Some Issues in Isentropic Coordinate Modelling and Modelling the Tropopause

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1. Introduction

The use of entropy or potential temperature as the vertical coordinate appears to be an attractive option for modelling the stratosphere. Diabatic heating is small in the stratosphere, and, to the extent that it is small, the flow becomes quasi-horizontal along isentropic surfaces, so that any numerical errors associated with vertical advection should be reduced. Moreover, the difficulties of handling the bottom boundary at the Earth's surface, for example through the use of massless layers or through a hybrid terrain-following coordinate, are far removed. In view of the growing importance of a good representation of the stratosphere for numerical weather prediction and data assimilation, it is appropriate to review both the difficulties and advantages of using an isentropic coordinate.

In the early 1990s a hybrid-isentropic coordinate general circulation model, known as the HIGCM, was developed under the UK Universities Global Atmospheric Modelling Programme (UGAMP). Section 2 below briefly reviews some of the lessons learned from that effort, and, in particular, highlights the difficulties of using a Lorenz vertical grid with an isentropic coordinate. Some new results on the numerical wave dispersion properties of different vertical grid arrangements reinforce the argument in favour of a Charney-Phillips vertical grid.

Despite the difficulties, some clear benefits of using an isentropic coordinate have been found by various groups. Section 3 below focuses on two in particular. The first benefit is a reduction in a cold bias around the extratropical tropopause, which appears to be at least partly associated with an improved representation of propagation of planetary waves into the stratosphere. The second benefit is improved stratospheric tracer transport, with more realistic "tape recorder" signal and age of air. An idealized one-dimensional advection problem shows how this benefit can be related to the expected reduction in vertical advection errors.

The mixing ratio of water vapour in the stratosphere is believed to be controlled by dehydration processes near the tropical tropopause, though the details are still controversial. Section below discusses some results from a cloud-resolving simulation, which support the hypothesis that the final stage of dehydration, which controls the stratospheric water vapour content, occurs in slow large-scale ascent rather than in cumulus clouds. The implications of this result for accurately modelling stratospheric water vapour are discussed.

2. Some lessons learned from the UGAMP HIGCM

2.1 The HIGCM

The HIGCM (Zhu et al. 1992) was developed from the ECMWF cycle 27 weather forecast model. Its earliest implementation used the same suite of physical parameterizations. The dynamical core integrated the hydrostatic primitive equations using an Eulerian spectral transform method with semi-implicit time stepping. The vertical scheme used an extension of the Simmons and Burridge (1981) vertical scheme on a Lorenz vertical grid. Figure 1 shows the arrangement of variables on the vertical grid.

————————	$k-1/2$	p, ω
– – – – – – – –	k	u, v, T, ω
————————	$k+1/2$	p, ω

Figure 1. Schematic showing the arrangement of variables on the HIGCM vertical grid. $\omega = Dp/Dt$ *and* ω *is the cross-level vertical mass flux in the same units as* ω*; other notation is standard.*

The pressure on any model half-level is defined in terms of the surface pressure p_* and the half-level temperature by

$$p_{k+1/2} = \frac{a_{k+1/2} + b_{k+1/2}p_*}{d_{k+1/2} + c_{k+1/2}T_{k+1/2}^{-1/\kappa}}, \tag{1}$$

where a, b, c, d are four constants for that level and $\kappa = R/C_p$ is the ratio of the gas constant to the specific heat capacity at constant pressure for dry air. Note that (1) involves half-level temperature, which must be obtained by averaging the full-level temperature predicted by the model:

$$T_{k+1/2} = \frac{1}{2}(T_k + T_{k+1}). \tag{2}$$

If c is set to zero then (1) reduces to the hybrid sigma-pressure formula of Simmons and Burridge (1981). On the other hand, if b and d are set to zero then $\theta \propto Tp^{-\kappa}$ will be constant on that model level. More generally, by a suitable choice of the coefficients for each level a hybrid coordinate that changes smoothly from one type (such as sigma at the ground) to another (such as isentropic in the stratosphere) can be obtained.

An important feature of this hybrid vertical coordinate is that the mass continuity and thermodynamic equations become a coupled system for the two unknowns ω and $\partial T/\partial t$:

$$\hat{\omega}_{k+1/2} = -E_{k+1/2}\frac{\partial p_*}{\partial t} - F_{k+1/2}\frac{\partial T_{k+1/2}}{\partial t} + \sum_{r=1}^{k} \nabla \cdot (\mathbf{v}_r \Delta p_r) \tag{3}$$

where E and F are known coefficients, and

$$\frac{\partial T_k}{\partial t} + \frac{1}{2\Delta p_k}\{\hat{\omega}_{k-1/2}(T_k - T_{k-1}) + \hat{\omega}_{k+1/2}(T_{k+1} - T_k)\} = \text{known} \tag{4}$$

Because of the averaging in (2) the coupling is non-local in the vertical and leads to a tridiagonal system to be solved for each column. Also, the “known” terms on the right hand side of (4) involve the diabatic heating, so that the diabatic tendencies due to all parameterized processes must be computed before this system can be solved and the dynamical step completed.

The feasibility of this approach depends on the fact that the centred difference vertical advection scheme in (4) is linear in ω. Many advection schemes (including semi-lagrangian) are not linear in the advecting velocity; the use of such a scheme in (4) would lead to a coupled non-local and non-linear system for ω and temperature tendency, which would be difficult to solve and make the scheme unattractive. Thus the HIGCM formulation leads to a restriction on the choice of vertical advection scheme.

2.2 Inflexibility of the HIGCM vertical coordinate

There is a limit to how quickly the model coordinate can make the transition from σ at the ground to θ higher up. For example, if the potential temperature at the ground is 335K (it is about this on the Tibetan Plateau in July) then the transition to θ levels obviously cannot be made before 335K, and in practice it must be made much more slowly to maintain smooth level spacing. A limitation of the formulation (1) is that conditions at a single location, such as the Tibetan Plateau, restrict the transition globally, so that we were unable to run the HIGCM with a lowest pure θ level below 350K. The hybrid coordinate formulation of Konor and Arakawa (1997) is more flexible in this respect: a high potential temperature at the ground at one location does not limit the transition to θ levels at remote locations.

2.3 Conservation

When the model layer thickness $\partial p/\partial\eta$ varies in the horizontal, application of a scale-selective dissipation term to the advective-form equation for moisture or any chemical mixing ratio

$$\frac{\partial\chi}{\partial t} + \ldots = \kappa\nabla^{2n}\chi \tag{5}$$

implies a flux-form equation in which the dissipation term is not a divergence of a flux, and is therefore not conservative. In addition, because layer thickness depends on temperature, scale-selective dissipation acting on temperature can change the tracer mass distribution in a non-conservative way. Both contributions are significant. At low horizontal resolution (T21) tracer mass was found to drift by as much as 40% over a year (Thuburn 1993). Scale-selective dissipation acting on temperature when $\partial p/\partial\eta$ varies in the horizontal can also act as a spurious source of internal energy (Webster et al. 1999). The problem is expected to be reduced at higher horizontal resolution.

If the model formulation were closer to that currently employed at ECMWF, e.g. with a semi-lagrangian advection scheme that is not inherently conservative to begin with, but without explicit scale-selective dissipation, it is not clear whether horizontal variations in $\partial p/\partial\eta$ would adversely affect conservation.

2.4 Lorenz grid

The vertical averaging of temperature (2) required in the HIGCM formulation made that model intolerant of noise in the vertical temperature profile: it was possible for p at level $k-1/2$ to become greater than the pressure at level $k+1/2$, i.e. for model levels to cross, causing the model to "blow up".

More generally, it has been argued by Arakawa (2000) that the Charney Phillips vertical grid is "almost unquestionably the best choice for quasigeostrophic models", and that the discretization for more complete equation sets, such as the primitive equations, should be a generalization of, not a departure from, the best discretization for quasi-geostrophic models, provided that quasigeostrophy is a good approximation.

A comparison of normal modes for the continuous and discretized equations in isentropic coordinates (Tim Woollings, personal communication) adds further weight to the argument for the Charney-Phillips grid. For the hydrostatic primitive equations and for the fully compressible Euler equations, Woollings computed the frequencies of normal modes of oscillation of a resting isothermal atmosphere on a beta-plane for various choices of thermodynamic variable and vertical grid staggering, and compared the results with the analytical solutions. In the hydrostatic case the Charney-Phillips grid (with p staggered relative to u and v) is clearly superior: its dispersion properties are better, and it does not have a computational mode with zero frequency, whereas the Lorenz grid does. For the fully compressible Euler equations three configurations were found that have good dispersion properties, no unstable or missing modes, and no computational mode. These all have w as a prognostic variable staggered relative to u and v, and when z is used as a prognostic variable it should be staggered relative to u and v, while other thermodynamic variables (p or $\partial p/\partial\theta$) should not be staggered.

Despite these arguments in favour of a Charney-Phillips grid, it may be that a hybrid isentropic coordinate scheme based on the new ECMWF vertical discretization (Untch and Hortal 2003), which is a type of Lorenz discretization but with pressure stored at full levels, would avoid the need for vertical averaging of temperature that led to the worst of the problems in the HIGCM discretization.

3. Benefits of an isentropic coordinate

Several studies have claimed to find improvements in accuracy in at least some aspects of model performance through the use of isentropic coordinates (e.g. Black 1987, Johnson et al. 2000). Here we note two of particular relevance to modelling the stratosphere.

Webster et al. (1999), using the HIGCM, found that using a hybrid isentropic coordinate, rather than the usual sigma-pressure coordinate, significantly reduced a cold bias around the extratropical tropopause. Several factors probably contributed to the reduction of the cold bias, including the extra vertical diffusion employed to control the problem of levels crossing mentioned in section 2. However, at least part of the improvement seems to have come from an improvement in the simulation of planetary waves propagating into the winter stratosphere. The Eliassen-Palm flux convergence is significantly strengthened through much of the lower stratosphere, which would lead to increased extratropical downwelling and a warmer tropopause. (See Thuburn and Craig 2000 for a discussion of the sensitivity of tropopause temperature and height to dynamical or other imposed warming.) Moreover, Webster et al. found that the Eliassen-Palm flux convergence field contained apparently unphysical features near the model top when the uppermost model levels were pressure levels, but not when they were isentropic levels. Again this is suggestive of improved handling of planetary waves in an isentropic coordinate. The detailed mechanism behind the improvement, however, was not understood.

A more clear cut advantage of isentropic coordinates is seen in offline simulations of stratospheric transport (e.g. Mahowald et al. 2002). Non-isentropic coordinate models tend to underestimate age of air and to have an excessively fast “tape recorder” signal, while isentropic coordinate models are much more realistic. The benefits are expected to be seen in online transport simulations too. A simple one-dimensional advection test problem, described by Gregory and West (2002) and recreated here, illustrates the likely reason for the improvement in isentropic coordinates. The parameters are chosen to be representative of the tropical lower stratosphere in a GCM. The grid spacing is 1km. The advected field is initially zero, and its value at the lower boundary varies sinusoidally in time with a period of one year. The advection scheme here is a third-order upwind scheme. To begin with, the advecting velocity is a constant 1km per month, comparable to the tropical mean ascent rate. The results (figure 2a) are quite good: there is only a slight numerical damping of the peak of the signal as it propagates upwards, and it propagates at the correct speed. Now, Gregory and West noted that the cross-level vertical velocity in the tropical lower stratosphere of the version of the Met Office Unified Model that they were studying had very large variations, two orders of magnitude bigger than the mean ascent, on diurnal time scales. They therefore repeated the advection test with a diurnal oscillation in ascent rate 80 times greater than the mean ascent rate superposed on the mean ascent (figure 2b). The results are significantly worse: the signal is damped much more quickly and it propagates too fast. It is clear that the task of handling the repeated back-and-forth advection across model levels is a much greater challenge for an advection scheme than handling advection by the mean ascent alone..Gregory and West showed that the original (lower order) advection scheme in the Unified Model performed even worse, and used their results to argue for a higher-order (at least fifth-order) advection scheme. However, their one-dimensional test problem also helps to explain why non-isentropic coordinate models underestimate age of air and have too fast a tape recorder signal. In isentropic coordinate models, however, the diurnal variations in cross-level flow should be comparable to the mean ascent, not two orders of magnitude greater, so the problem illustrated in figure 2b should be much reduced.

4. Water transport across the tropical tropopause

It is widely accepted that the extreme dryness of the stratosphere is due to dehydration of air as it enters the stratosphere through the cold tropical tropopause region. However, the details of the final stage of dehydration, which sets the water vapour mixing ratio for air entering the stratosphere, are still uncertain.

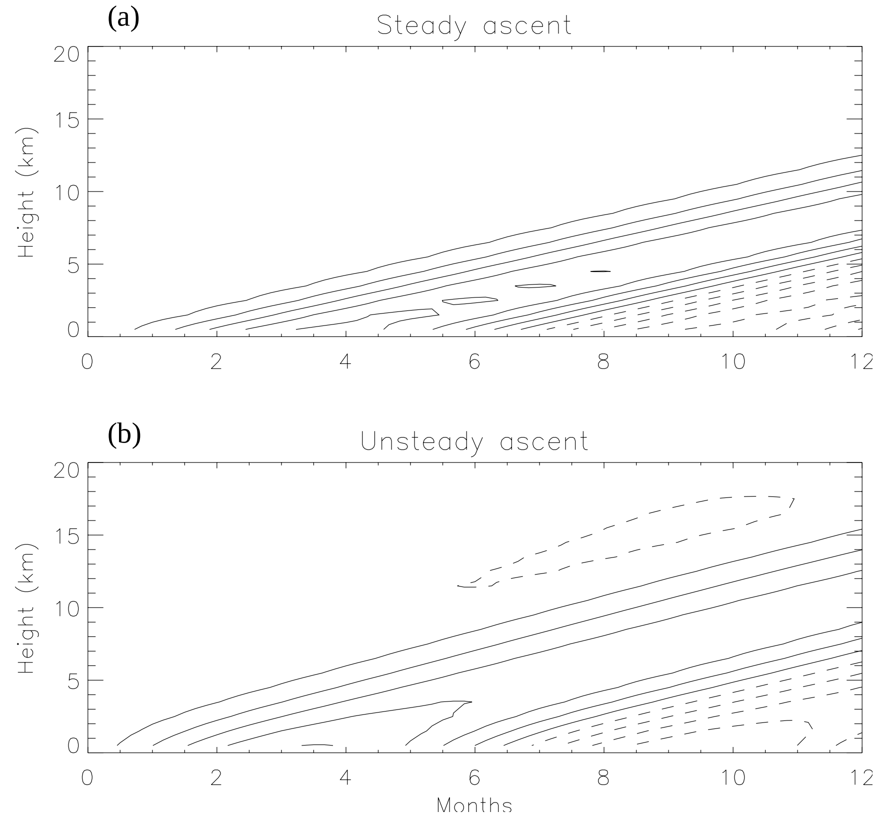

Figure 2. Contours of tracer mixing ratio versus time and height in an idealized one-dimensional advection problem. (a) with weak steady ascent. (b) with large diurnal oscillations superposed on weak mean ascent.

Figure 3 illustrates schematically three scenarios that have been proposed. In scenario 1 the final stage of dehydration occurs in or associated with cumulus clouds overshooting the large-scale temperature minimum. In scenario 2 the final stage of dehydration occurs at the large-scale temperature minimum in slow mean ascent. In scenario 3 the final stage of dehydration occurs in cumulus clouds overshooting their level of neutral buoyancy (and hence colder than their environment) but below the large-scale temperature minimum.

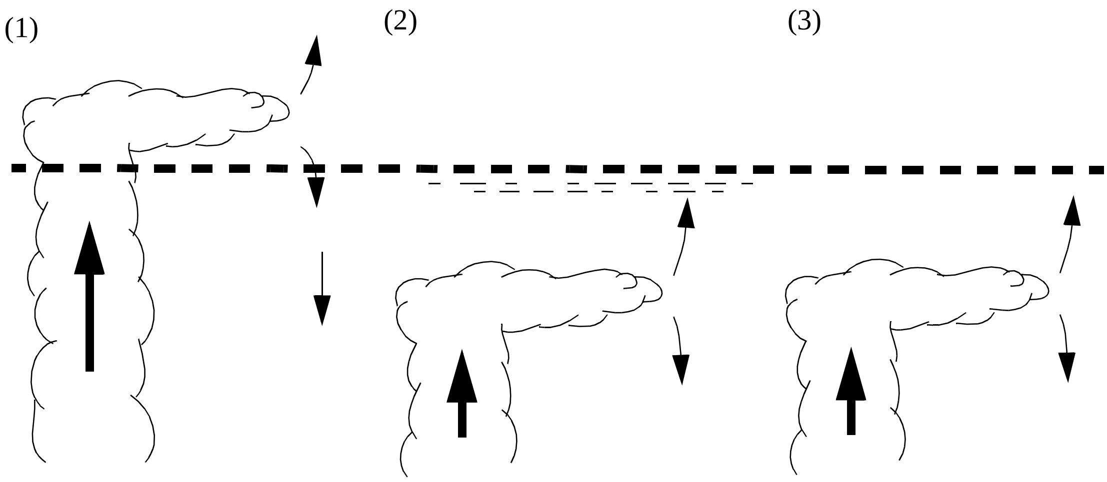

Figure 3. Three proposed scenarios for the final stage of dehydration of air entering the tropical stratosphere. The heavy dashed line indicates the large-scale temperature minimum. (1) Dehydration in cumulus clouds overshooting the large-scale temperature minimum. (2) Dehydration in slow mean ascent across the large-scale temperature minimum, forming ultra-thin cirrus clouds. (3) Dehydration in cumulus clouds followed by slow ascent across the large-scale temperature minimum.

Observations and numerical simulations all indicate that the cumulus mass flux falls off rapidly with height above about 12km (figure 4). One way to distinguish between the three scenarios described above is in terms of the matching altitude at which the cumulus mass flux equals the upward mass flux required by the wave-driven Brewer-Dobson circulation: In scenario 1 the matching altitude is above the large-scale temperature minimum; in scenarios 2 and 3 it is below. Unfortunately, the cumulus mass flux is extremely difficult to estimate, and the available observational estimates do not constrain it sufficiently to be able to distinguish the scenarios (figure 4).

Küpper et al. (2003) have looked for the matching altitude in a cloud-resolving simulation, intended to be representative of the mean tropical atmosphere. They imposed a mean ascent typical of the Brewer-Dobson circulation, and ran the simulation to radiative-convective equilibrium. The resulting cumulus mass flux profile is shown by the asterisks in figure 4. In the simulation the matching altitude is at 14.3km, well below the large-scale temperature minimum at 16km, and the final stage of dehydration occurs in slow mean ascent, as in scenario 2. The large-scale dehydration gives rise to a layer of ultra-thin cirrus at the large-scale temperature minimum, similar to those observed (e.g. Winker and Trepte 1998), but in this case extending across the entire model domain. The water vapour mixing ratio in the simulation is not reduced to its saturation value at the large-scale temperature minimum, but remains about 5% supersaturated.

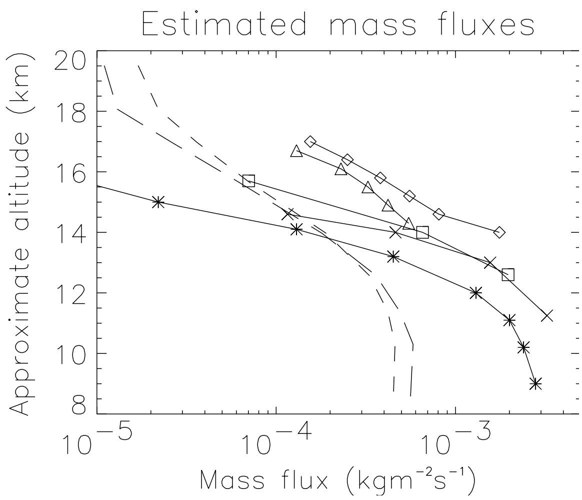

Figure 4. Dashed curves show tropical average residual mean ascent rates from ERA-15 data for January (short dash) and July (long dash). Solid curves show estimates of cumulus mass flux: diamonds and triangles based on O_3 and CO budgets (Dessler 2002); squares based on cloud imagery (Gettelman et al. 2002); crosses based on a radiative calculation (Folkins 2002 - this is actually cumulus flux minus mean ascent); asterisks based on a cloud-resolving simulation (Küpper et al. 2003).

Some caveats remain in interpreting these results for the real atmosphere. In particular, the tropical atmosphere is not locally in radiative-convective equilibrium, and in regions where large values of CAPE can build up cumulus clouds can overshoot more strongly, so that the dehydration mechanism might more closely resemble scenario 1 or 3. Also, variability in the large-scale temperature minimum due to tropical waves and zonal inhomogeneities are expected to modify the picture and lead to (more realistic) decreased stratospheric water vapour mixing ratios and patchiness in the ultra-thin cirrus.

Nevertheless, if scenario 2 is indeed the dominant final dehydration scenario then this has implications for what numerical models must get right in order to simulate stratospheric water vapour accurately. They need to simulate well the mean ascent rate and the large-scale temperature minimum (these two are, of course, related), and probably the variability in the large-scale temperature minumum. They need an accurate representation of vertical advection (see section 3 above). And they need a microphysical scheme that allows for the possibility of supersaturation with respect to ice. However, it is less important to capture accurately the details of transport and dehydration by cumulus clouds, unless the schemes are so grossly in error that the model simulates entirely the wrong dehydration scenario.

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