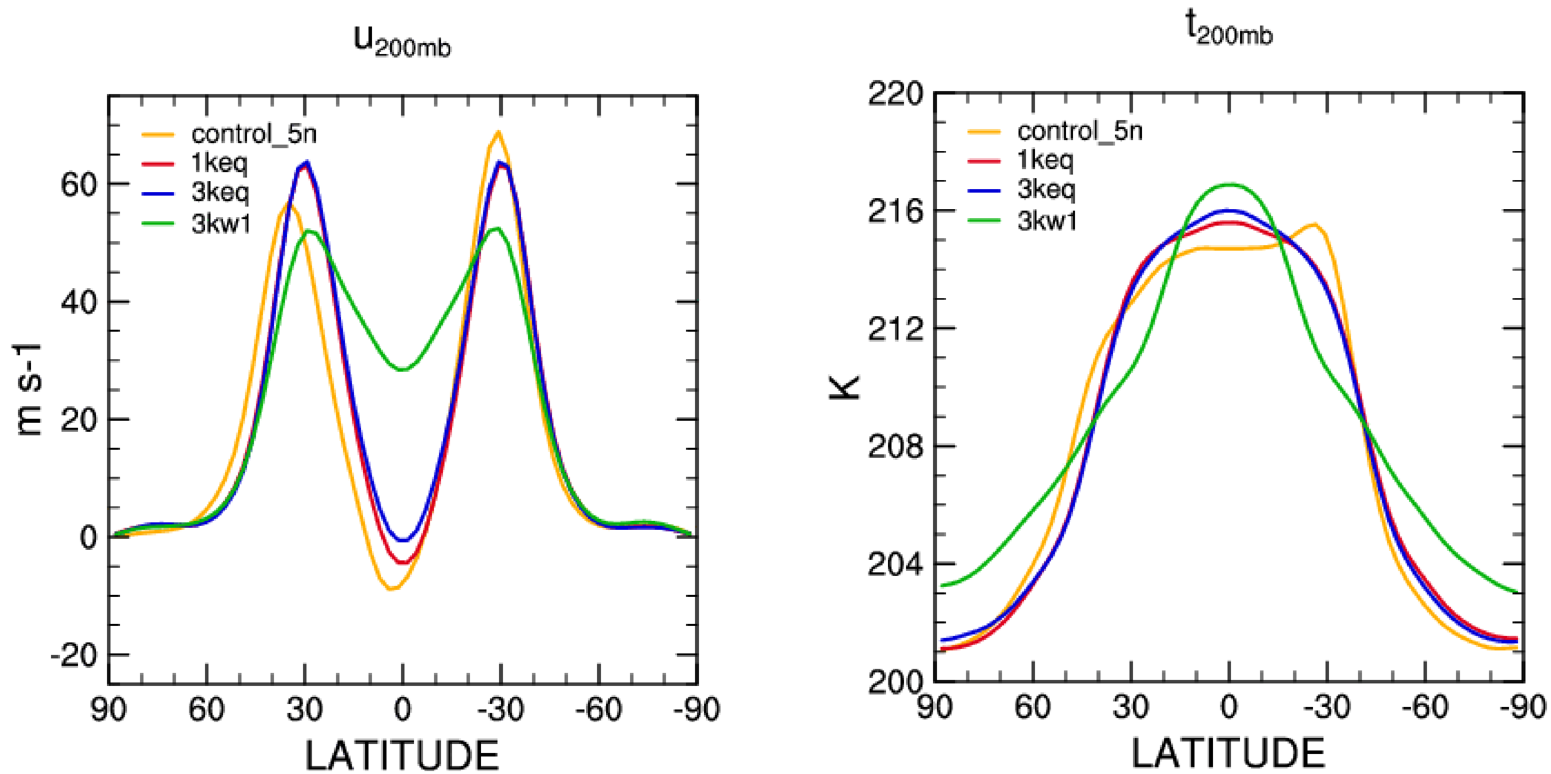
Neale, R.B. and B.J.Hoskins, 2000a:
A standard test for AGCMs including their physical parameterizations. I: The proposal. Atmos. Sci. Lett, Vol.1, No.2, pp. 101-107.

- **Different SST distributions force the flow**
- **3 year “model climate”**
- **APE intercomparison project with an ATLAS of many different models/parameters in preparation**
  http://www.met.reading.ac.uk/~mike/APE/

APE Aqua-Planet Experiment Project



Courtesy of D. Williamson

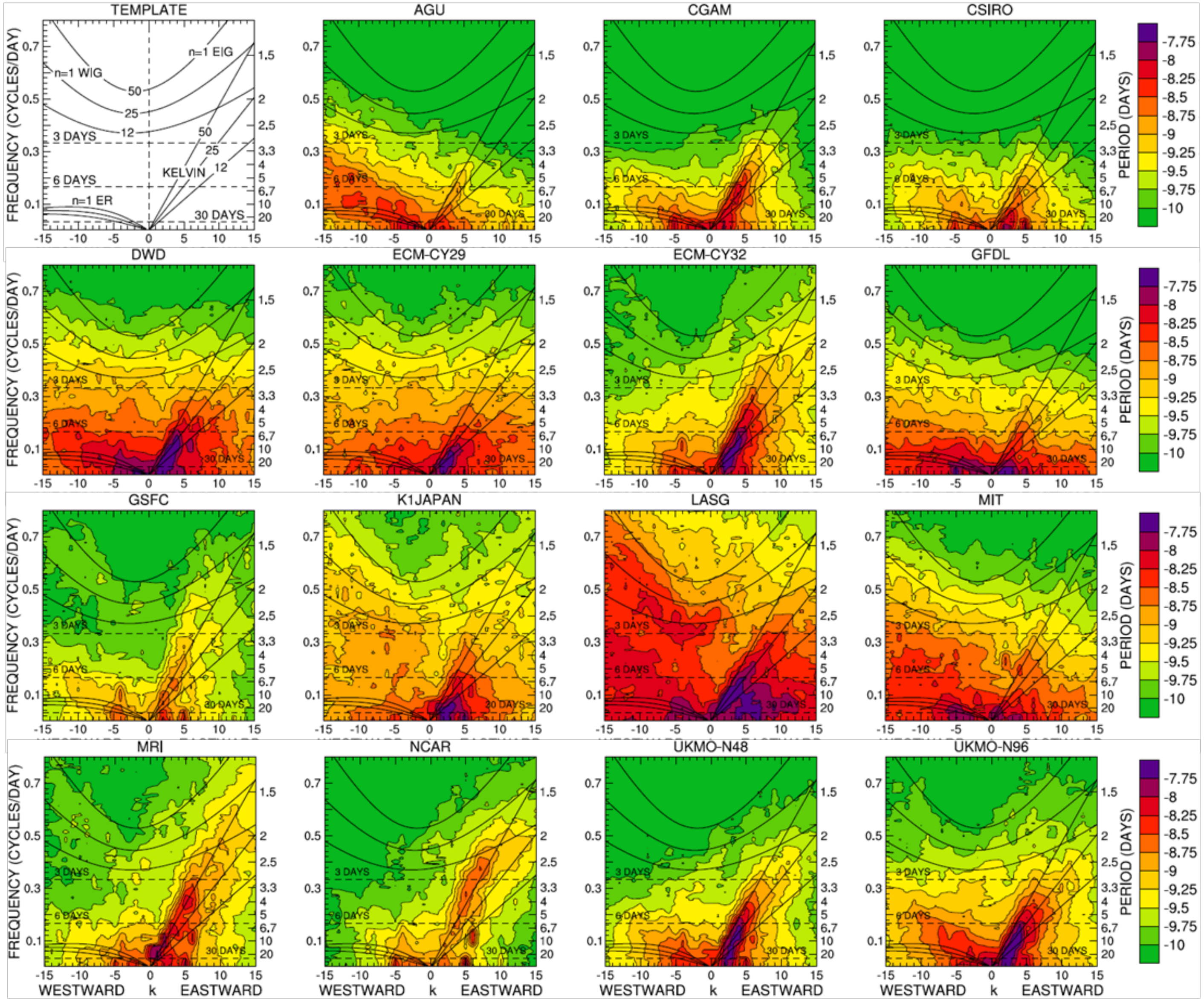


Courtesy of D. Williamson

Tropical Wave Spectra (control)

Courtesy of Mike Blackburn

Aqua-Planet Experiment Project

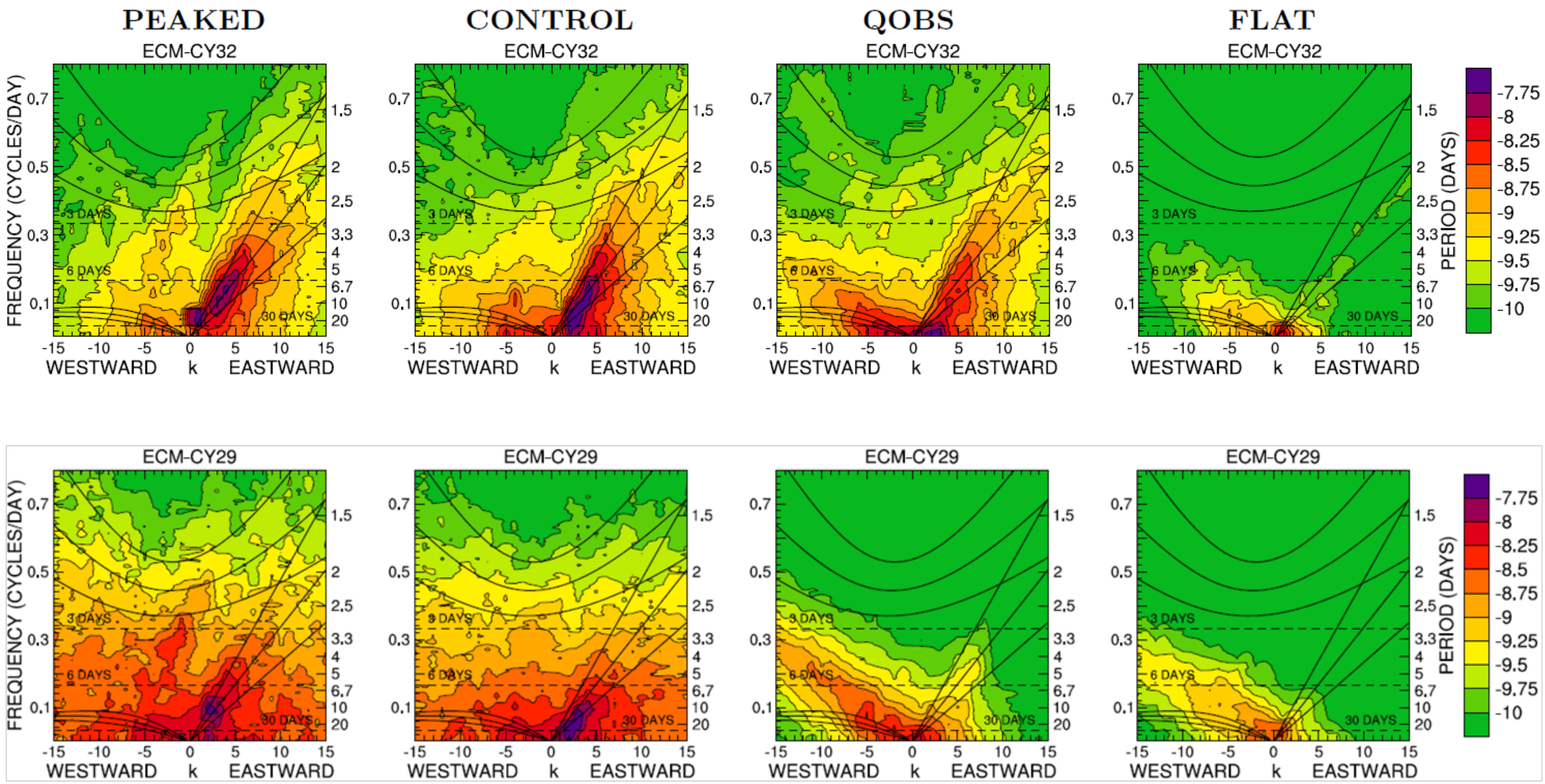


Log-power of symmetric modes of equatorial precipitation (10°N - 10°S average)

Tropical Wave Spectra (ECMWF)

Courtesy of Mike Blackburn

APE Aqua-Planet Experiment Project



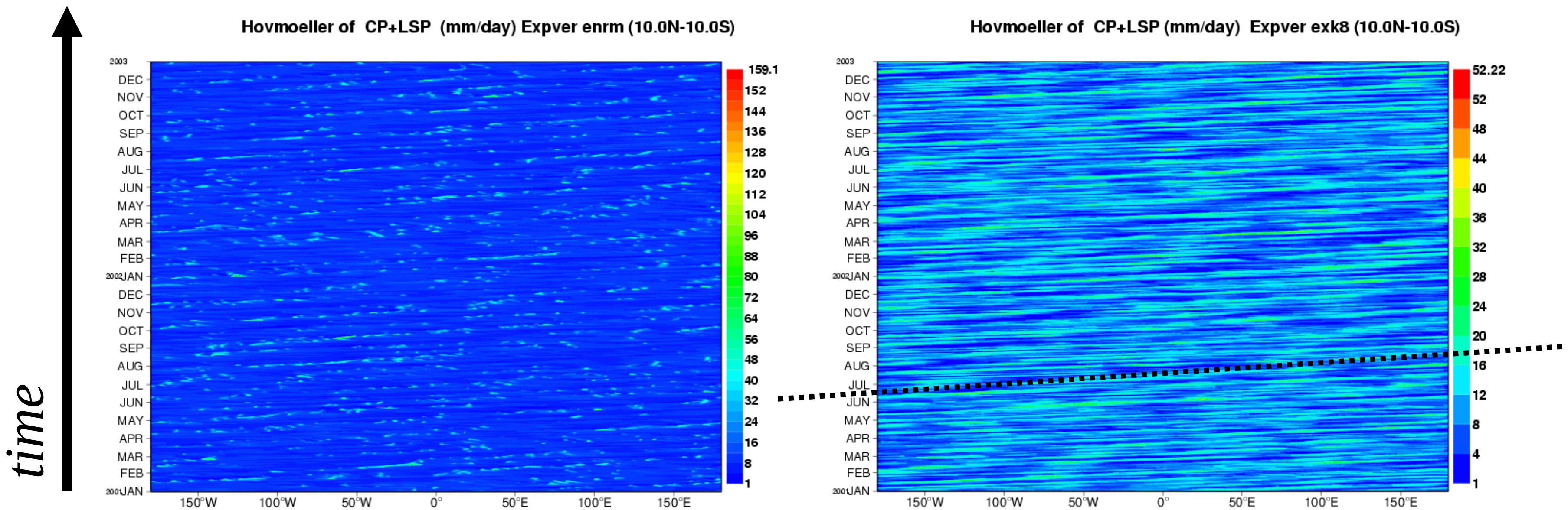
- Recent model changes to the convection parametrization (cy32r3, November 2007, *Bechtold et al. 2008*)
- Precipitation organised strongly in equatorial waves

Aquaplanet control: precipitation

Aqua-Planet Experiment Project

(Courtesy of Peter Bechtold)

29R2 32R3



Note the unrealistically high zonally averaged total precipitation rates (mm/day) in 29r2.

The theoretical phase speed for Kelvin waves is 15-20 m/s, here we roughly see 18 m/s in 32r3.

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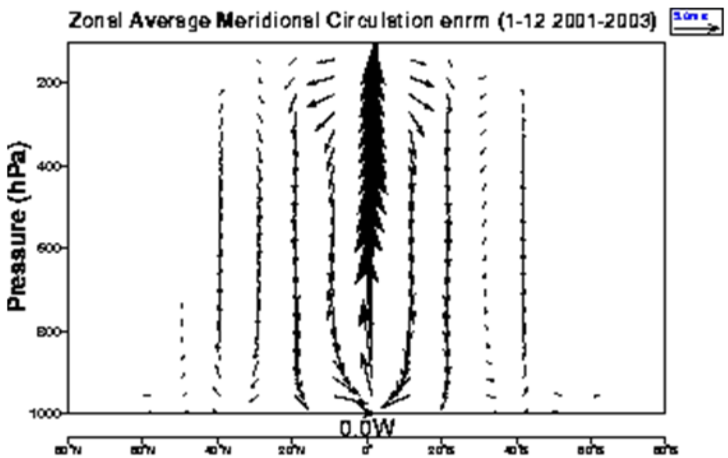
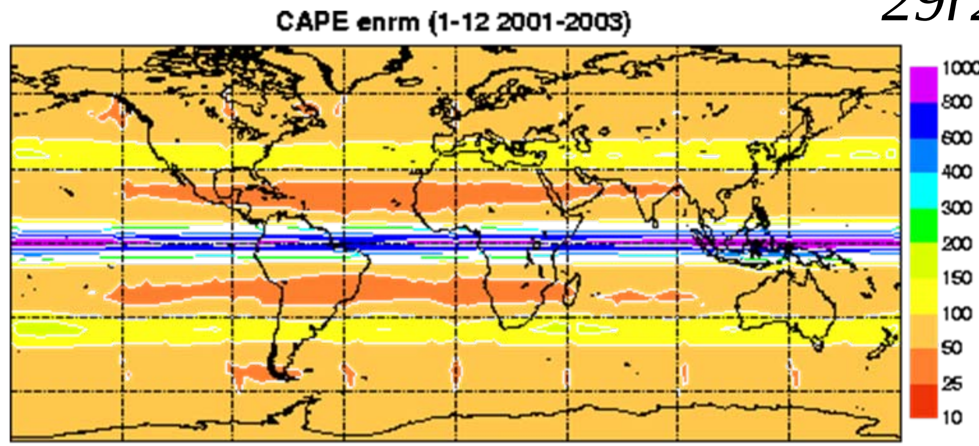
Aquaplanet control: Hadley cell and CAPE

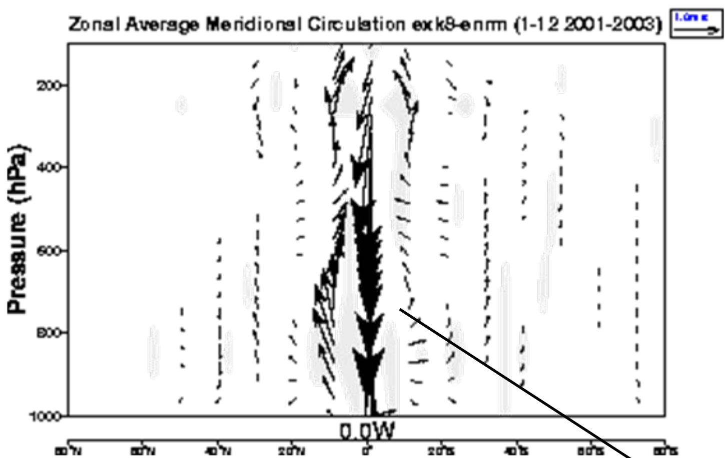
(Courtesy of Thomas Jung)

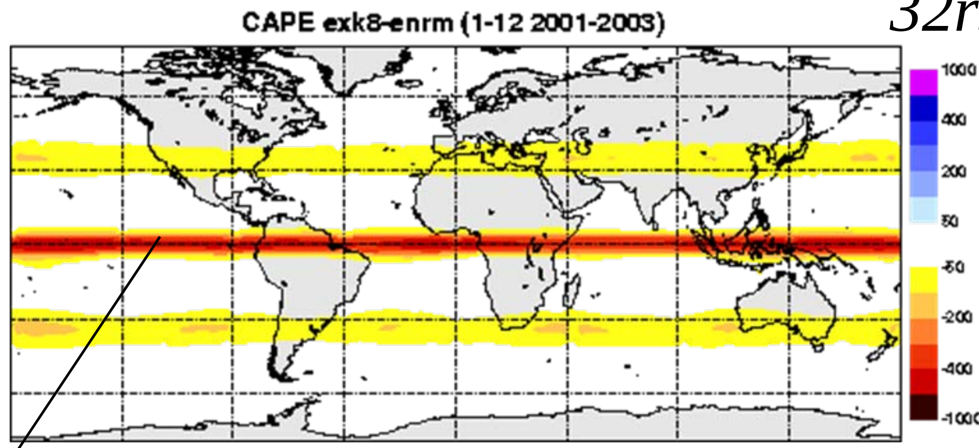
29r2





32r3





$[\overline{v}][\overline{\omega}]$

$$CAPE = \int_{z_f}^{z_n} g \left(\frac{T_{parcel} - T_{env}}{T_{env}} \right) dz$$

weakening

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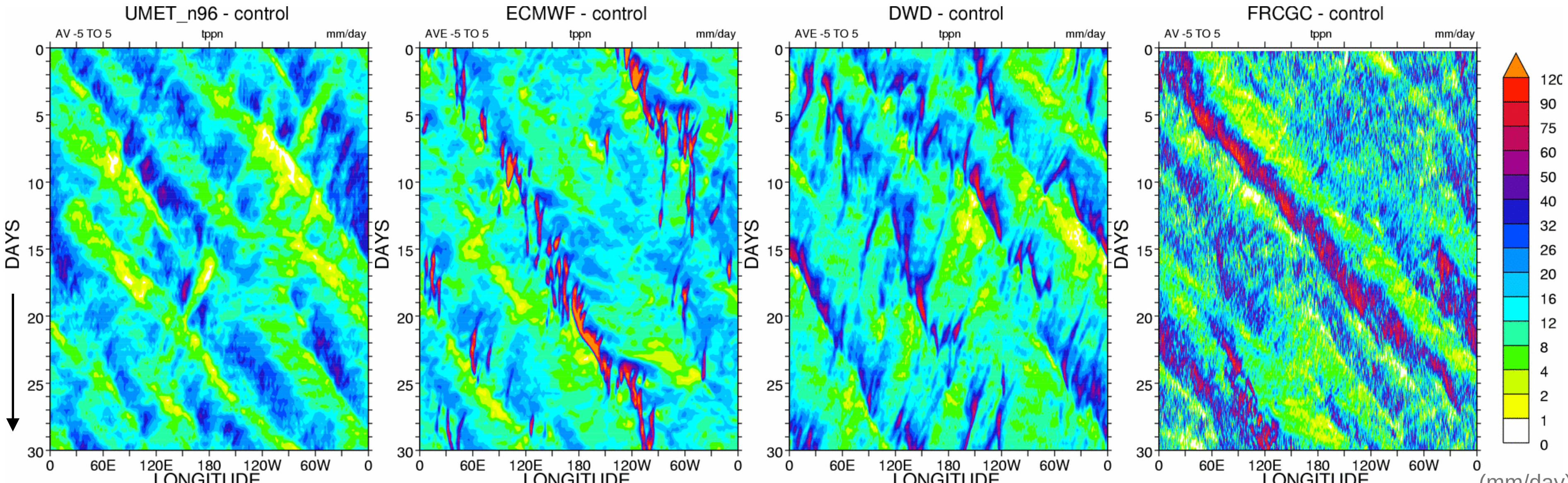
 Courtesy of D. Williamson

APE Aqua-Planet Experiment Project

Higher resolution models

average 5ºN-5ºS



pre-HadGAM1	*IFS Cy29r2*	*GME*	*NICAM*
N96 L38	T_L159 L60	Icosahedral L31	Icosahedral L54
1.25° x 1.875°	~125km grid	~100km grid	7km grid

non-hydrostatic
no convective param.

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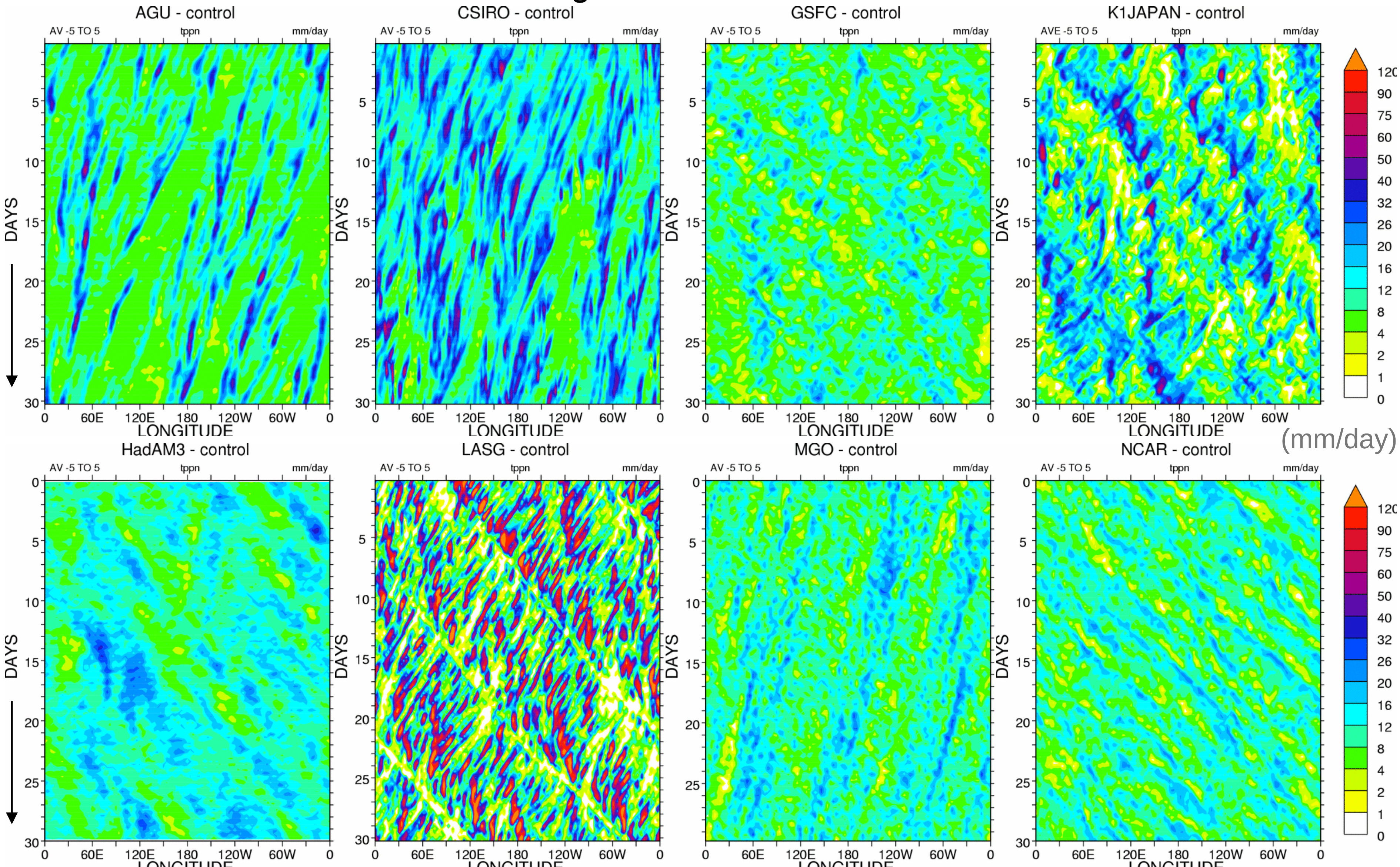
Tropical Variability (precipitation)

Courtesy of D. Williamson

average 5ºN-5ºS



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Variability of organized convection

- **The formation of organized convection is sensitive to explicit or implicit viscosity in under-resolved simulations.**
- **The sensitivity is consistent with linear theory adapted to include the effect of anisotropy of viscosity (horizontal vs. vertical) at moderate Rayleigh number.**

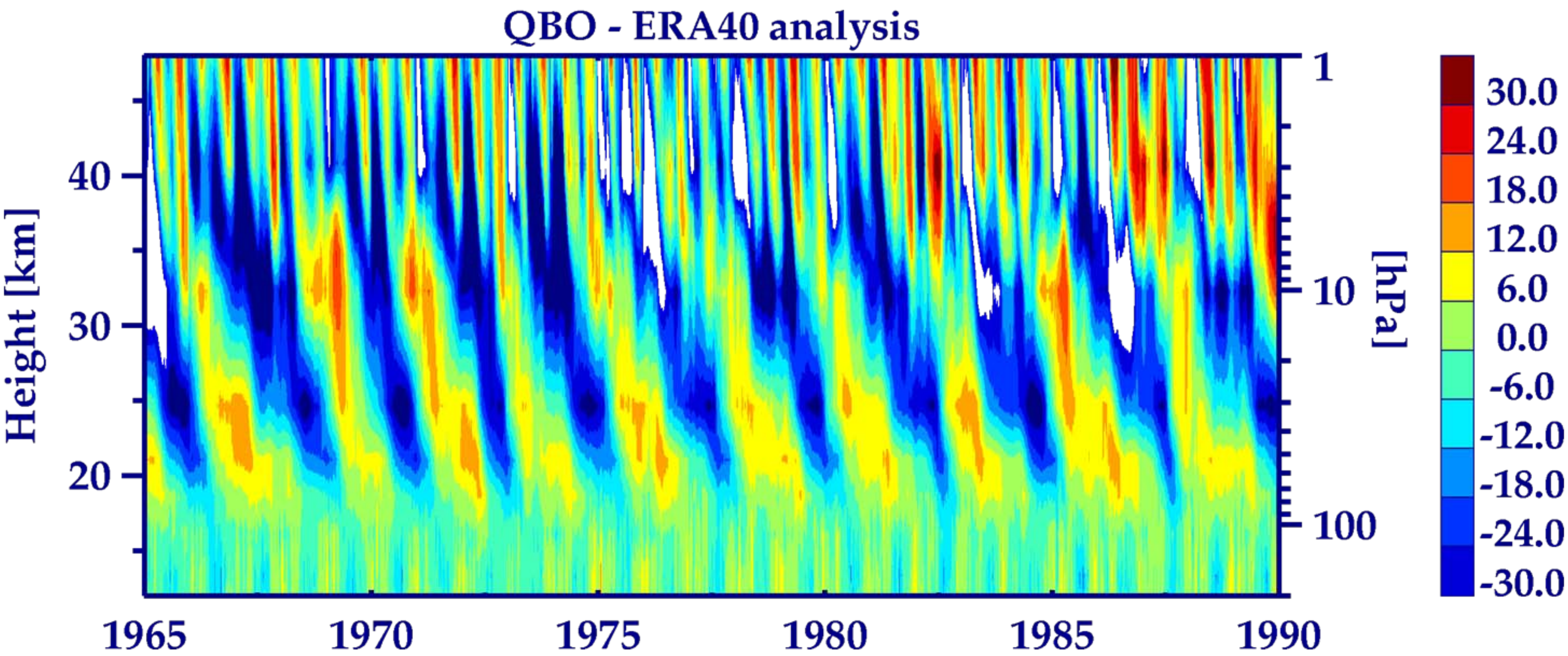
  (Piotrowski et al., J. Comput. Phys. 2009)

- **It suggests a careful control of the effective numerical viscosity in the numerical core ! (such as the dependence of the truncation error on the derivatives of the flow variables rather than the flow variables themselves).**

DNS of laboratory experiment: The QBO

- **The laboratory experiment shows the pure gravity wave acceleration of the zonal mean zonal flow.**
- **The setup represents a stringent test of the model numerical core.**

The stratospheric QBO



- westward

\+ eastward

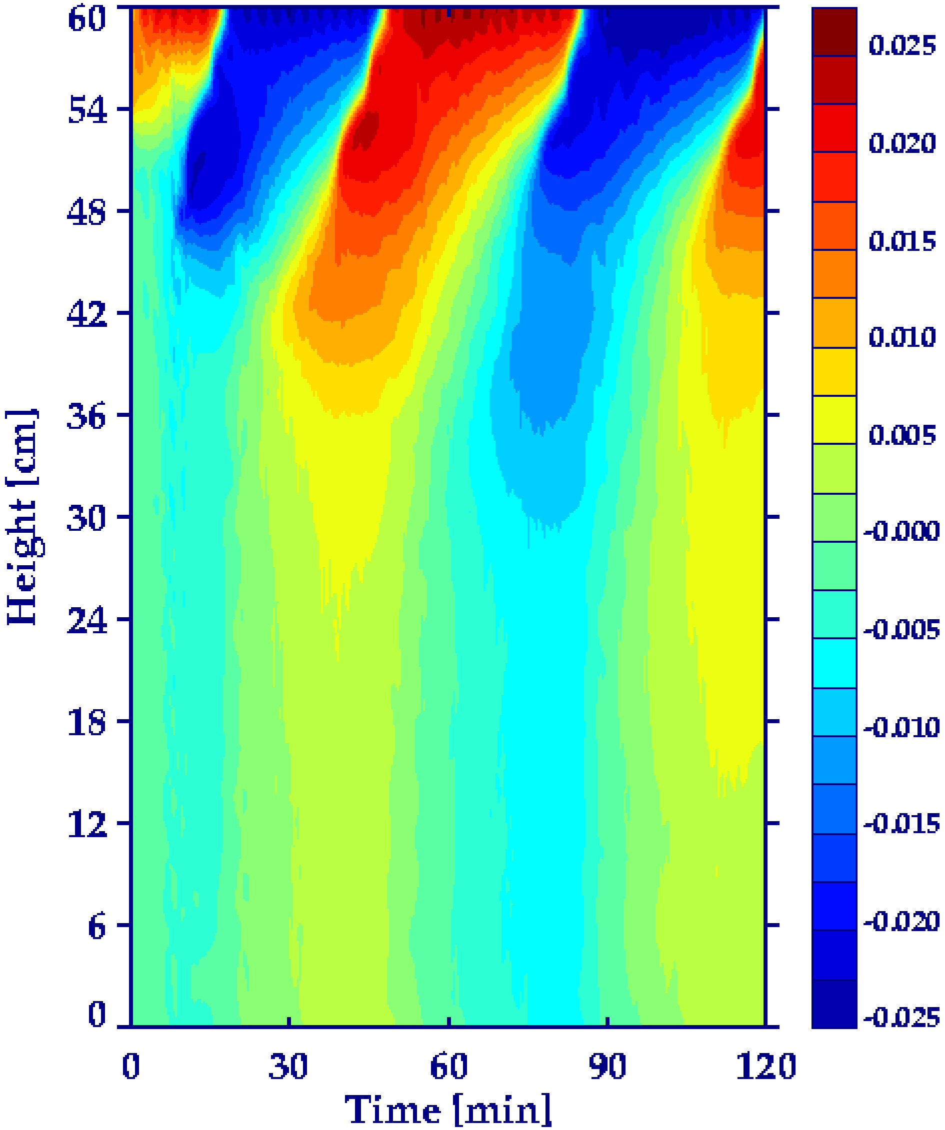
(unfiltered) ERA40 data *(Uppala et al, 2005)*

The laboratory experiment of Plumb and McEwan

- **The principal mechanism of the QBO was demonstrated in the laboratory** *Plumb and McEwan, J. Atmos. Sci. 35 1827-1839 (1978)*
- **University of Kyoto**
  http://www.gfd-dennou.org/library/gfd_exp/exp_e/index.htm

Animation:

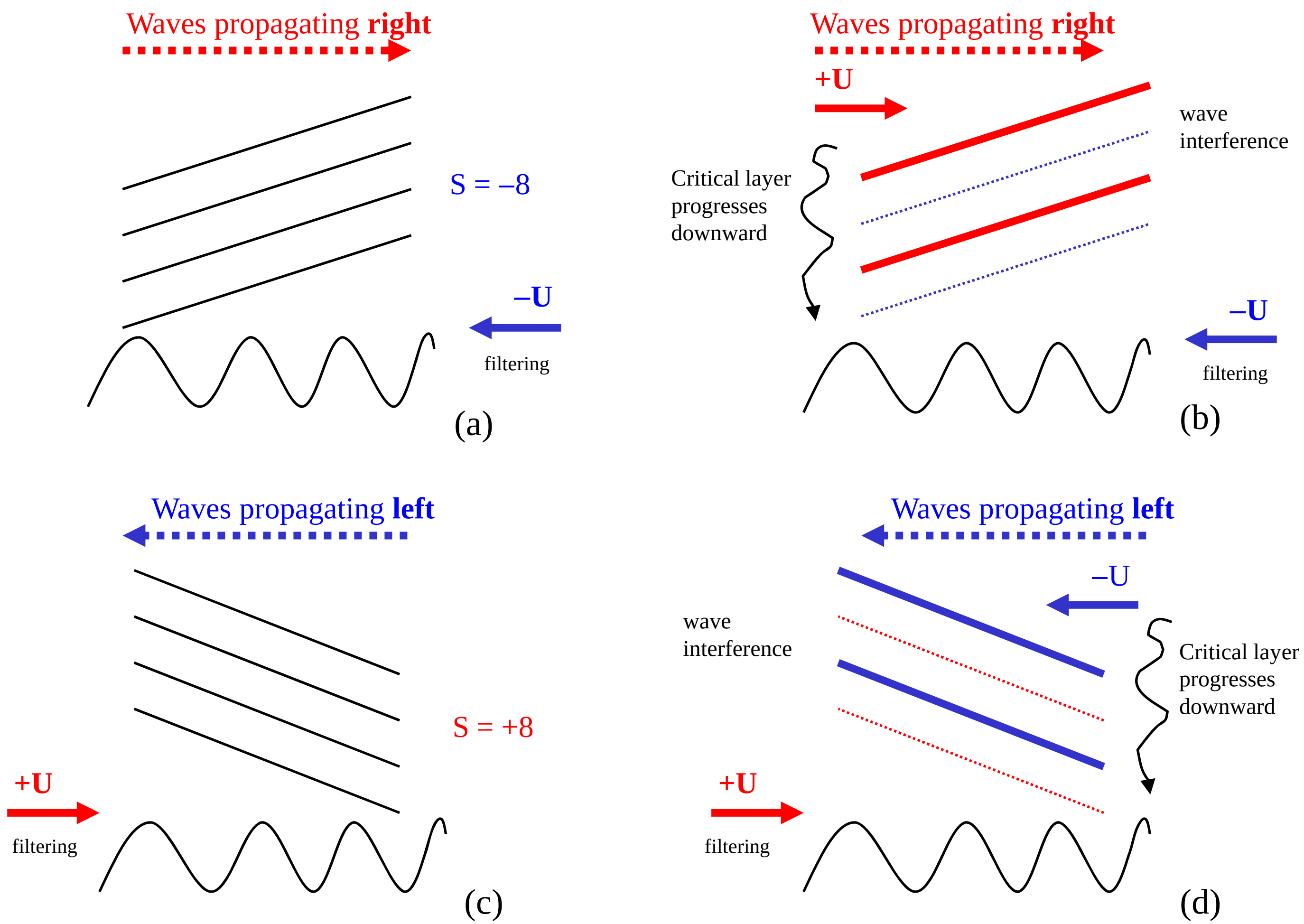
(Wedi and Smolarkiewicz, J. Atmos. Sci., 2006)



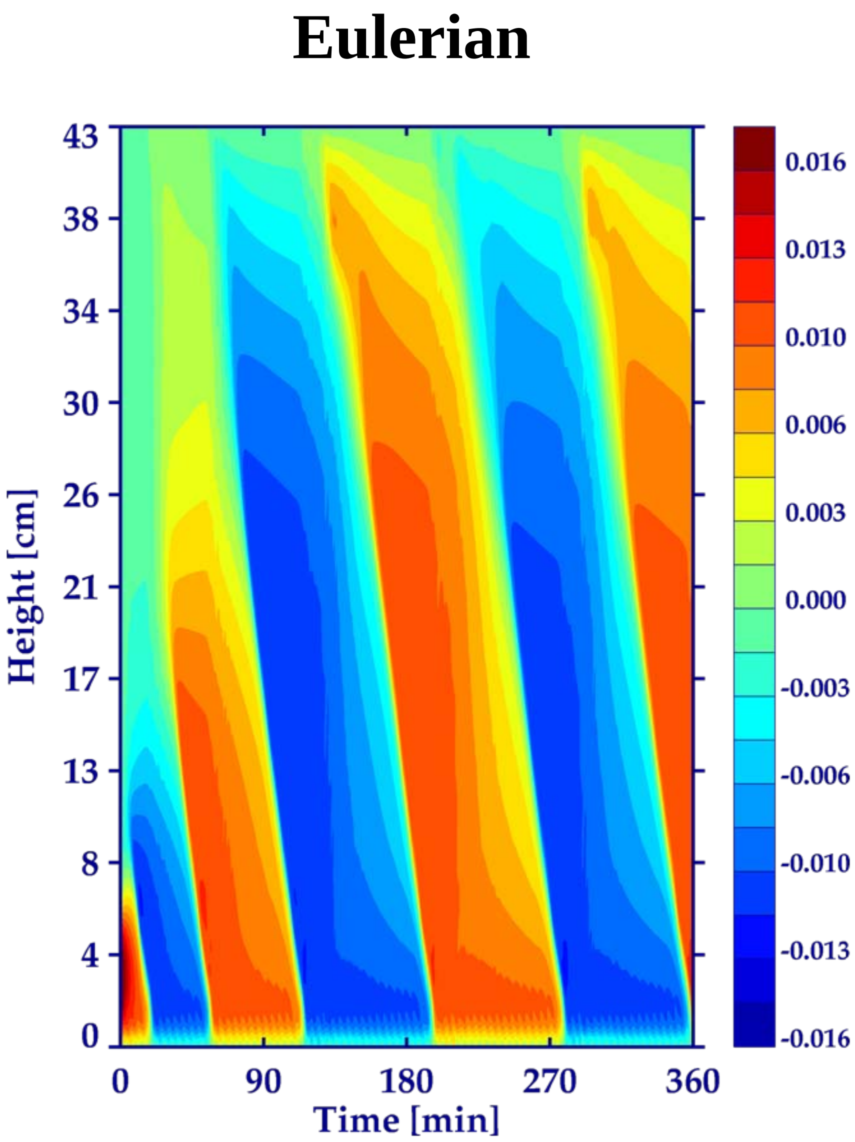
Time – height cross section of the mean flow U in a 3D simulation

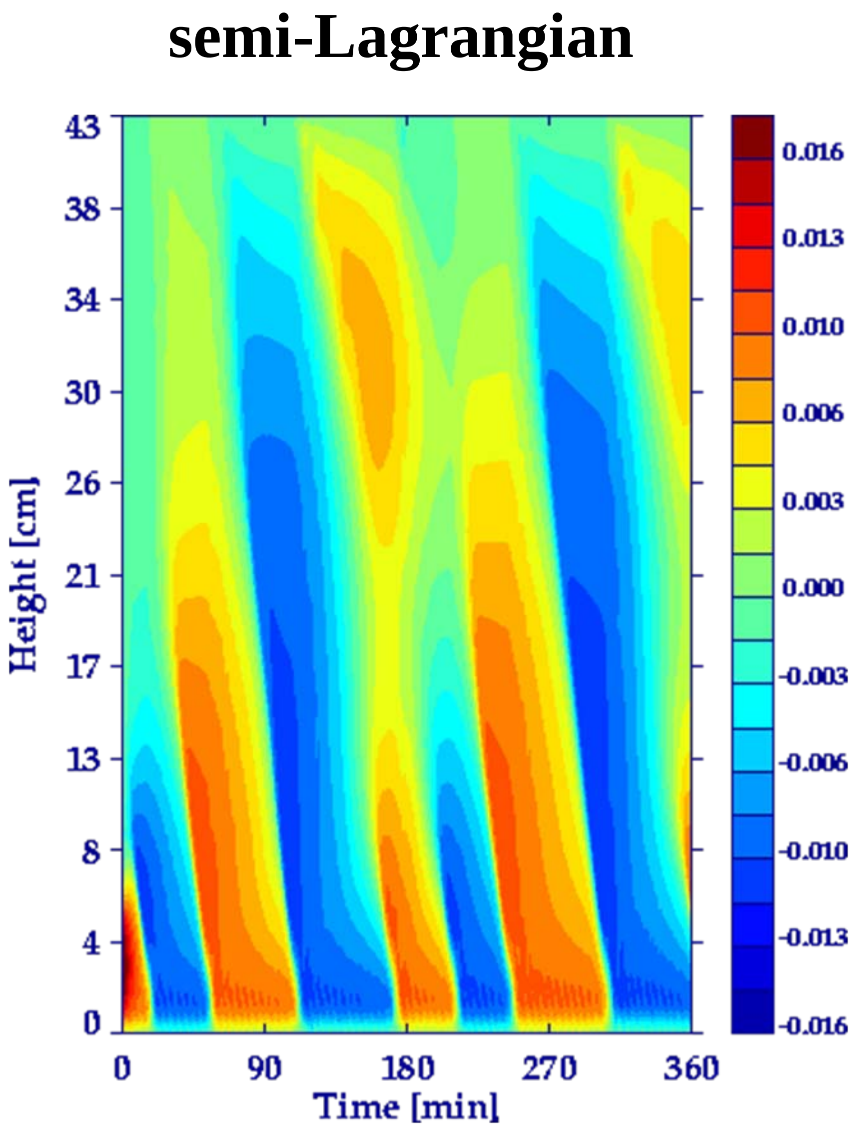
Animation

Schematic description of the QBO laboratory analogue



Viscous simulation





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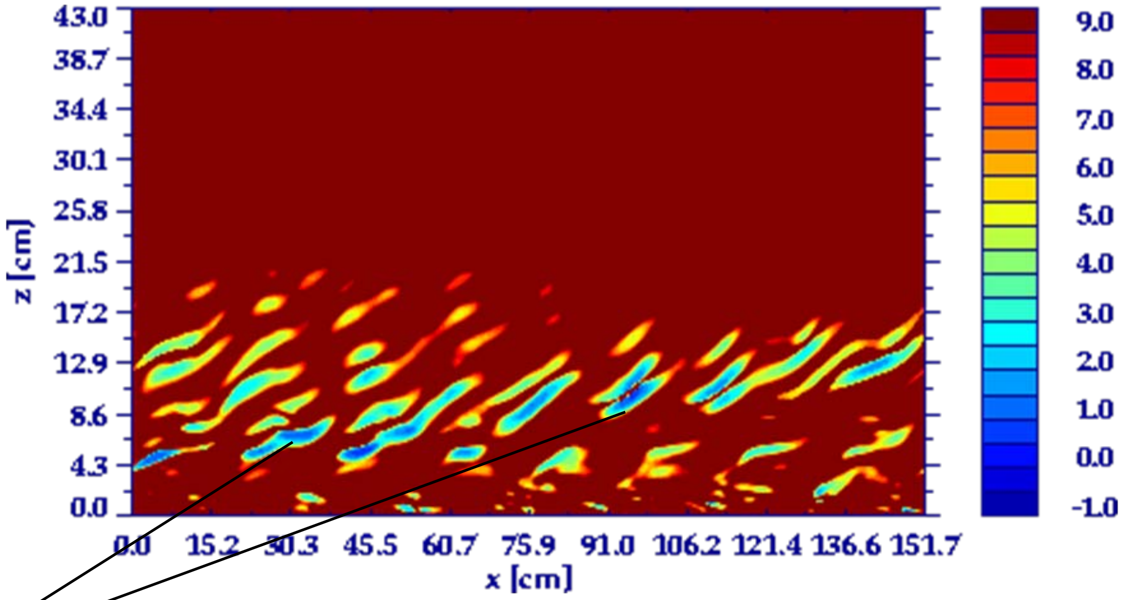
Local Richardson number analysis

$$Ri = -\frac{g}{\rho}\frac{\partial \rho}{\partial z} \Big/ \left(\frac{\partial u}{\partial z}\right)^2$$



Regions of wave overturning and breaking

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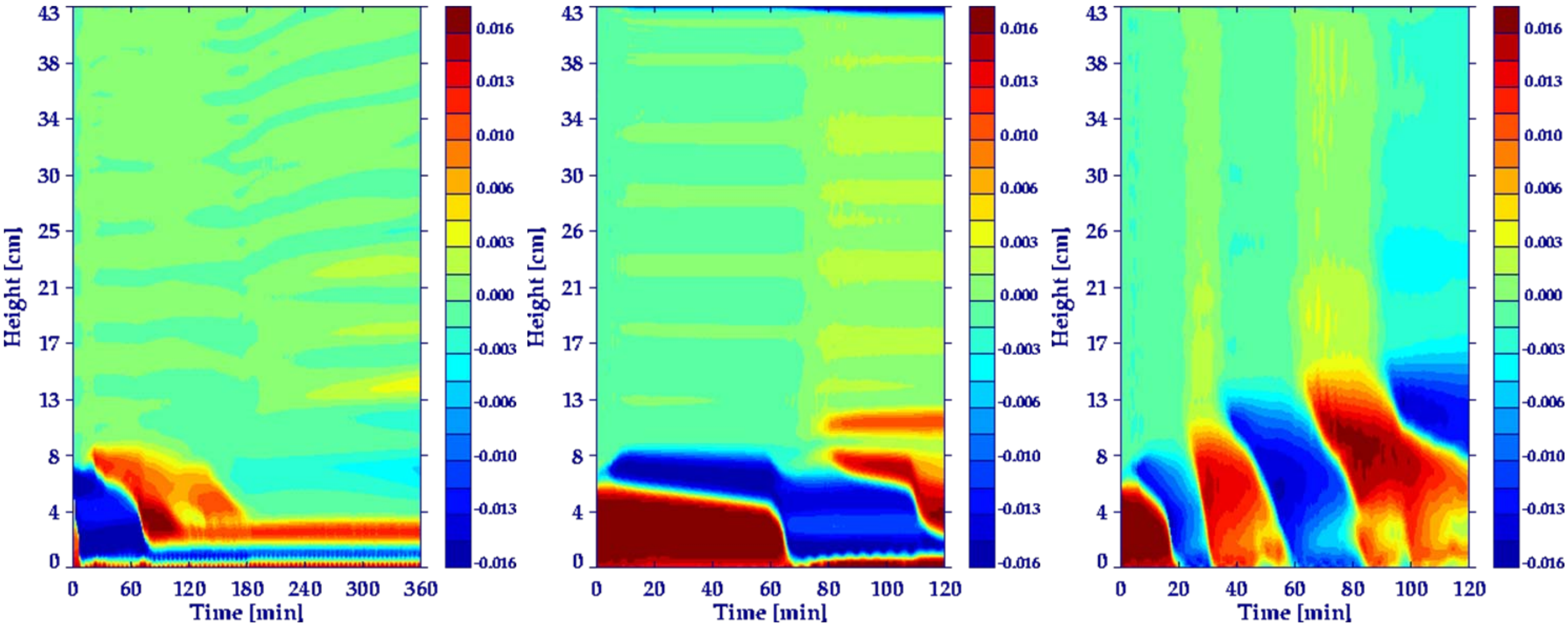
Inviscid simulation



Eulerian
St=0.36, n=640
top absorber

Eulerian
St=0.25, n=384
top rigid, freeslip

semi-Lagrangian
St=0.25, n=384
top rigid, freeslip

Numerical realisability of wave driven flows

Influence on the period and the vertical extent of the resulting zonal mean zonal flow changes

- **Lower horizontal resolution results in increasing period (~16 points per horizontal wavelength still overestimates the period by 20-30%)**
- **Lower vertical resolution results in decreasing period and earlier onset of flow reversal as dynamic or convective instabilities develop instantly rather than previously described wave-wave mean flow interaction *(need ~10-15 points per vertical wavelength, <5 no oscillation observed)***
- **First or second order accurate *(e.g. rapid mean flow reversals with 1st order upwind scheme)***
- **A low accuracy of pressure solver may result in spurious tendencies with a magnitude similar to physical buoyancy perturbations and are due to the truncation error of the Eulerian scheme; equally explicit vs. implicit formulation of the thermodynamic equation results in distorted longer mean flow oscillation *(explicit may be improved by increasing the vertical resolution)***
- **Choice of advection scheme *(flux-form Eulerian more accurate)***
- **Upper boundaries and stratification changes *(may catalyze the onset of flow reversal; also in 2D Boussinesq experiments due to wave reflection, in atmospheric conditions also changing wave momentum flux with non-Boussinesq amplification of gravity waves)***

Summary

- **Introduced the concept of reductionism and a hierarchy of reduced models with gradually increased complexity to test a numerical model core but even more so to test the model assumptions and our understanding !**

Additional slides

Conclusions

- **Suitable for testing 3D non-hydrostatic developments in an existing hydrostatic technical framework with minimal effort and computational cost.**
- **Reduced-size planet experiments offer exciting new avenues for research with complex NWP models, while further testing the code for potential future applications.**

see also Q. J. R. Meteorol. Soc. 135: 469-484 (2009) for more details!

Some characteristics of IFS and EULAG

IFS *www.ecmwf.int/research*	**EULAG** *http://box.mmm.ucar.edu/eulag*
Time integration is semi-implicit or iterative-centred-implicit (ICI), coupled to a two-time-level semi-Lagrangian advection on a reduced Gaussian grid, using the spectral transform method and a direct solver for the resulting Helmholtz equation.	**Time integration is an implicit, two-time-level, flux-form Eulerian or semi-Lagrangian fluid solver on a regular grid, with an iterative, preconditioned conjugate residual algorithm for solving the resulting elliptic pressure equation.**
Terrain-following, mass-based coordinate	**Terrain-following, height coordinate**
Fully compressible	**Anelastic/Boussinesq/other**
Spectral horizontal representation with spherical harmonics basis functions; finite-difference (NH) / finite element (H) vertical discretization	**Non-staggered/staggered gridpoint representation, finite-differences**
Deep/shallow atmosphere	**Deep/shallow atmosphere**

Cost-effectiveness

- **The cost of a resolution of 1km globally, *O*(~T_L20000), is approximately 300 x T_L2047 (approx. 10 km) and currently not affordable.**
- **However, reducing the radius by a factor 10 (even accounting for a corresponding reduction in time-step) reduces the problem size to make 1km globally affordable now, which in return opens up intriguing opportunities for research in the future on initial and boundary conditions for small planets… ?!**

A multiphase example (Sylvie Malardel)

$$\rho = \rho_d + \rho_v + \rho_c + \rho_i + \rho_r + \ldots$$

$$c_V = (1 - q_v - q_c - q_i - q_r) c_{Vd} + q_v c_{Vv} + (q_c + q_r) c_{liq} + (q_i + \ldots) c_{sol}$$

$$c_p = (1 - q_v - q_c - q_i - q_r) c_{pd} + q_v c_{pv} + (q_c + q_r) c_{liq} + (q_i + \ldots) c_{sol}$$

$$R = R_d - (R_v - R_d) q_v - R_d (q_c + q_i + q_r + \ldots)$$

$$p = \rho R T$$

Hydrostatic Model

$$\frac{Dv}{Dt} = \ldots$$

$$\frac{D}{Dt} q_{(k=v,c,i,r,\ldots)} = P_{phys,q_k}$$

$$\frac{DT}{Dt} - \frac{RT}{c_p p} \frac{Dp_{hyd}}{Dt} = \frac{Q}{c_p} = P_{phys,T}$$

$$\frac{Dp_{hyd}}{Dt} = -\int \nabla_p \cdot v$$

A multiphase example (Sylvie Malardel)

Nonhydrostatic Model

(note the additional prognostic equations for pressure and vertical velocity)

$$\frac{Dv}{Dt} = \ldots, \frac{Dw}{Dt} = \ldots$$

$$\frac{D}{Dt} q_{(k=v,c,i,r,\ldots)} = P_{phys,q_k}$$

$$\frac{DT}{Dt} - \frac{RT}{c_p p}\frac{Dp}{Dt} = \frac{Q}{c_p} = P_{phys,T}$$

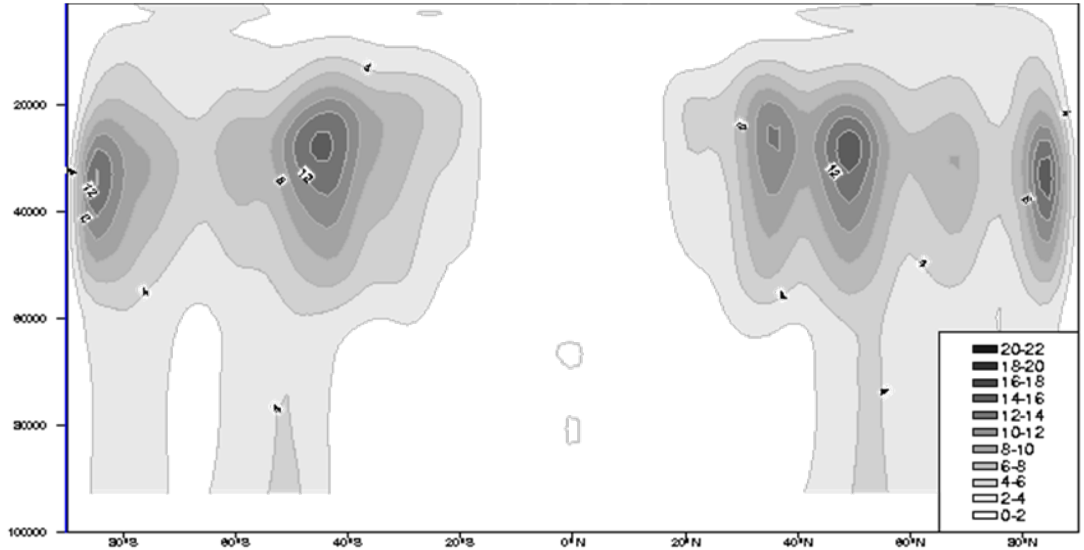
$$\frac{D\rho}{Dt} = -\rho \nabla \cdot v$$

$$\Rightarrow \frac{1}{p}\frac{Dp}{Dt} + \frac{c_p}{c_v}\nabla \cdot v = \frac{c_p}{c_v}\frac{1}{R}\frac{DR}{Dt} + \frac{Q}{c_v T}$$

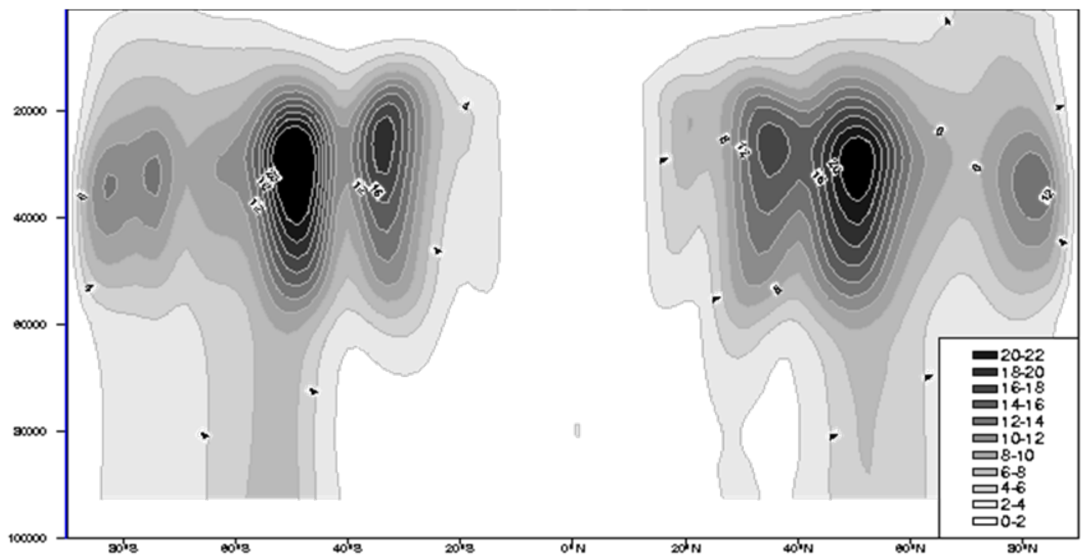
currently set to 0 in IFS

Physics or Dynamics ?

Held-Suarez: Time variance of stationary zonal mean zonal flow



T_y=60K



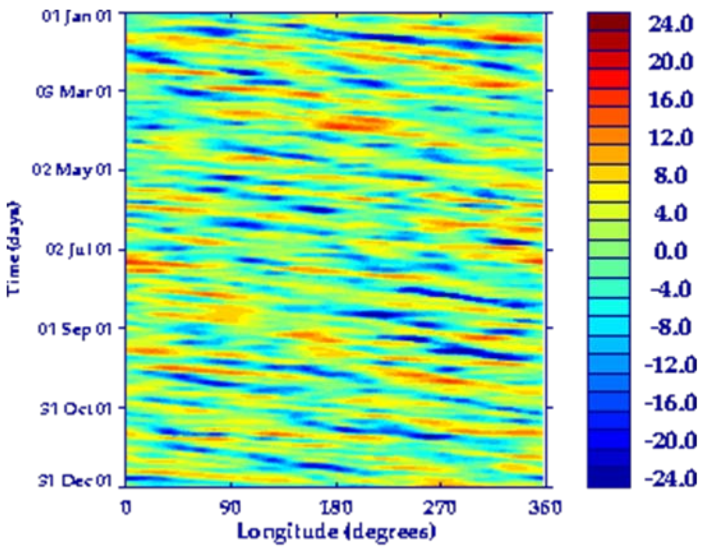
T_y=80K

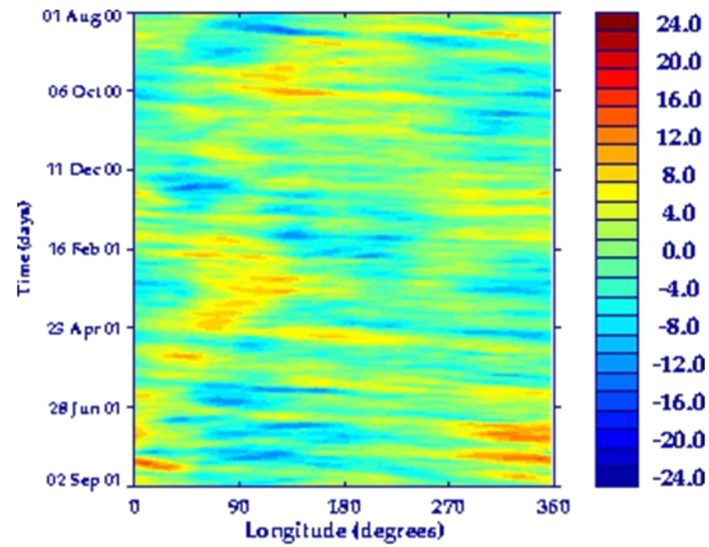
29r2

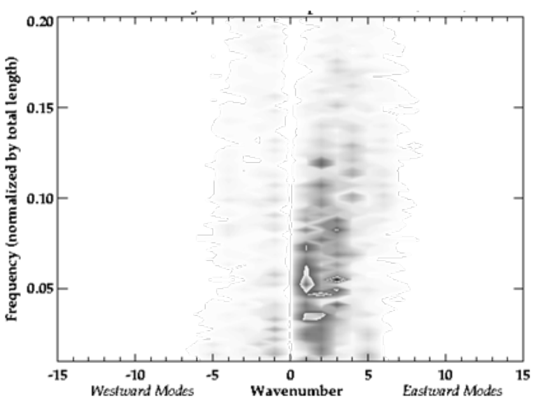
mf_tm_uu

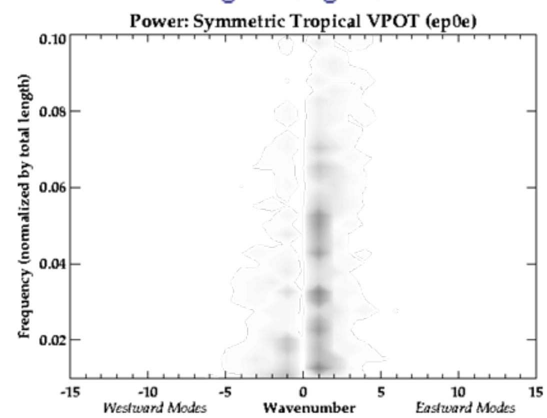
(temporal) anomaly of velocity potential 200hPa

time









29r2 *Aquaplanet Control* *Full IFS 'climate' simulation*

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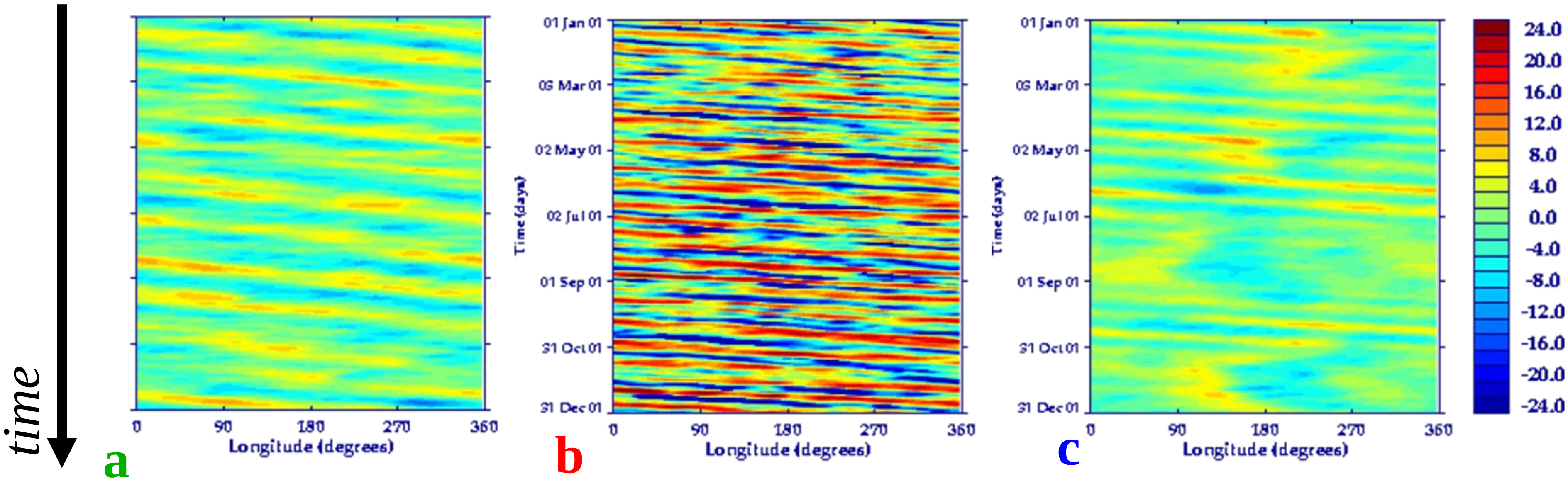

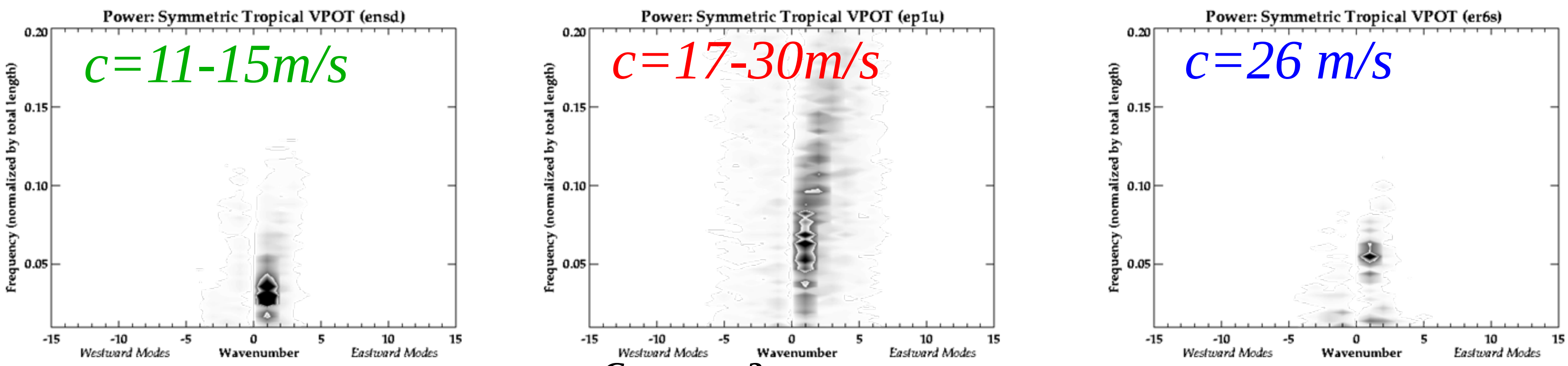

Contour x 2

Aqua-planet experiments with **(a) flat**, **(b) peaked** and **(c) constant** SST distribution.

(5- day low-pass filtered)