Humidity Control Variable and Supersaturation

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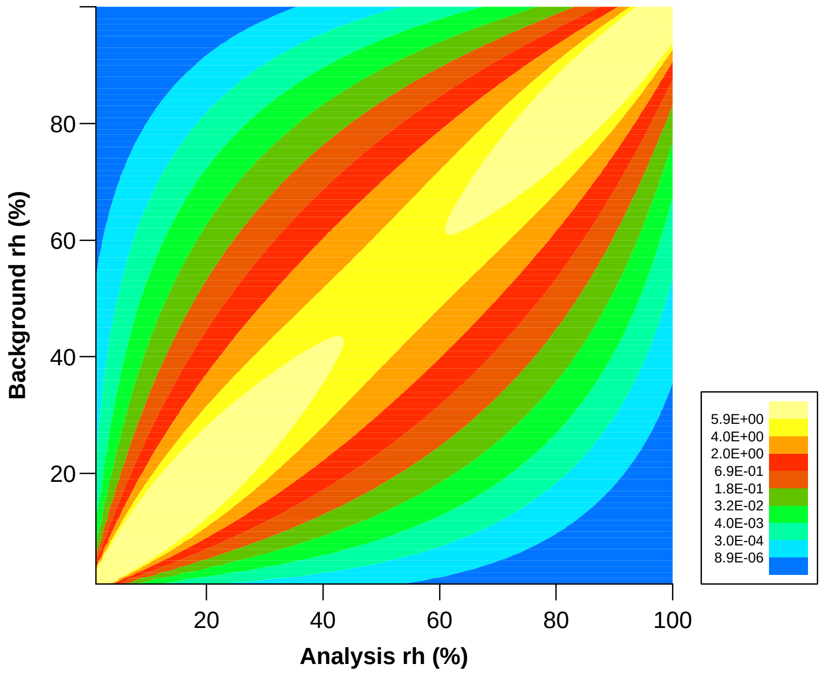
1. Inhomogeneity and asymmetry of humidity variances
2. Supersaturation and ice: different regime?

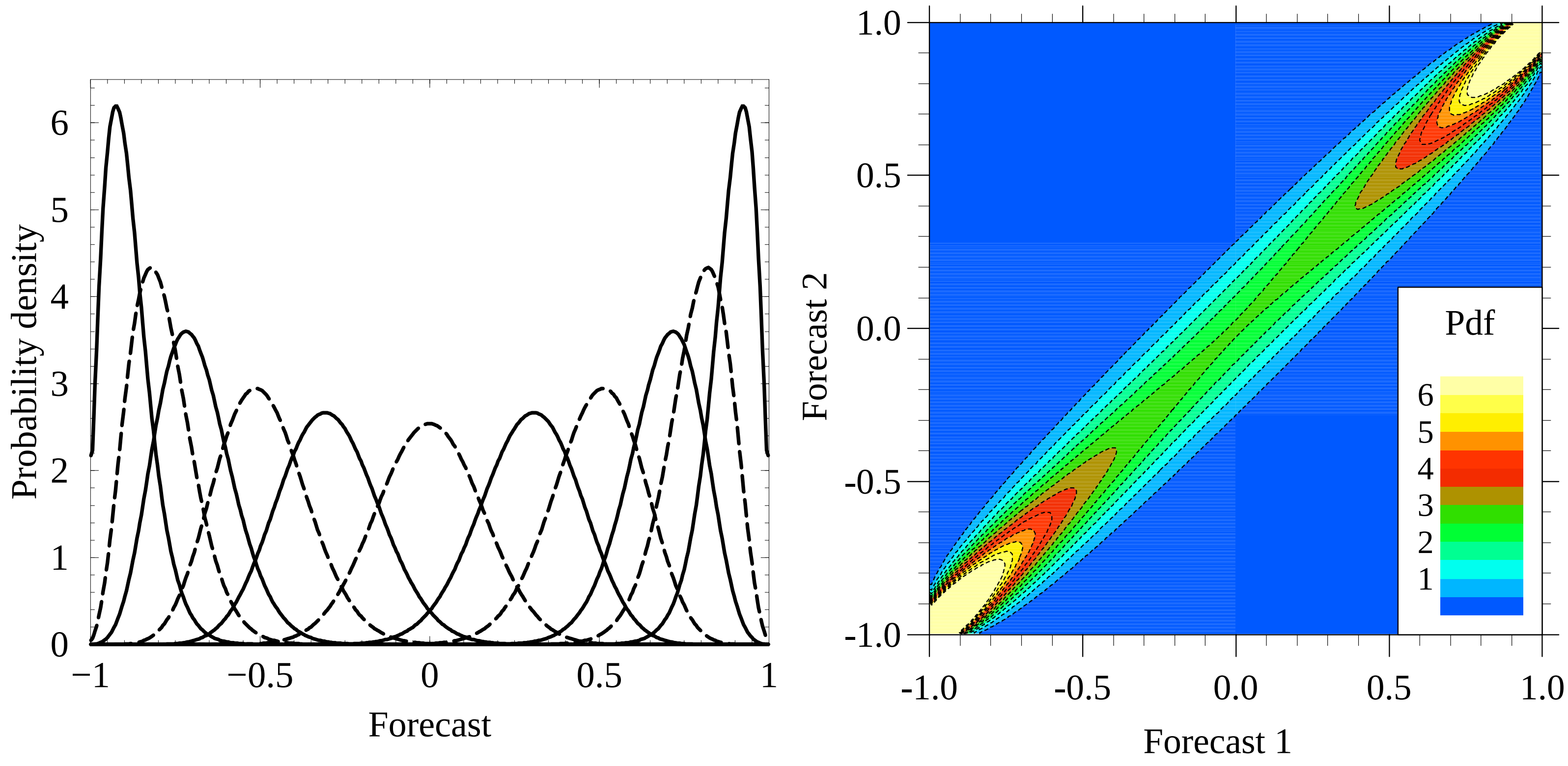
The joint pdf $P(rh^b, rh^{an})$



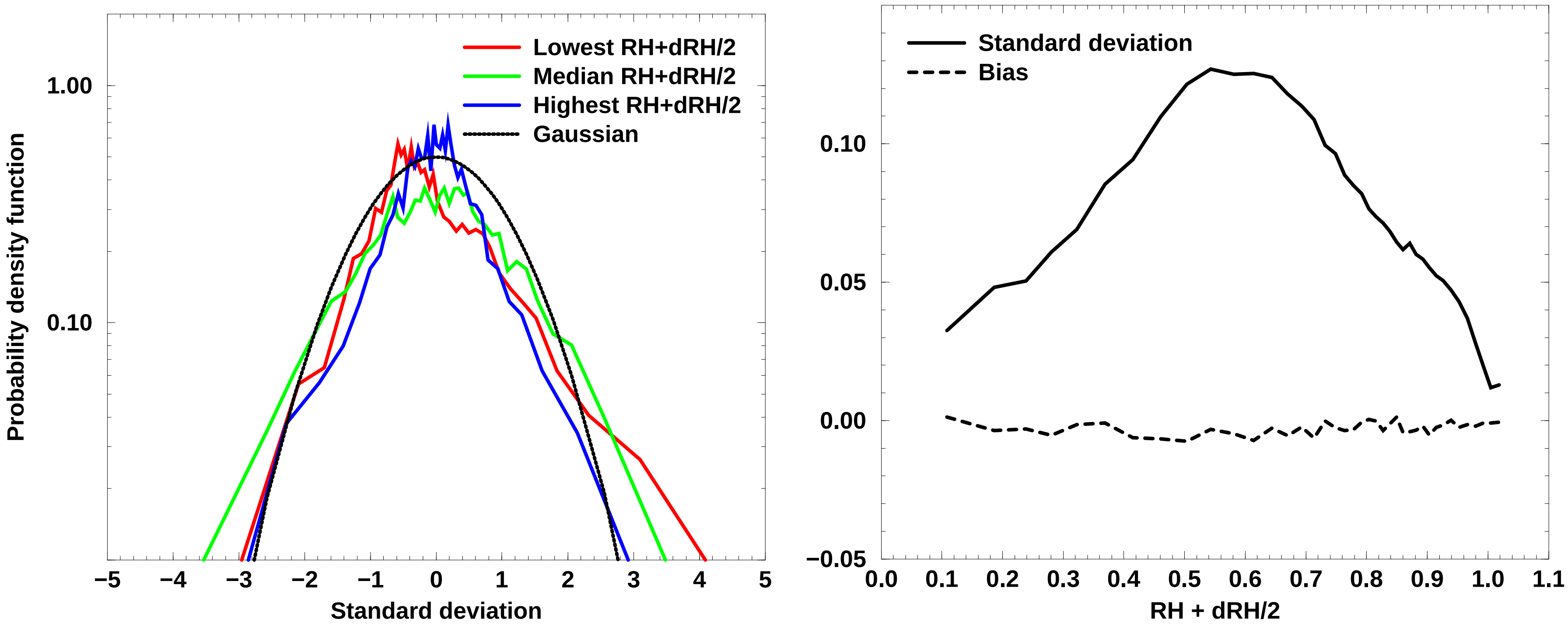
The joint pdf (from forecast differences) is asymmetric w. r. t. rh^b, but symmetric w. r. t.

$$\frac{1}{2}(rh^b + rh^{an}) = rh^b + \frac{1}{2}\delta rh$$

Background error for a harmonic oscillator



The symmetric pdf $P(\delta rh | rh^b + \frac{1}{2}\delta rh)$

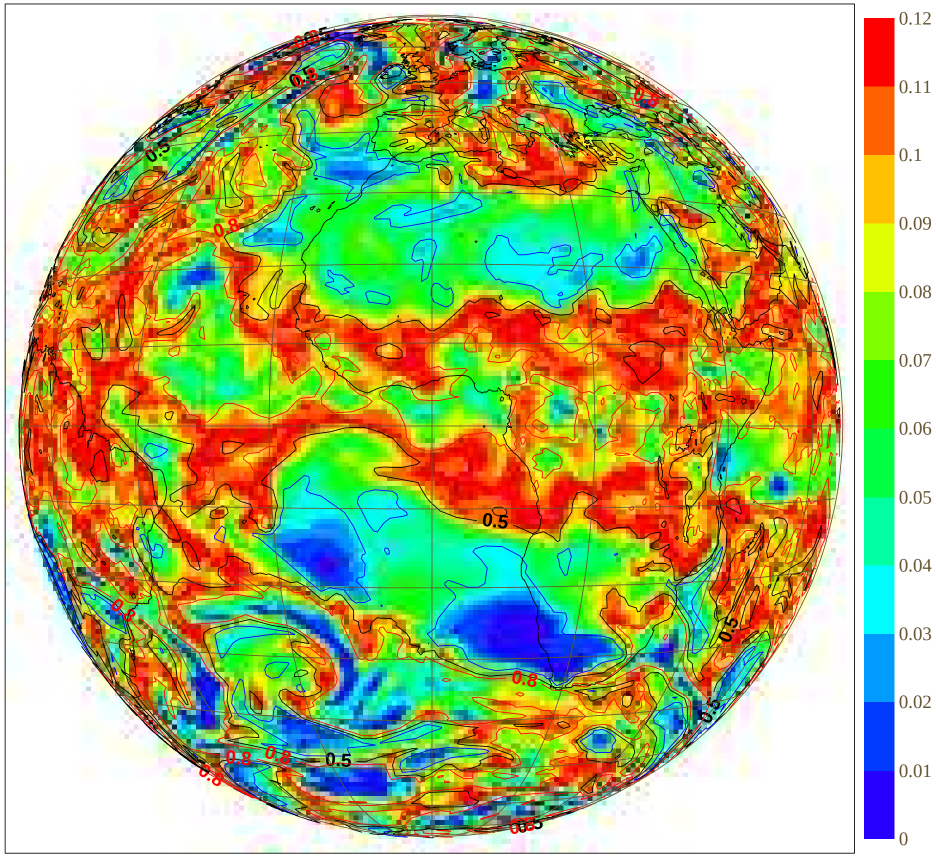


The symmetric pdf $P(\delta rh | rh^b + \frac{1}{2}\delta rh)$ can be modelled by a normal distribution.

The variance changes with $rh^b + \frac{1}{2}\delta rh$, and the bias is zero. A control variable with approximately a normal distribution $\mathcal{N}(0, 1)$ is obtained by nonlinear normalization

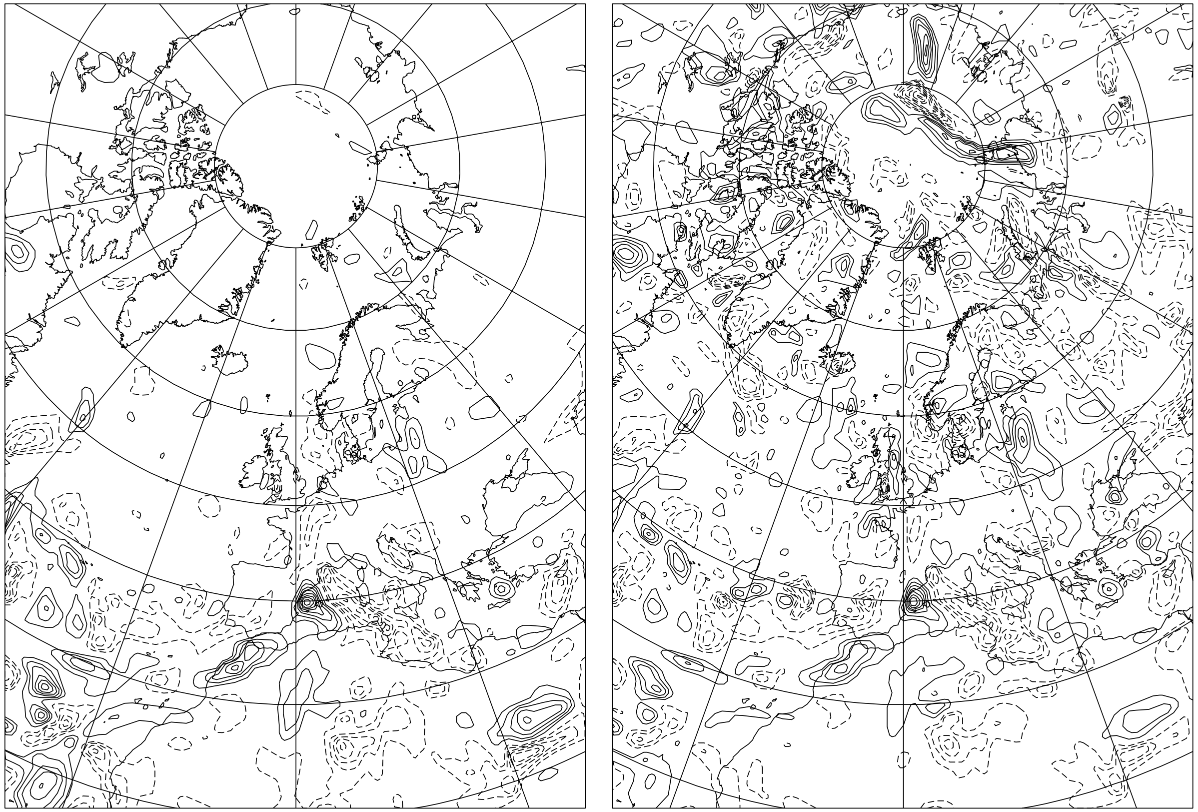
$$\widetilde{\delta rh} = \frac{\delta rh}{\sigma(rh^b + \frac{1}{2}\delta rh)}$$

Background error standard deviation $\sigma_{rh}(\frac{1}{2}(rh^b + rh^{an}))$



Isolines of cloudcover (o.2=blue, 0.5=black, 0.8=red)

Normalization



- Forecast differences (ca. 850 hPa) for specific humidity δq (left) and normalized relative humidity $\widetilde{\delta rh}$ (right).
- The normalized relative humidity gives a more homogeneous field for the following normalization in spectral or wavelet space.

Implementing nonlinear humidity analysis

The background error costfunction J_b is now nonlinear

$$J_b = f^T(\delta rh) B^{-1} f(\delta rh)$$

$$f(\delta rh) = \frac{\delta rh}{\sigma(rh^b + \frac{1}{2}\delta rh)}$$

Implementation requires linear inner loops with nonlinearities in outer loops

- Inner loops use $\widetilde{\delta rh} = \delta rh / \sigma(rh^b)$.
- In outer loops $\widetilde{\delta rh}$ is given. Solve δrh from the nonlinear equation

$$\frac{\delta rh}{\sigma(rh^b + \frac{1}{2}\delta rh)} - \widetilde{\delta rh} = 0$$

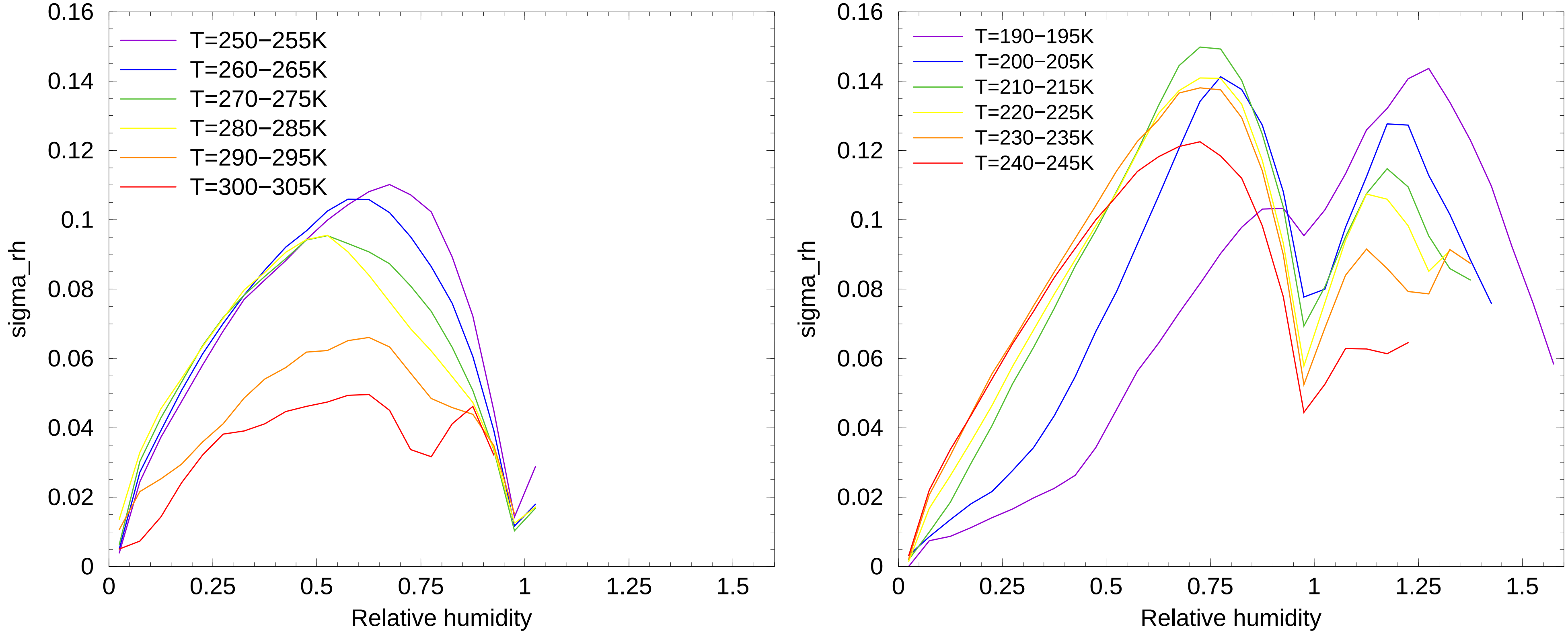
Extending humidity analysis to supersaturation

- Asymmetric behaviour of relative humidity errors close to zero and saturation is treated by a nonlinear inversion at outer loop level

$$\widetilde{\delta rh} - \delta rh/\sigma(rh^b + \frac{1}{2}\delta rh) = 0$$

- Here σ is approximated by a convex function with a minimum at zero and close to saturation.
- Now relative humidity in the range 1-2 frequently occurs in ice clouds.
- How do these statistics look?

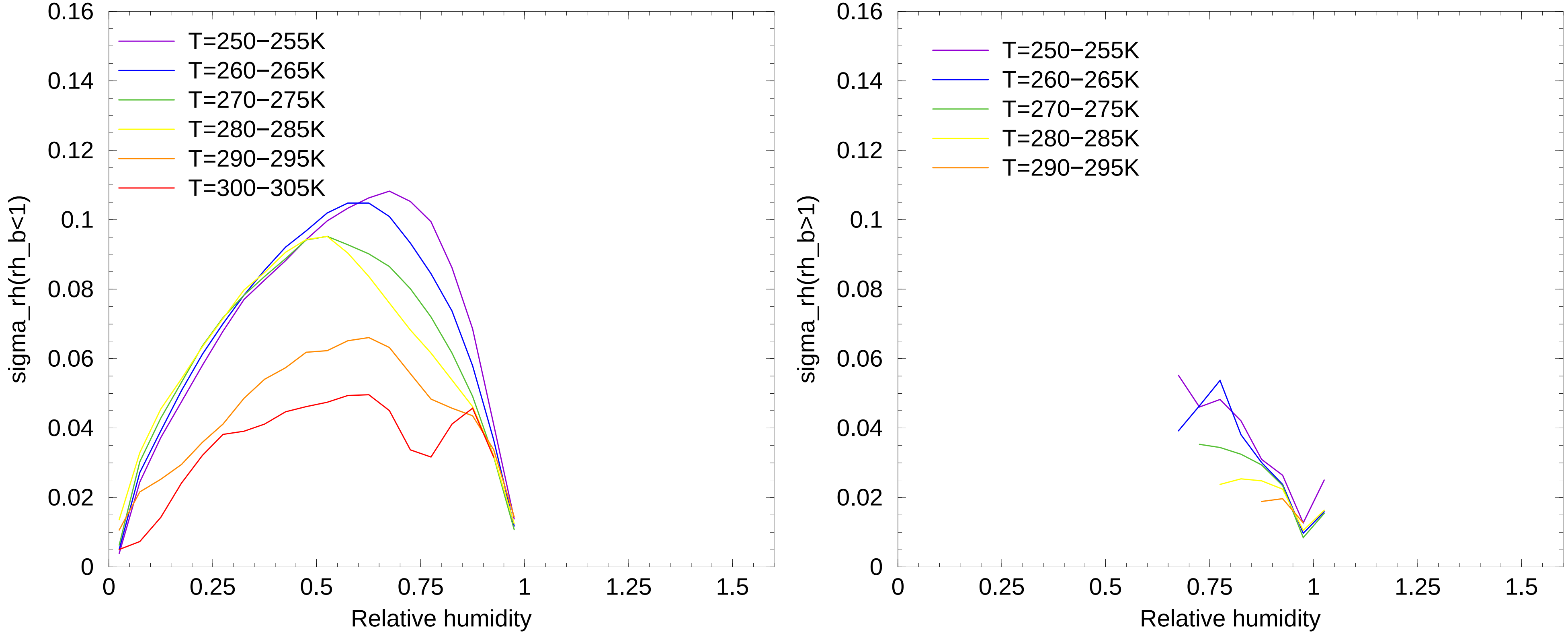
σ_{rh} for different temperature intervals



σ_{rh} characteristics

- In warm and mixed water-ice conditions ($T > 253K$) no significant supersaturation.
- Below $253K$ a second mode of icy supersaturated conditions appears. A minimum appears around $\frac{1}{2}(rh^b + rh^{an}) = 1$, giving non-convex $\sigma(rh)$.
- The nonlinear inversion which takes into account the asymmetry of the background error pdf's does not work for non-convex functions: increment not a smoothly increasing function of the increment.

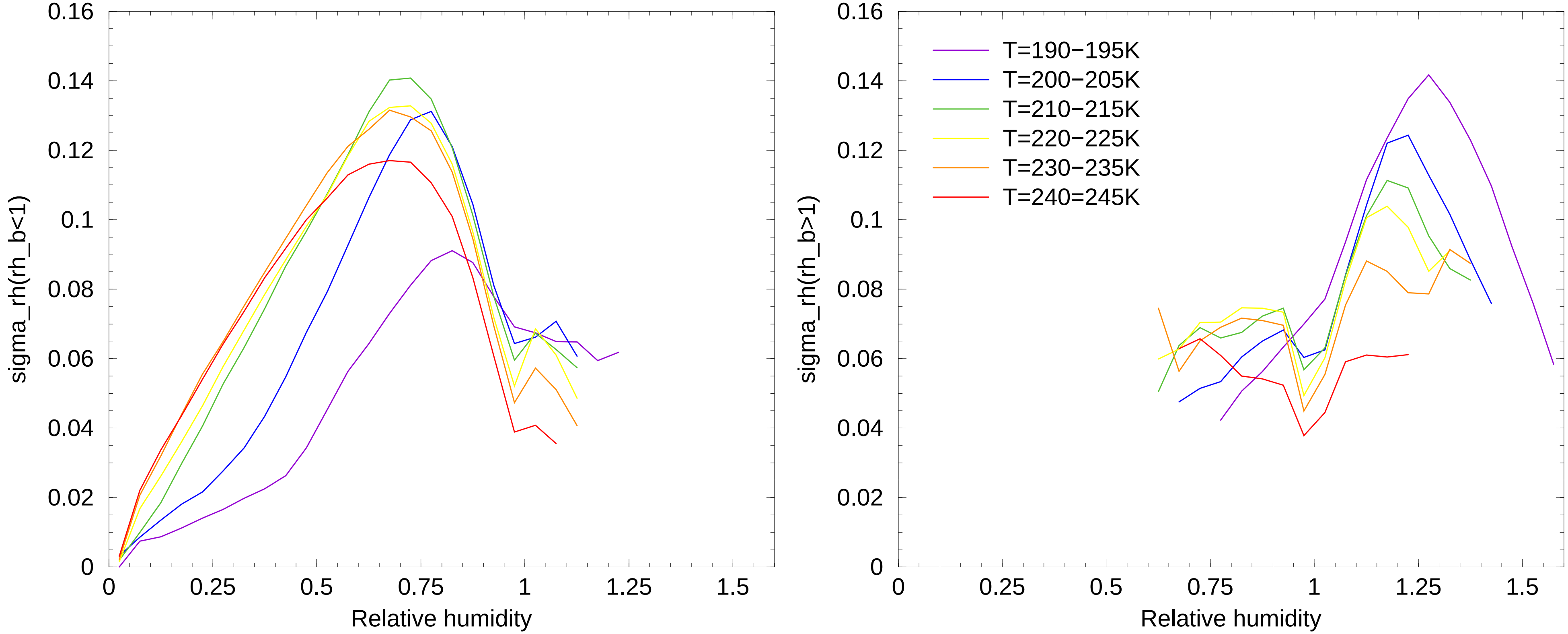
σ_{rh} for warm/mixed conditions $(T > 253K)$



Left: Background relative humidity below 1

Right: Background relative humidity above 1

σ_{rh} for ice conditions $(T < 253K)$



Left: Background relative humidity below 1

Right: Background relative humidity above 1

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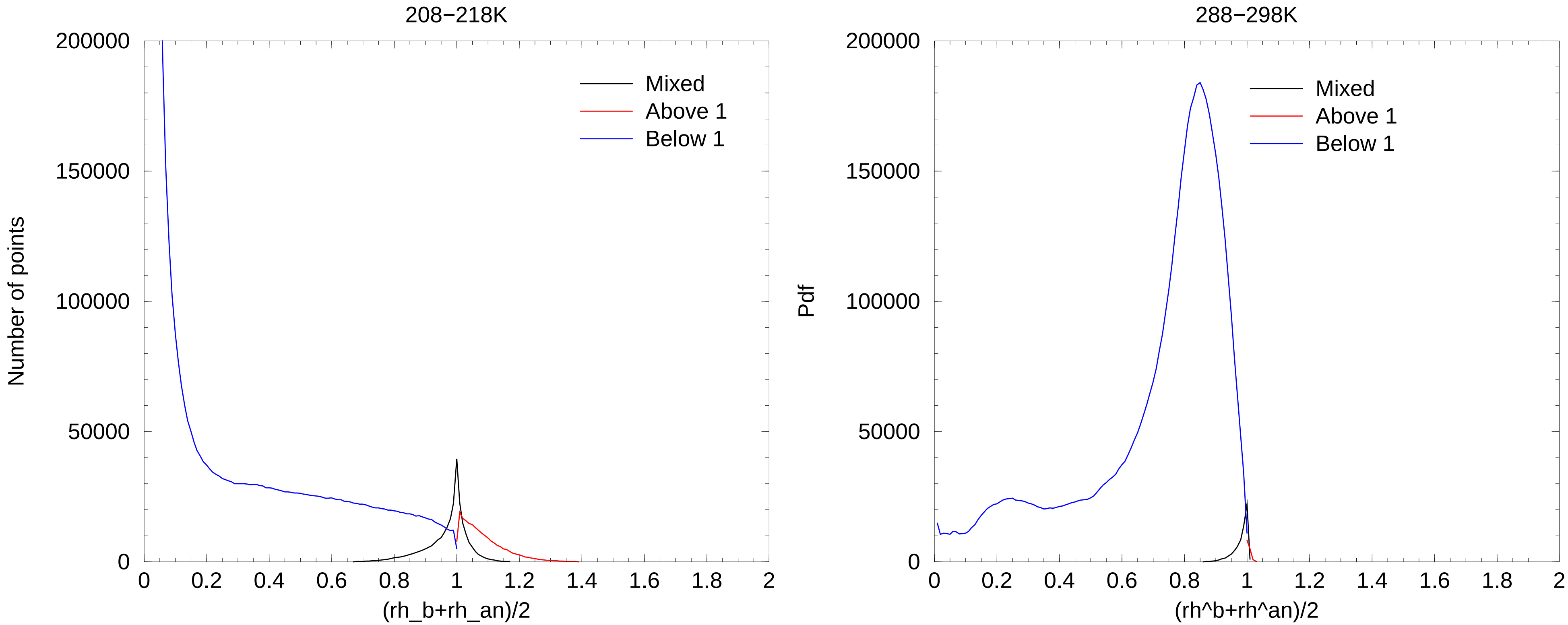
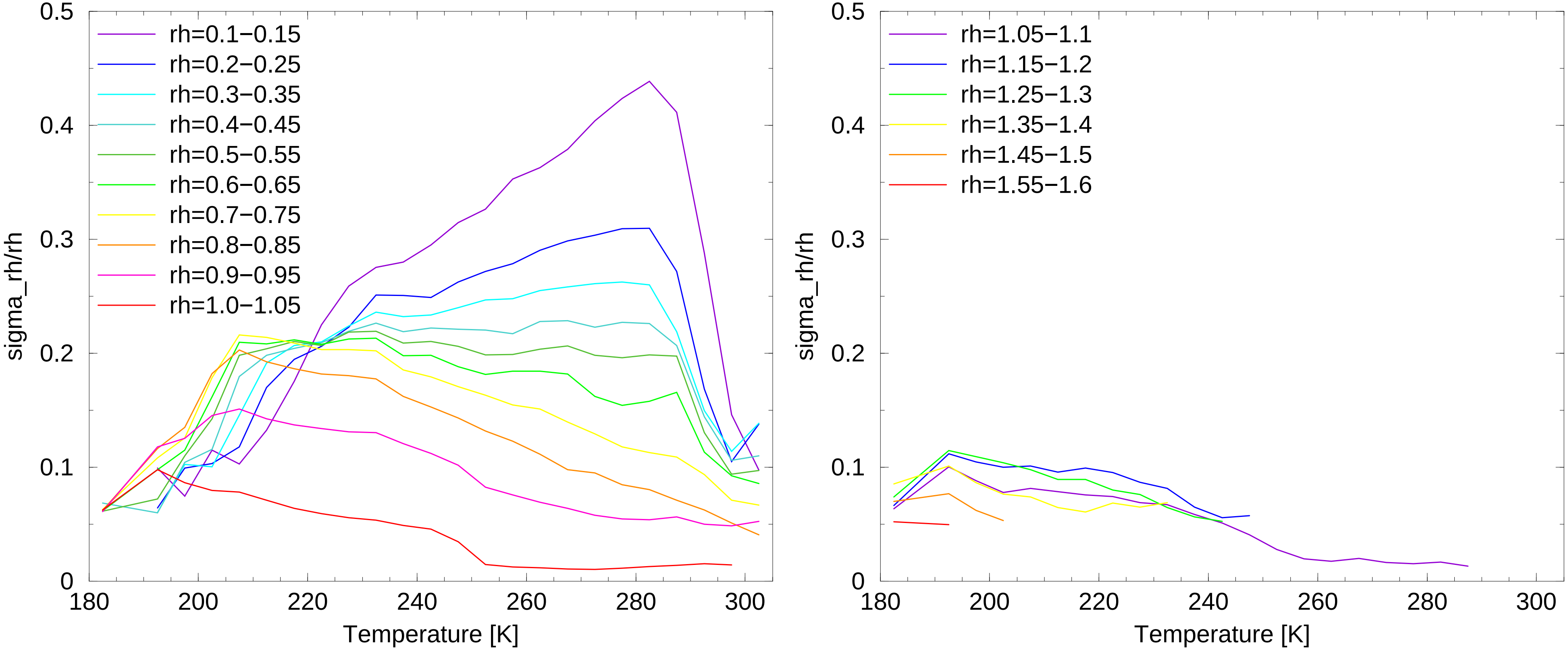
The transition of increments at different temperatures



The conditional pdf's for background errors

- Model by two conditional distributions: one for $rh^b < 1$ and another for $rh^b > 1$, both convex.
- The pdf for $rh^b < 1$ can go up to $\frac{1}{2}(rh^b + rh^{an}) \approx 1.2$ ($rh^{an} < 1.4$), thus allowing transition from sub to supersaturated conditions.
- The pdf for $rh^b > 1$ can go down to $\frac{1}{2}(rh^b + rh^{an}) \approx 0.8$ ($rh^{an} < 0.6$), thus allowing transition from super to subsaturated conditions.
- At each outer loop iteration the choice of conditional pdf is determined by the value of the nonlinear trajectory, thus allowing different choice on different iterations as needed.

σ_{rh}/rh for different relative humidity intervals



Temperature or level based covariance modelling?

- Temperature dependent covariance modelling blends stratosphere and troposphere with different characteristics. Correct limiting behaviour for low rh essential, otherwise the background error could easily be overestimated by several orders of magnitude, resulting in a very bad condition number for the analysis if for example a humidity sensitive radiance is assimilated in the area!

- Model level dependent modelling is still possible, as long as one uses two conditional pdf's for each level (sub/supersaturated rh^b

- Model by two conditional distributions: one for $rh^b < 1$ and another for $rh^b > 1$, both convex.