# Accounting for Model Biases in 4D-Var\*

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### Variational Data Assimilation

Variational data assimilation comprises minimising a cost function which measures the discrepancy between an estimated atmospheric state x and available observations y. In the most general form, x is the four dimensional state of the atmosphere during the assimilation window. With the additional assumption that the forecast model is perfect, x is only a function of the initial condition  $x_0$  and the size of the control variable reduces to a three dimensional state using the forecast model  $\mathcal{M}$ . This approach is known as strong constraint 4D-Var or simply 4D-Var. It is used in most operational implementations of 4D-Var.

However, in practice, the model equations are only known and solved inexactly. They can be imposed as a weak constraint in the optimisation problem. Model error  $\eta$ , can defined at each time step by:

$$\eta_i = x_i - \mathcal{M}_i(x_{i-1}).$$

It is also possible to define model error as the error relative to the model solution integrated from the initial condition  $x_0$ . It is denoted  $\beta$  defined by:

$$\beta_i = x_i - \mathcal{M}_{i,0}(x_0)$$

With the assumption that it is constant in time,  $\beta$  is a good representation of the model bias.

In principle, the choice of x,  $(x_0, \eta)$  or  $(x_0, \beta)$  as a control variable for weak constraint 4D-Var are equivalent. However, the choice of approximation that  $\eta$  or  $\beta$  are constant in time are not. A description of the properties of 4D-Var with these control variables is given by Trémolet (2006).

#### **Experimental results**

Weak constraint 4D-Var is being implemented in the IFS. Preliminary experiments have been performed and showed encouraging results. For example, taking model error into account as a constant forcing ( $\eta$  constant over the assimilation window) reduces the initial condition increment over Antarctica. This is confirmed by the mean first guess departure for AMSU-A on NOAA-16 shown on figure 1. The bias with respect to AMSU-A is more uniform over both hemispheres and, in the southern hemisphere, more data is used while the background departures standard deviation is reduced. This is confirmed by the statistics of the fit to radiosonde data over Antarctica (figure 2) which show that the vertical oscillations in bias are reduced and standard deviation is reduced above 50 hPa for both the background and analysis. This shows that weak constraints 4D-Var does capture some model errors such as the temperature bias in the winter stratosphere.

Preliminary experiments have also shown that, in addition to capturing model errors, weak constraint 4D-Var can capture observation bias (Trémolet (2005)). As 4D-Var only sees the difference between model and

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Figure 1: NOAA-16 AMSU-A channel 14 mean first guess departures for July 2004.



Figure 2: Fit to radiosonde temperature observations for the South Pole area in strong constraint 4D-Var (red) and weak constraint 4D-Var (black).

observations, the distinction between model bias and observation bias can only be achieved through the prior knowledge built into the formulation of 4D-Var. Two tools are available to define the errors to be determined: the model error covariance matrix which can be tuned to selectively capture scales representative of certain types of error and the model for model error which can be more or less relevant to various forms of error. That includes the choice of the most appropriate formulation of weak constraint 4D-Var in terms of the control variable  $\eta$ ,  $\beta$  or x. Interactions with variational observation bias correction will also have to be studied and taken into account. More research is still needed in these areas to exploit the full potential of weak constraint 4D-Var, both for operational data assimilation and reanalysis.

In addition to lifting the questionable assumption that the model is perfect, model error is valuable information which can be used in several ways. It can be added as forcing in the forecast model, or at the post-processing stage if a model bias was determined, to improve the forecast. It should also help identify model deficiencies and improve the model. In the context of reanalysis, weak constraint 4D-Var should allow for the use of longer assimilation windows which can take full advantage of future observations.

# References

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