

Numerics of the Physics and the Physics of Numerics

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1. Introduction
2. Time integration
3. Processes in the ECMWF model
4. Conclusions



Introduction

General considerations:

- ◆ Parametrization packages have some level of modularity
- ◆ Explicit time integration is the preferred option; implicit schemes are used if necessary for stability
- ◆ Time steps can be large (in the IFS, 15 minutes for T511 and 1 hour for the seasonal forecasts at T95)
- ◆ Vertical resolution is often not sufficient to resolve relevant processes (e.g. sharp inversions, layered clouds)
- ◆ Scheme has to be compatible with dynamics; IFS uses 2 time level time integration
- ◆ Accuracy of the numerics of parametrization is often ignored and parametrizations are sometimes optimized for a given vertical resolution and time step
- ◆ A high level of modularity (i.e. different processes are handled independently) is desirable from code maintenance point of view, but not always desirable from numerical point of view

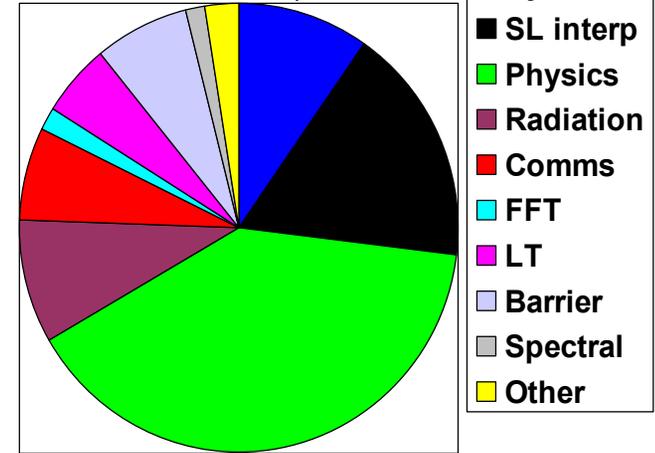


Time stepping

Requirements for time stepping:

1. **Stability (requires implicit solution for some processes)**
2. **Balance (correct steady state for long time steps)**
3. **Modularity of code**
4. **Accuracy**

T511- L60, HPCD, CY28R2,
Radiation 3-hourly at T255



Papers on time stepping of equations with multiple time scales (stiff equations):

Beljaars(1991): Numerical schemes for parametrization (ECMWF seminar)

Browning (1994): Splitting methods for problems with different time scales

Caya et al. (1998): Splitting methods

McDonald (1998): Numerical methods for atmospheric models (ECMWF seminar)

Wedi(1999): Physics dynamics coupling

Sportisse (2000): Operator splitting for stiff problems

Williamson (2002): Sequential-Split versus Parallel-split in the NCAR model

Cullen and Salmand (2003): Predictor-corrector for parametrization

Ropp et al (2003): Time integration of reaction-diffusion equations

Dubal et al. (2004): Parallel versus sequential splitting

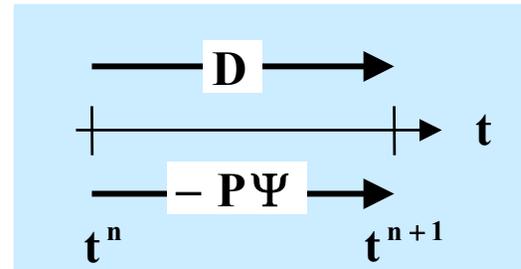


Time stepping: Process split / Parallel split

$$\frac{d\Psi}{dt} = \mathbf{D}(\Psi) - \mathbf{P}(\Psi)\Psi$$

Steady state solution with linear physics (i.e. $\mathbf{P}=\text{constant}$): $\Psi = \frac{\mathbf{D}}{\mathbf{P}}$

Different terms are computed independently:



Dynamics (explicit in this example):

$$\frac{\Psi_D^{n+1} - \Psi^n}{\Delta t} = \mathbf{D}(\Psi^n)$$

Physics (implicit):

$$\frac{\Psi_P^{n+1} - \Psi^n}{\Delta t} = -\mathbf{P}(\Psi^n)\Psi_P^{n+1} \Rightarrow \frac{\Psi_P^{n+1} - \Psi^n}{\Delta t} = \frac{-\mathbf{P}(\Psi^n)\Psi^n}{1 + \Delta t \mathbf{P}(\Psi^n)}$$

Total tendency:

$$\frac{\Psi^{n+1} - \Psi^n}{\Delta t} = \frac{\Psi_D^{n+1} - \Psi^n}{\Delta t} + \frac{\Psi_P^{n+1} - \Psi^n}{\Delta t} = \mathbf{D}(\Psi^n) - \frac{\mathbf{P}(\Psi^n)\Psi^n}{1 + \Delta t \mathbf{P}(\Psi^n)}$$

Steady state: $\Psi^n = \frac{\mathbf{D}(\Psi^n) \{1 + \Delta t \mathbf{P}(\Psi^n)\}}{\mathbf{P}(\Psi^n)}$

is time step dependent unless $\Delta t \mathbf{P}(\Psi^n) \ll 1$

Advantage of process split: Processes can be handled independently!



Time stepping: Time split / Sequential split / Fractional step

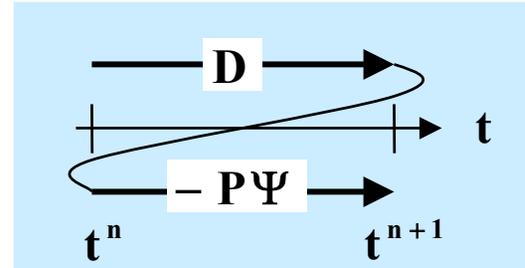
$$\frac{d\Psi}{dt} = \mathbf{D}(\Psi) - \mathbf{P}(\Psi)\Psi$$

Steady state solution with linear physics (i.e. $\mathbf{P}=\text{constant}$): $\Psi = \frac{\mathbf{D}}{\mathbf{P}}$

Processes are used incrementally:

Dynamics (explicit in this example):

$$\frac{\Psi^* - \Psi^n}{\Delta t} = \mathbf{D}(\Psi^n)$$



Physics (implicit) starts from dynamics:

$$\frac{\Psi^{n+1} - \Psi^*}{\Delta t} = -\mathbf{P}(\Psi^n)\Psi^{n+1} \quad \text{or} \quad \frac{\Psi^{n+1} - \Psi^n}{\Delta t} = \mathbf{D}(\Psi^n) - \mathbf{P}(\Psi^n)\Psi^{n+1}$$

Total tendency:

$$\frac{\Psi^{n+1} - \Psi^n}{\Delta t} = \frac{\mathbf{D}(\Psi^n) - \mathbf{P}(\Psi^n)\Psi^n}{1 + \Delta t \mathbf{P}(\Psi^n)}$$

Steady state: $\Psi^{n+1} = \Psi^n = \frac{\mathbf{D}(\Psi^n)}{\mathbf{P}(\Psi^n)}$ is correct and independent of time step

Note: - Nonlinear physics term is at time level \mathbf{n} and not at $\mathbf{*}$
 - Implicit process has to be last



Example with sequential split: condensation

$$\frac{dT}{dt} = D + \frac{L}{C_p} c = D + \frac{L}{C_p} \frac{q - q_{\text{sat}}(T)}{\tau}$$

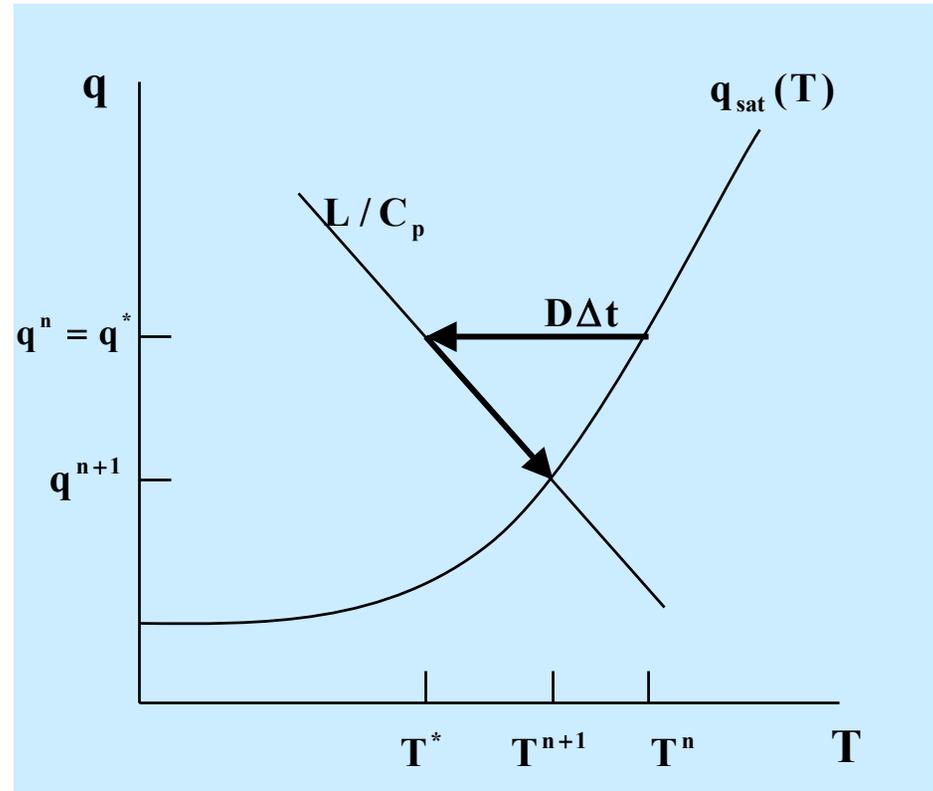
$$\frac{dq}{dt} = -c = -\frac{q - q_{\text{sat}}(T)}{\tau}$$

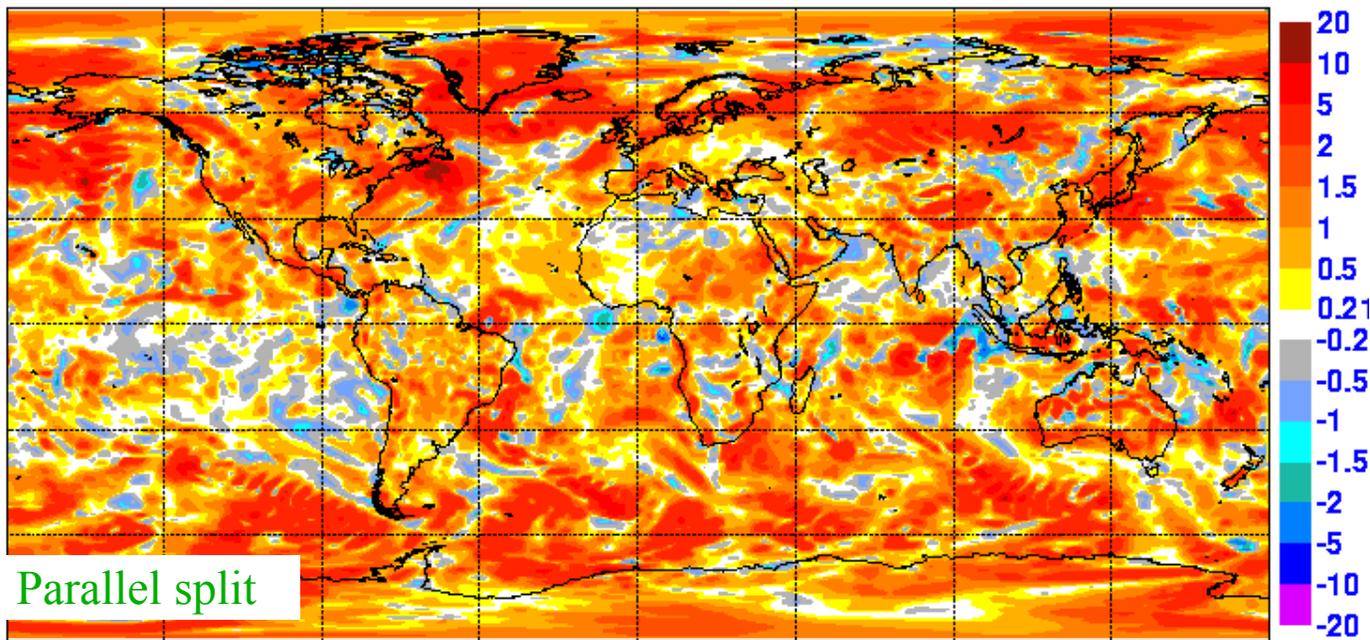
- Assume saturated air
- D is negative e.g. large scale lifting
- q-tendency only from condensation
- Condensation time constant is very small

Traditional procedure:

1. Compute T^* (after dynamics)
2. Assume $\tau \ll \Delta t$
3. Set $q^{n+1} = q_{\text{sat}}(T^{n+1})$
4. Use $T^{n+1} - T^* = -(q^{n+1} - q^*)L / C_p$
5. Iterate towards solution

Note: Iterative procedure towards saturation has to be last process; without applying D, condensation will not occur (or only in the next time step)

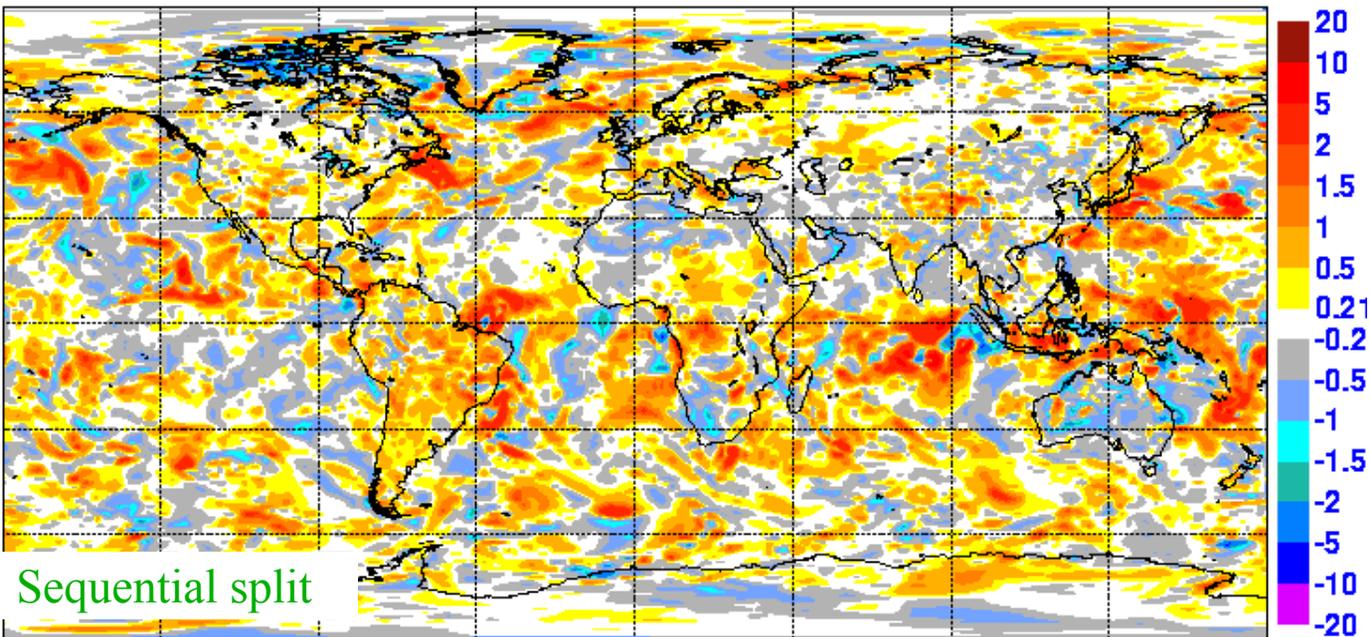




Example with splitting of dynamics and vertical diffusion.

Errors in 10m wind speed (with respect to 5 min time step).

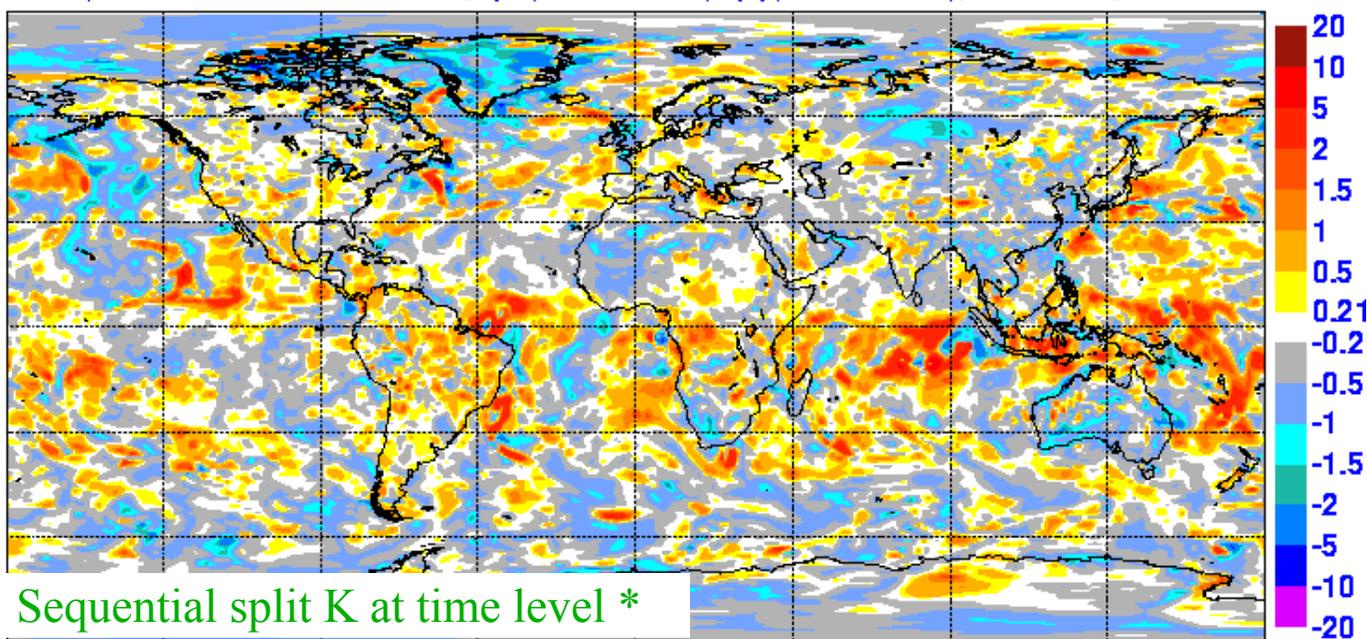
Time step: 60 min
Date: 20020115
Resolution: T159
Forecast: 24 hours



Sequential split guarantees balance between Coriolis term, pressure gradient and turbulent stress divergence.



Wind speed diff 24-fcts from 20020115; ej4x(m60R1btsV1F)-ej4y(m05R1btsV1F); Mean=-0.11; RMS=0.83

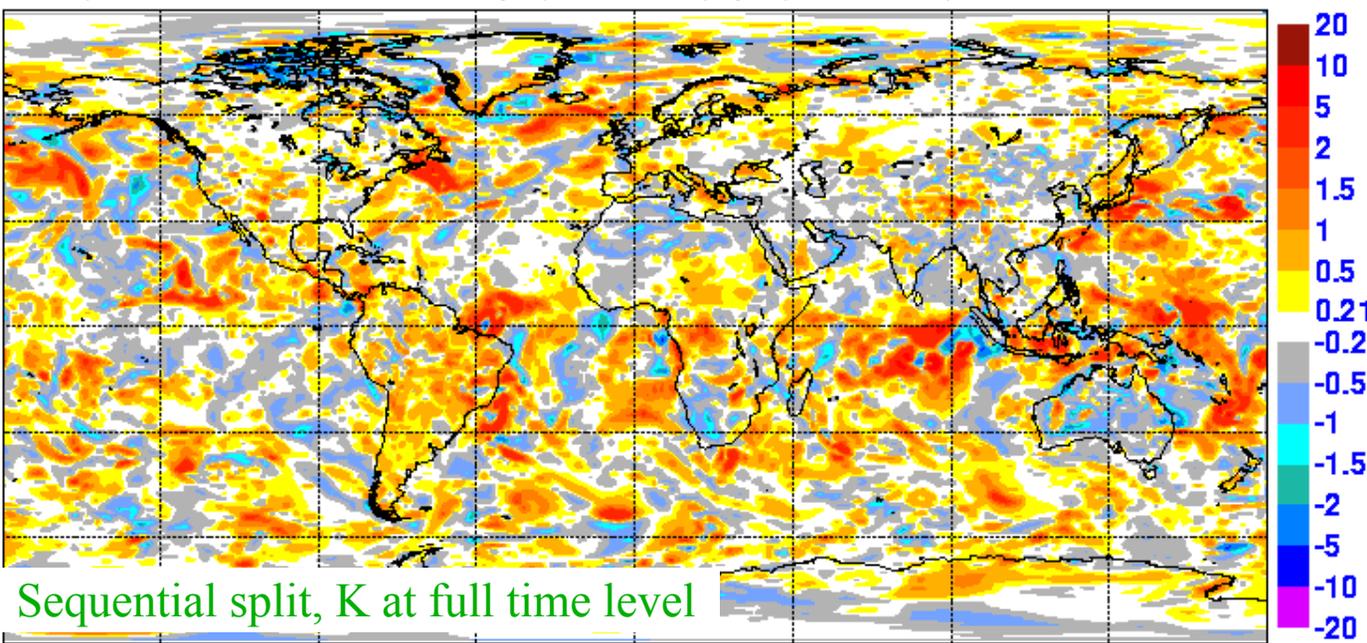


Example with splitting of dynamics and vertical diffusion.

Errors in 10m wind speed (with respect to 5 min time step).

Time step: 60 min
Date: 20020114
Resolution: T159
Forecast: 24 hours

Wind speed diff 24-fcts from 20020115; ej4n(m60R1tsV1F)-ej4m(m05R1tsV1F); Mean=0.11; RMS=0.82



Evaluation of diffusion coefficient at “in between time level” lowers wind speed by 0.2 m/s



Parallel split versus sequential split: summary

- ◆ Some form of splitting is necessary with current parametrizations; “fully unified physics packages” do not exist.
- ◆ Parallel split allows for maximum code modularity but steady state solutions are time step dependent if time step is not small compared to time scale of process.
- ◆ Sequential split is preferred option
- ◆ Order of processes is important:
 1. First: slow explicit processes
 2. Last: fast implicit processes (in principle only one implicit process is allowed)

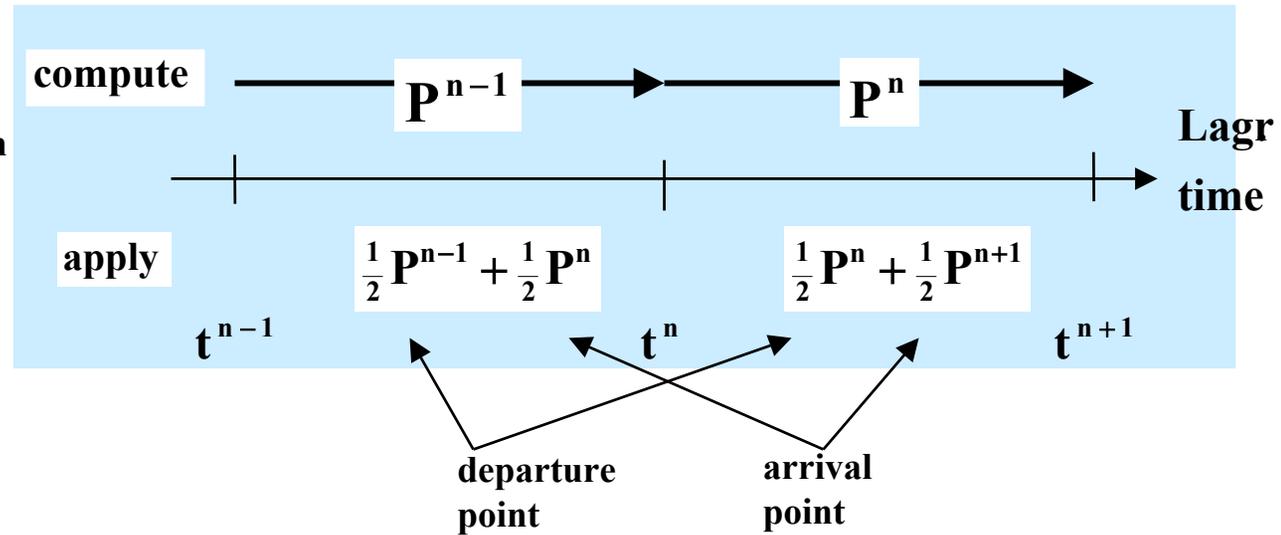


Towards 2nd order accuracy

$$\frac{d\Psi}{dt} = \mathbf{D} + \mathbf{P}$$

\mathbf{D} : dynamics without advection

Compute physics as an average between departure and arrival points of semi-Lagrangian trajectory



However, some processes are evaluated “implicitly” on the new time level, therefore:

$$\frac{\Psi^{n+1} - \Psi^n}{\Delta t} = \mathbf{D} + \frac{1}{2} \mathbf{P}_{\text{rad} + \text{cnv} + \text{cld}}^{n+1} + \frac{1}{2} \mathbf{P}_{\text{rad} + \text{cnv} + \text{cld}}^n + \mathbf{P}_{\text{vdf} + \text{sgoro}}^{n+1}$$

For time level $n+1$ use Ψ^* that is as close as possible to the new time level, e.g. for vdf+sgoro as 1st process:

$$\Psi^* = \Psi^n + \Delta t \left(\mathbf{D} + \mathbf{P}_{\text{rad} + \text{cnv} + \text{cld}}^n + \mathbf{P}_{\text{vdf} + \text{sgoro}}^{n+1} \right)$$

Wedi(1999): The numerical coupling of the physical parametrization to the “dynamical” equations in a forecast model, ECMWF Tech Memo, No 274.

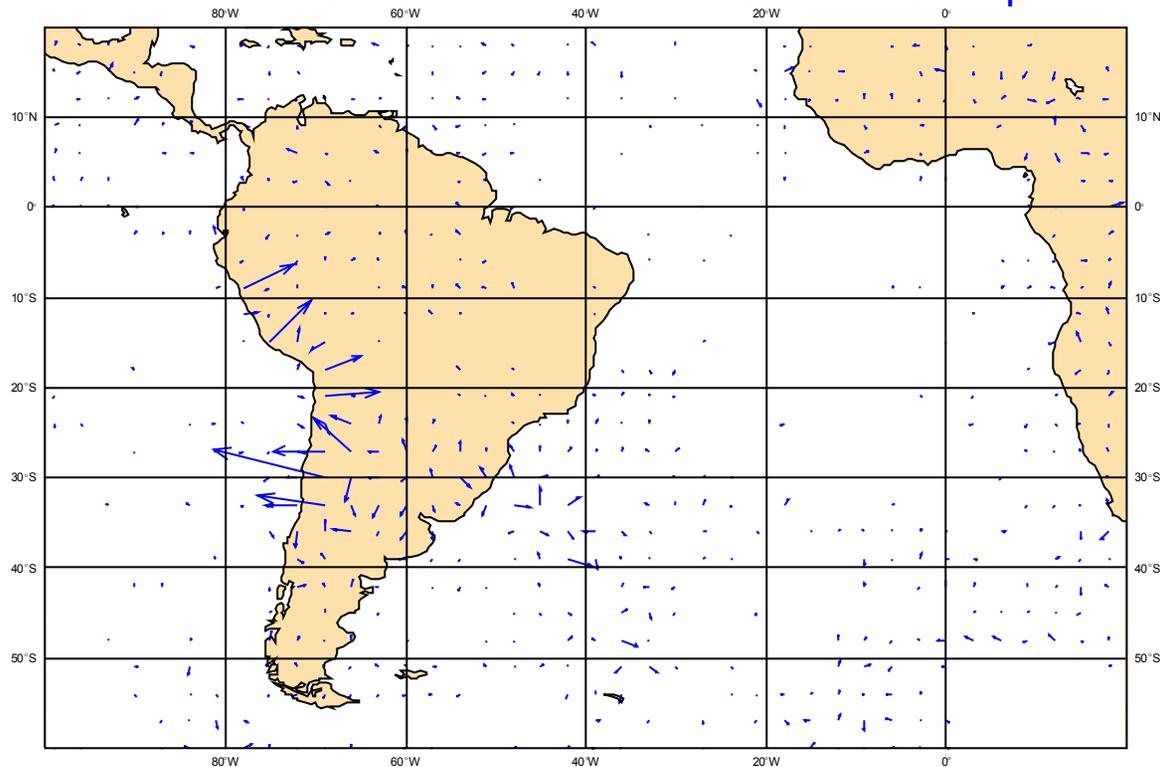


Use of
$$\frac{\Psi^{n+1} - \Psi^n}{\Delta t} = \mathbf{D} + \frac{1}{2} \mathbf{P}_{\text{rad} + \text{cnv} + \text{cld}}^{n+1} + \frac{1}{2} \mathbf{P}_{\text{rad} + \text{cnv} + \text{cld}}^n + \frac{1}{2} \mathbf{P}_{\text{vdf} + \text{sgoro}}^n + \frac{1}{2} \mathbf{P}_{\text{vdf} + \text{sgoro}}^{n+1}$$

Instead of
$$\frac{\Psi^{n+1} - \Psi^n}{\Delta t} = \mathbf{D} + \frac{1}{2} \mathbf{P}_{\text{rad} + \text{cnv} + \text{cld}}^{n+1} + \frac{1}{2} \mathbf{P}_{\text{rad} + \text{cnv} + \text{cld}}^n + \mathbf{P}_{\text{vdf} + \text{sgoro}}^{n+1}$$

Leads to big wind errors compared to short time step integrations (60 versus 5 minutes)

Level 31 U/V* 8/9/97 12h fc t+6 vt:8/9/1997 18h exp:ztcn



1



Towards 2nd order accuracy in the IFS

$$\frac{\Psi^{n+1} - \Psi^n}{\Delta t} = \mathbf{D} + \frac{1}{2} \mathbf{P}^{n+1} + \frac{1}{2} \mathbf{P}^n$$

In the IFS (CY28R1), “updated” profiles are supplied sequentially to the physics schemes:

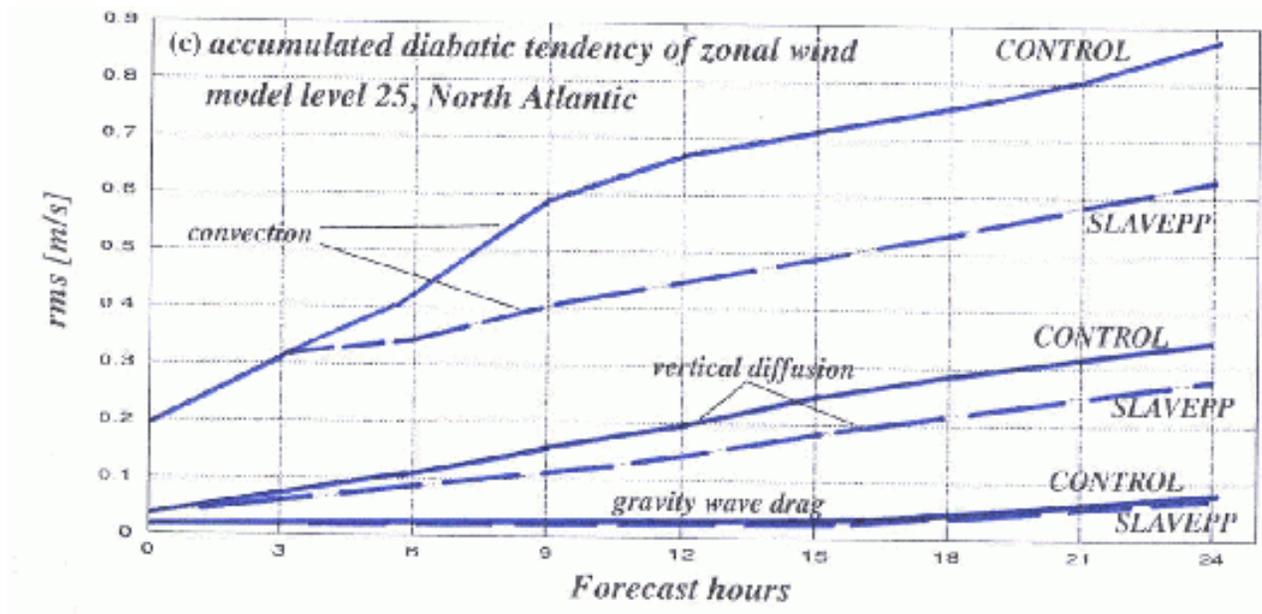
$$\begin{aligned} \mathbf{P}^{n+1} = & \mathbf{P}_{\text{rad}}^{n+1}(\Psi^n) \\ & + \mathbf{P}_{\text{vdf} + \text{sgoro}}^{n+1}(\Psi^n + \Delta t(\mathbf{D} + \mathbf{P}_{\text{rad}}^{n+1})) \\ & + \mathbf{P}_{\text{cnv} + \text{cld}}^{n+1}(\Psi^n + \Delta t(\mathbf{D} + \frac{1}{2} \mathbf{P}_{\text{rad}}^n + \frac{1}{2} \mathbf{P}_{\text{rad}}^{n+1} + \mathbf{P}_{\text{vdf} + \text{sgoro}}^{n+1} + \frac{1}{2} \mathbf{P}_{\text{cnv} + \text{cld}}^n)) \end{aligned}$$

Comments:

- ◆ RAD does not include guess from previous time level (technically difficult because radiation is computed on a low resolution grid)
- ◆ VDF+SGORO does not have guess from CNV+CLD (including these gives unrealistic boundary layers)
- ◆ CNV+CLD has only half of the tendency from the previous step (empirical choice to maintain sufficient convective activity)



2nd order physics reduces time truncation errors

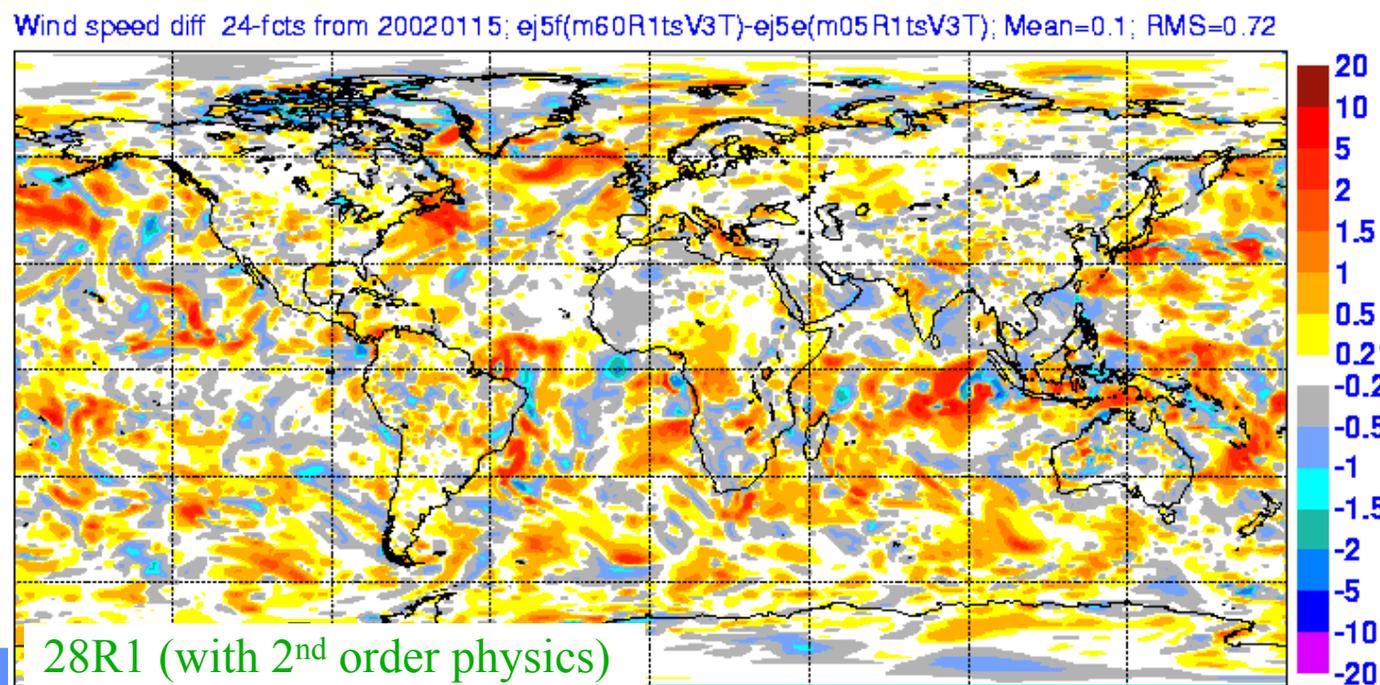
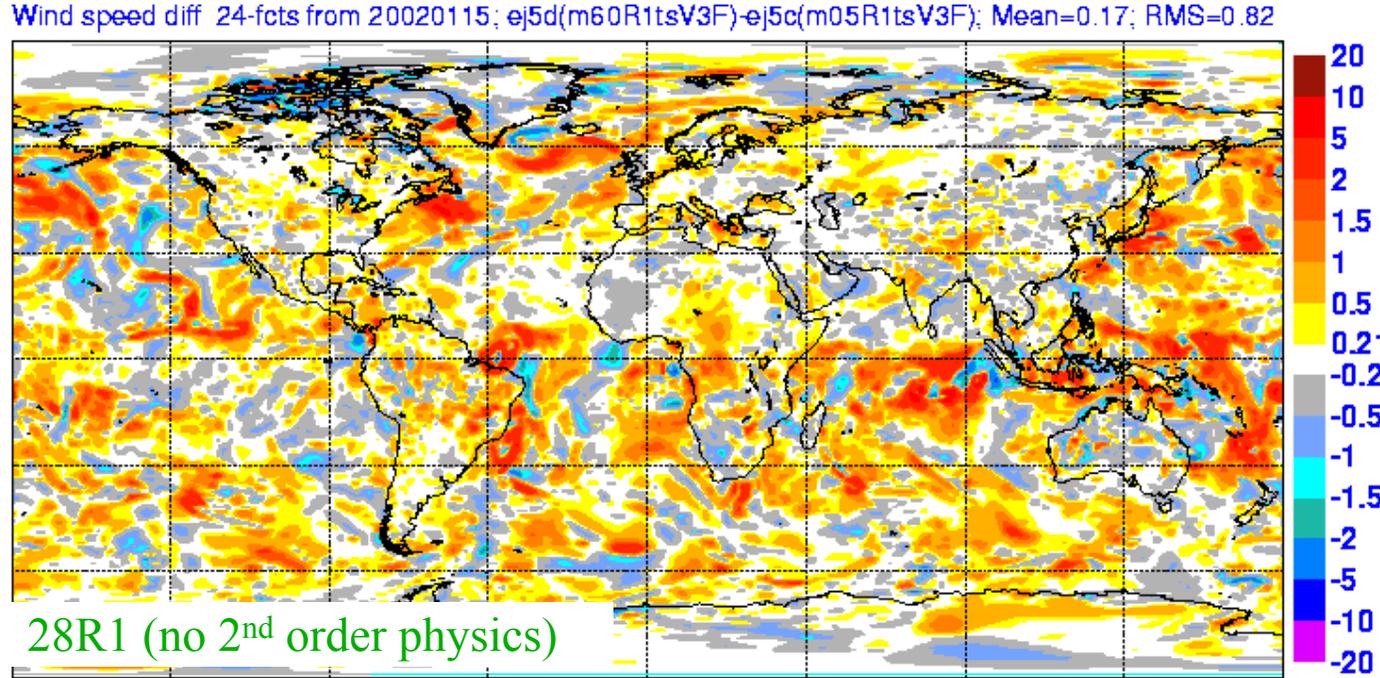


RMS difference of tendencies (cnv, vdf, sgoro) between integration with 60 minute time step and with 5 minute time step. CONTROL uses standard time integration; SLAVEPP uses the 28R1 2nd order physics.



2nd order physics
reduces time
truncation errors

Wind speed (10 m)
difference between
integration with 60
minute time step
and with 5 minute
time step.



Towards 2nd order accuracy in the IFS

$$\frac{\Psi^{n+1} - \Psi^n}{\Delta t} = \mathbf{D} + \frac{1}{2} \mathbf{P}^{n+1} + \frac{1}{2} \mathbf{P}^n$$

Modification in CY28R3 upgrade (the cloud scheme is also called before the convection to provide a guess of the cloud tendency:

$$\begin{aligned} \mathbf{P}^{n+1} = & \mathbf{P}_{\text{rad}}^{n+1} \left(\Psi^n \right) \\ & + \mathbf{P}_{\text{vdf + sgoro}}^{n+1} \left(\Psi^n + \Delta t (\mathbf{D} + \mathbf{P}_{\text{rad}}^{n+1}) \right) \\ & + \mathbf{P}_{\text{cnv}}^{n+1} \left(\Psi^n + \Delta t \left(\mathbf{D} + \frac{1}{2} \mathbf{P}_{\text{rad}}^n + \frac{1}{2} \mathbf{P}_{\text{rad}}^{n+1} + \mathbf{P}_{\text{vdf + sgoro}}^{n+1} + \frac{1}{2} \mathbf{P}_{\text{cldguess}}^{n+1} \right) \right) \\ & + \mathbf{P}_{\text{cld}}^{n+1} \left(\Psi^n + \Delta t \left(\mathbf{D} + \frac{1}{2} \mathbf{P}_{\text{rad}}^n + \frac{1}{2} \mathbf{P}_{\text{rad}}^{n+1} + \mathbf{P}_{\text{vdf + sgoro}}^{n+1} + \frac{1}{2} \mathbf{P}_{\text{cnv}}^{n+1} \right) \right) \end{aligned}$$

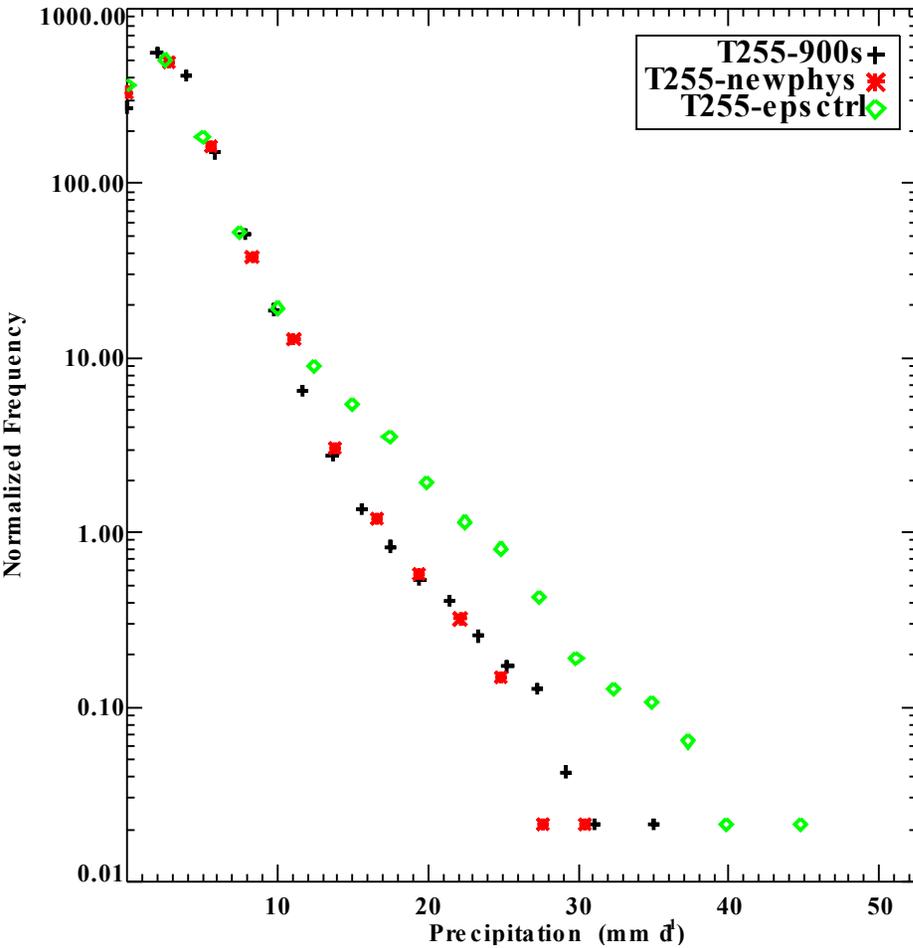
Comments:

◆ The extra call to the cloud scheme before the convection, provides more instability and therefore makes the convection scheme more active

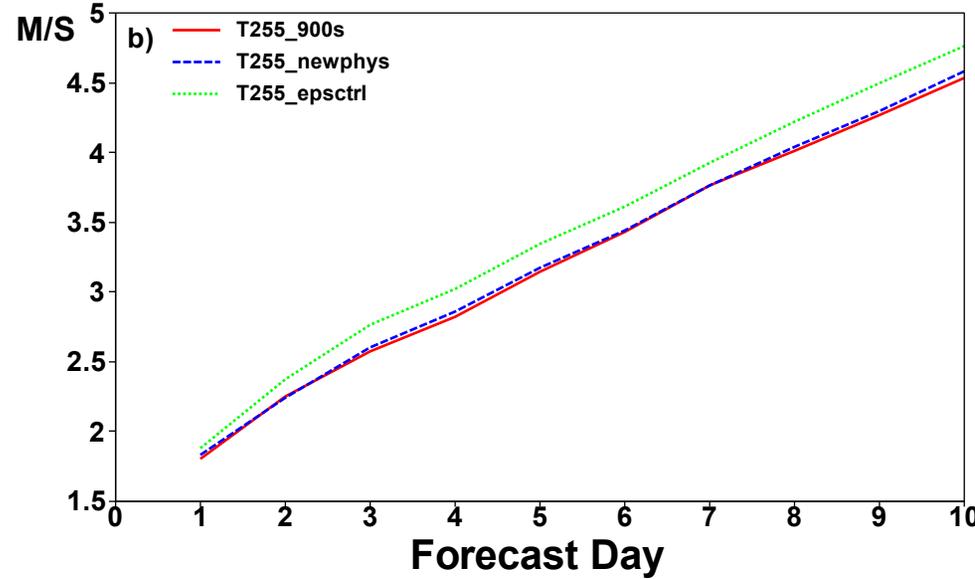


Improvement from CY28R3 time stepping compared to CY28R1

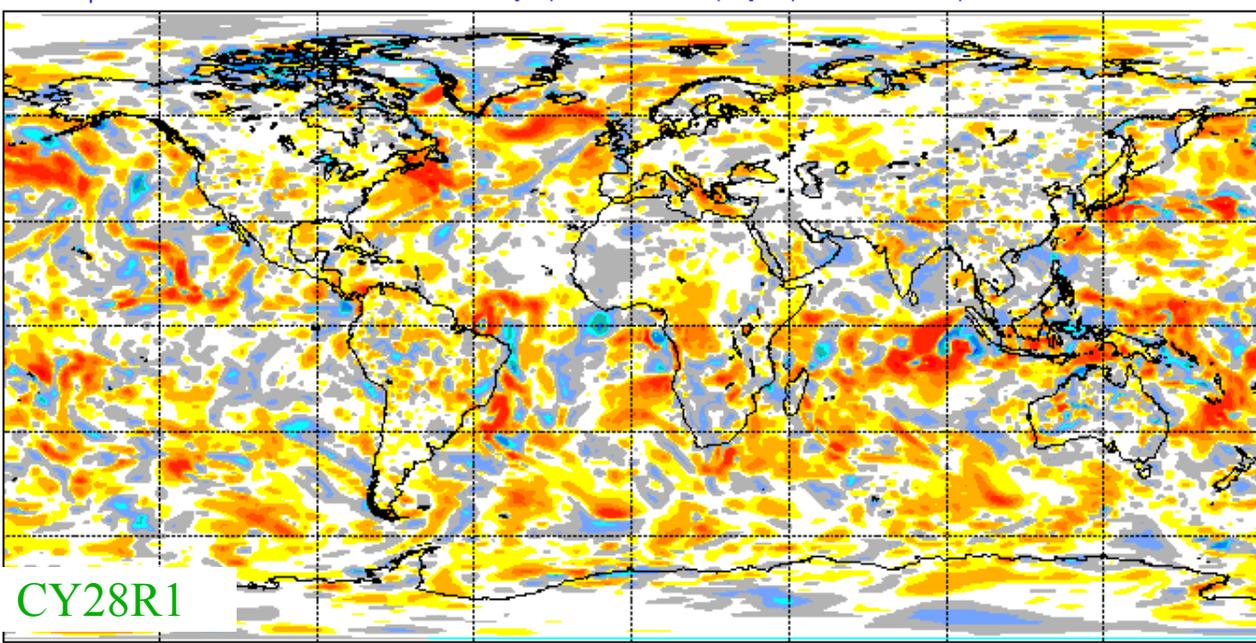
Histogram Precipitation rates



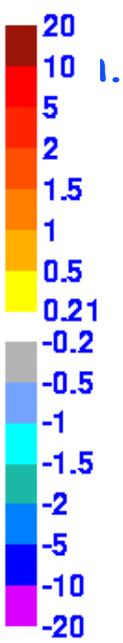
RMS ERROR 850 hPa VECTOR WIND TROPICS



Wind speed diff 24-fcts from 20020115; ej5f(m60R1tsV3T)-ej5e(m05R1tsV3T); Mean=0.1; RMS=0.72



CY28R1

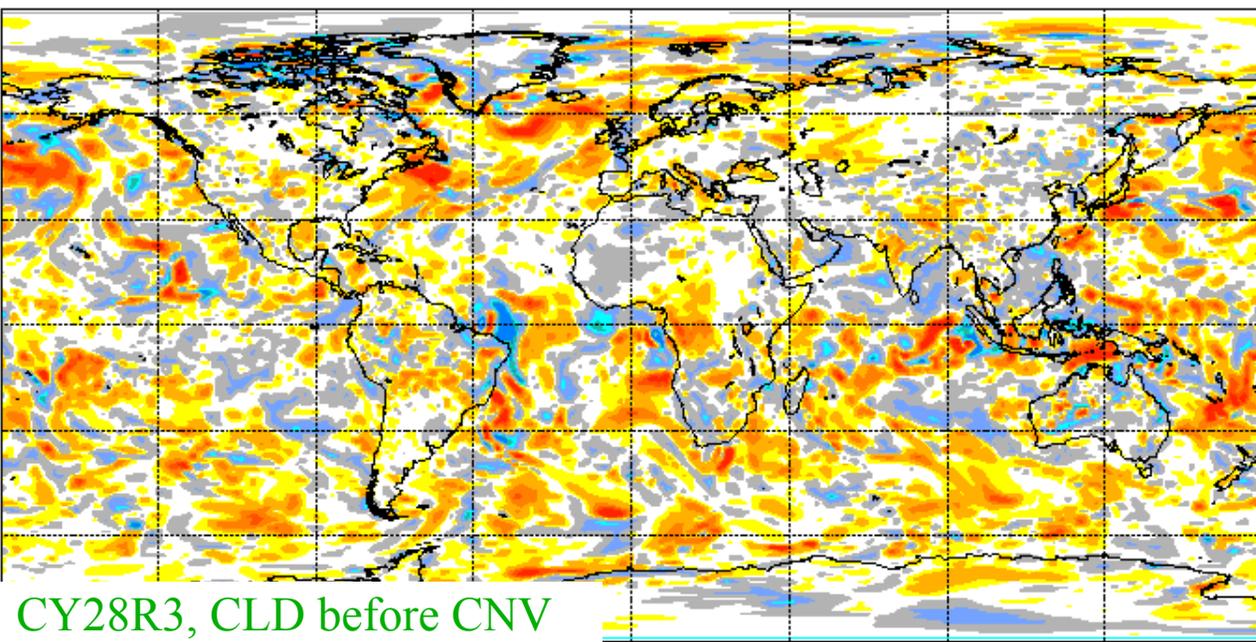


Effect of calling clouds before convection.

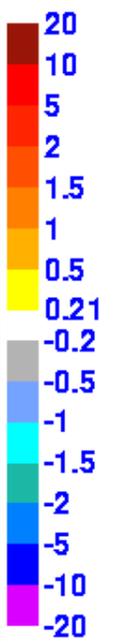
Errors in 10m wind speed (with respect to 5 min time step).

Time step: 60 min
Date: 20020114
Resolution: T159
Forecast: 24 hours

Wind speed diff 24-fcts from 20020115; ej5n(m60R3tsV3T)-ej5m(m05R3tsV3T); Mean=0.07; RMS=0.67



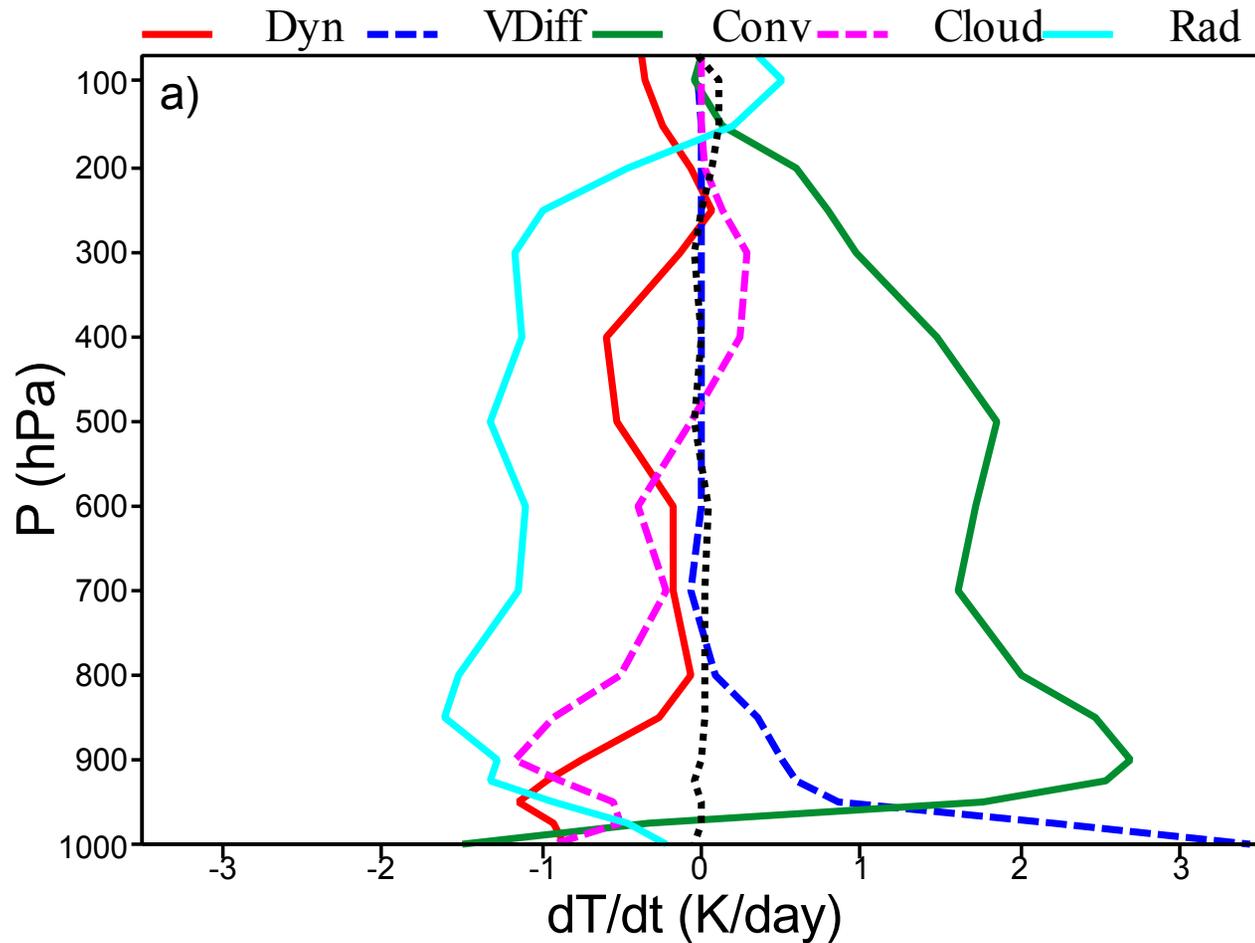
CY28R3, CLD before CNV



Calling clouds as a "first guess" before convection reduces wind errors in the tropics



Process tendencies averaged between 20S and 20N over a 5-day forecasts



Summary of time stepping procedures for long time steps

- ◆ Balance is a important consideration
- ◆ Ideal is to do explicit (slow) processes first and to have one implicit solver to take care of the remaining (fast) processes in a time (sequential) split way, i.e. the implicit solver takes the explicit term as part of the forcing
- ◆ Convection and clouds have the character of fast (implicit) processes.
- ◆ 2nd order time integration is still in its infancy
- ◆ Predictor corrector is an option but expensive (Cullen et al. 2002)



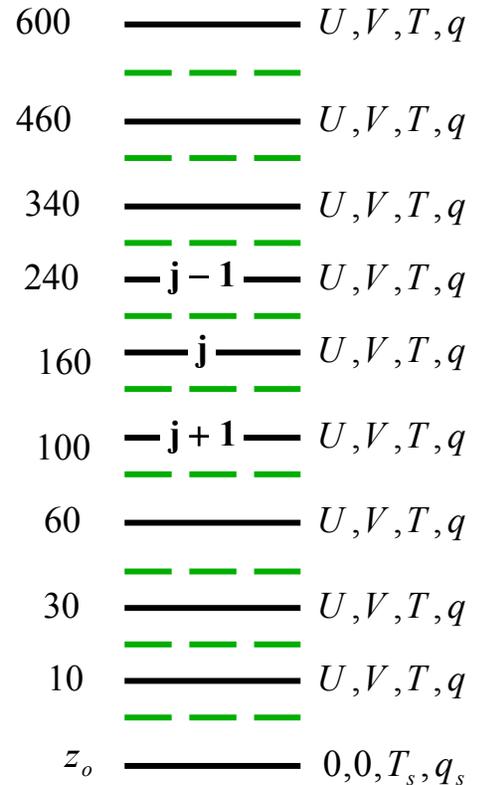
Processes in the IFS: Radiation

$$\frac{\partial T}{\partial t} = \frac{g}{C_p} \frac{dF}{dp}$$

$$(\Delta T_j)_{\text{rad}} = \Delta t \frac{g}{C_p} \frac{F_{j+1/2}^n - F_{j-1/2}^n}{P_{j+1/2} - P_{j-1/2}}$$

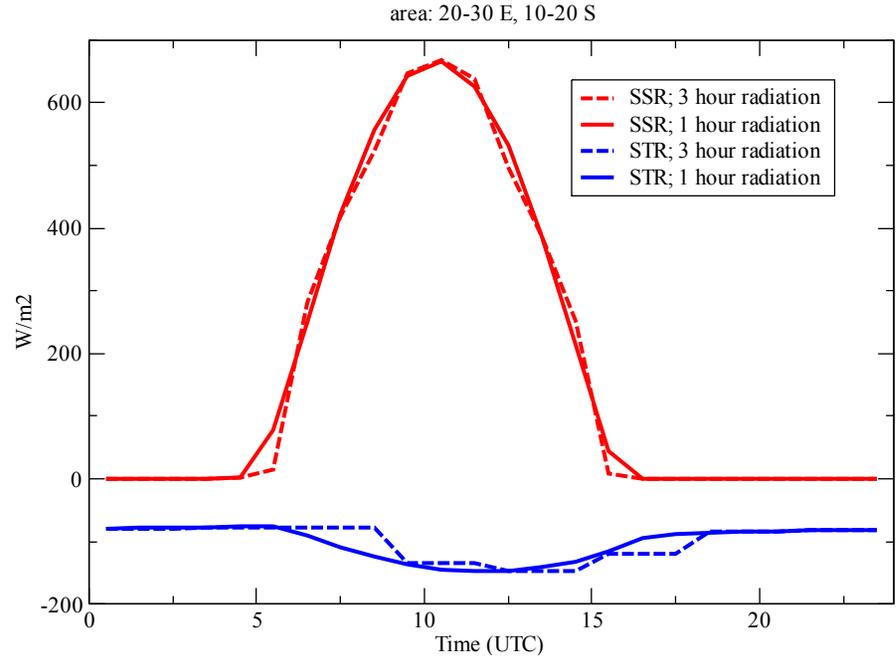
- ◆ Explicit numerics
- ◆ No update from dynamics (appropriate for explicit numerics)
- ◆ Low resolution grid for economy (T255 in T511):
This can lead to inconsistency between surface radiation and full resolution albedo field which can upset the surface scheme

60-level model



Processes in the IFS: Radiation

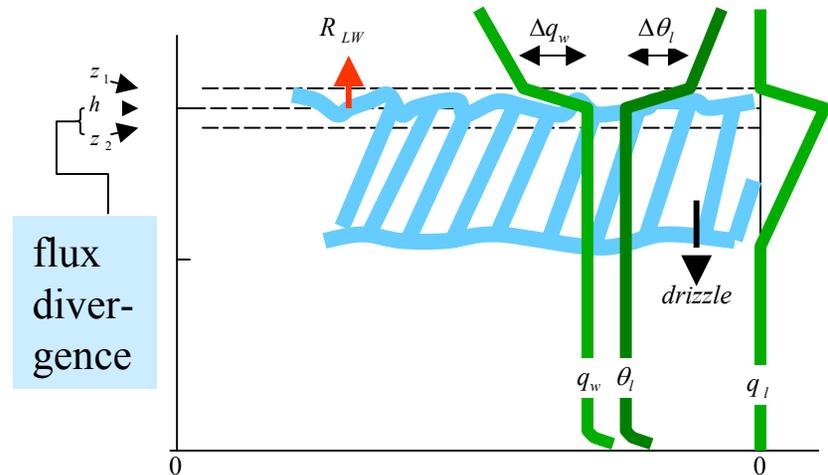
◆ Full radiation every 3 hours in 28R1 and hourly in 28R3



◆ Radiative tendency due to cloud top radiative cooling scales with layer depth:

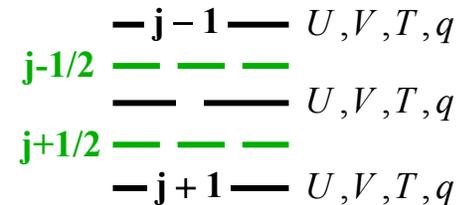
$$\left(\frac{\partial T}{\partial t}\right)_{\text{rad}} \approx \frac{g}{C_p} \frac{80}{P_{j+1/2} - P_{j-1/2}}$$

e.g. 35K/day for a 20hPa layer



Processes in the IFS: Vertical Diffusion

$$\frac{\partial \Psi}{\partial t} = \mathbf{g} \frac{d\mathbf{F}_\Psi}{d\mathbf{p}} \quad \mathbf{F}_\Psi = \mathbf{K}_\Psi \rho \frac{d\Psi}{dz}$$



$$(\Delta\Psi_j)_{\text{vdf}} + (\Delta\Psi_j)_{\text{dyn}} + (\Delta\Psi_j)_{\text{rad}} = \Psi_j^* - \Psi_j^n =$$

$$\frac{\Delta t \mathbf{g}}{\Delta \mathbf{p}_j} \left(\mathbf{K}_{j+1/2}^n \frac{\hat{\Psi}_{j+1} - \hat{\Psi}_j}{z_{j+1} - z_j} - \mathbf{K}_{j-1/2}^n \frac{\hat{\Psi}_j - \hat{\Psi}_{j-1}}{z_j - z_{j-1}} \right) + (\Delta\Psi_j)_{\text{dyn}} + (\Delta\Psi_j)_{\text{rad}}$$

$$\hat{\Psi} = \alpha \Psi^* + (1 - \alpha) \Psi^n \quad \alpha = 1.5$$

- ◆ Over-implicit numerics
- ◆ Balance with dynamics and radiation
- ◆ Specification of similarity profiles in the surface layer (exact finite differencing for a constant flux layer!)
- ◆ In the ECMWF model 3 VDF steps are made for every model time step
- ◆ Implicit coupling with surface tiles



Processes in the IFS: Vertical Diffusion

Issues:

- ◆ Non-linear instability

Comments:

- ◆ Predictor-corrector does not always give the correct result
- ◆ Different options exist, but a large implicitness factor is the more popular and robust option (Kalnay and Kanamitsu, 1988)
- ◆ More complicated methods are more expensive (Hammerstrand, 1997)
- ◆ Implicitness factor can be made flow dependent (Girard and Delage, 1990)
- ◆ Single point diagnostics is not sufficient

$$\frac{\partial T(t)}{\partial t} = -KT^P T(t) + D(t)$$

$T \equiv$ temperature difference between ground and air

$KT^P \equiv$ exchange coefficient, $K = 10, P = 3$

$D(t) \equiv 1 - \sin(2\pi n\Delta t / 24)$, diurnal cycle

$$\frac{T^{n+1} - T^n}{\Delta t} = -K(T^n)^P [\alpha T^{n+1} + (1 - \alpha)T^n] + D^n$$

$\alpha \geq P + 1$, unconditionally stable, over-implicit

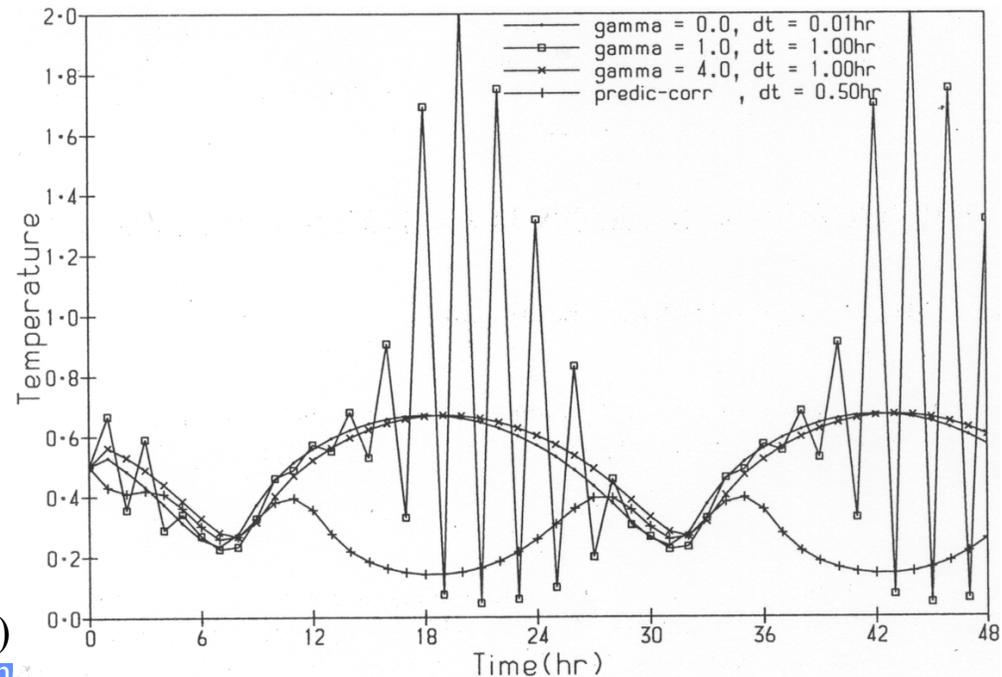


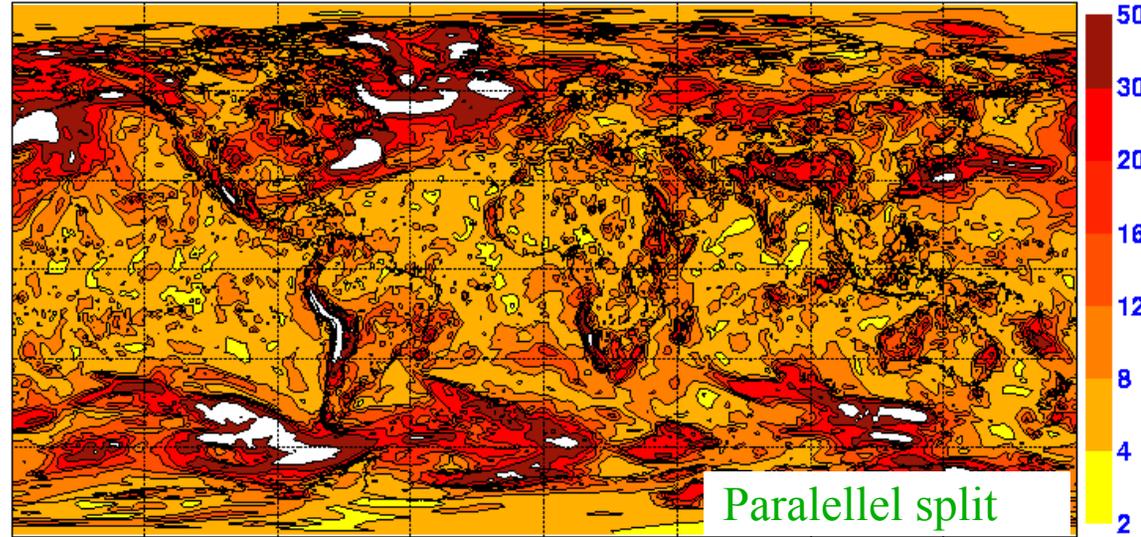
Figure: McDonald (1998)

Processes in the IFS: Vertical Diffusion

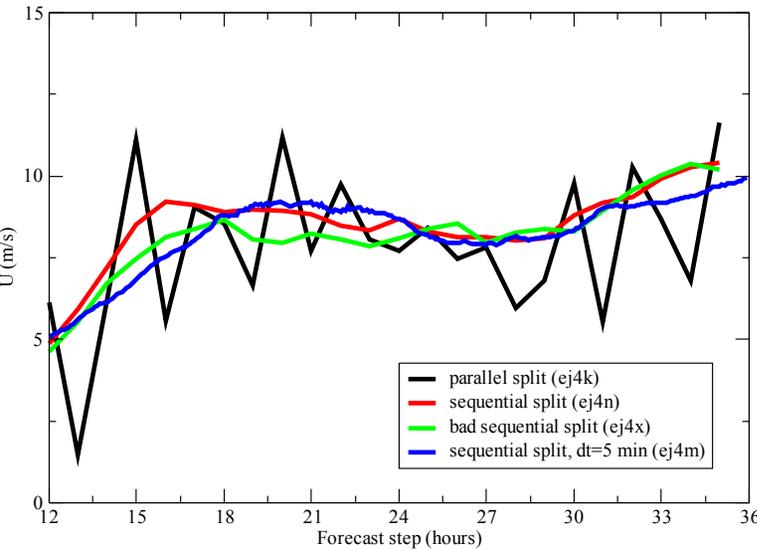
Time stepping of vertical diffusion affects noise

RMS(dU/dt) (m/s/day) at 10 m level

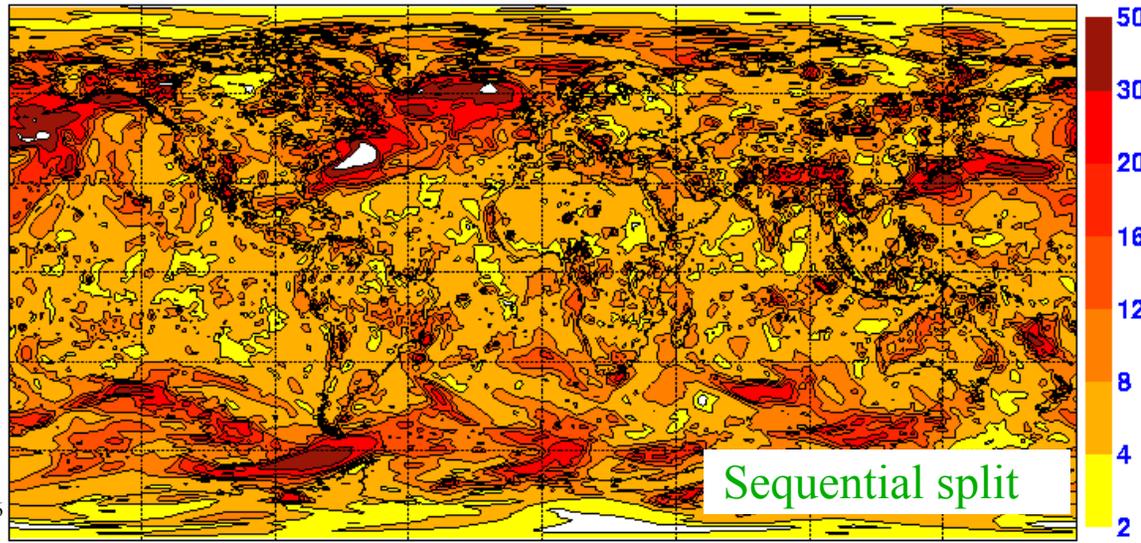
RMS(U-tend); 24-hr-av; L60; 20020115; ej4k(m60R1psV1F); Mean=12.69; RMS=17.53



(90 W, 60 S) T159 forecasts 2002011512, dt=60 min



RMS(U-tend); 24-hr-av; L60; 20020115; ej4n(m60R1tsV1F); Mean=8.9; RMS=10.92



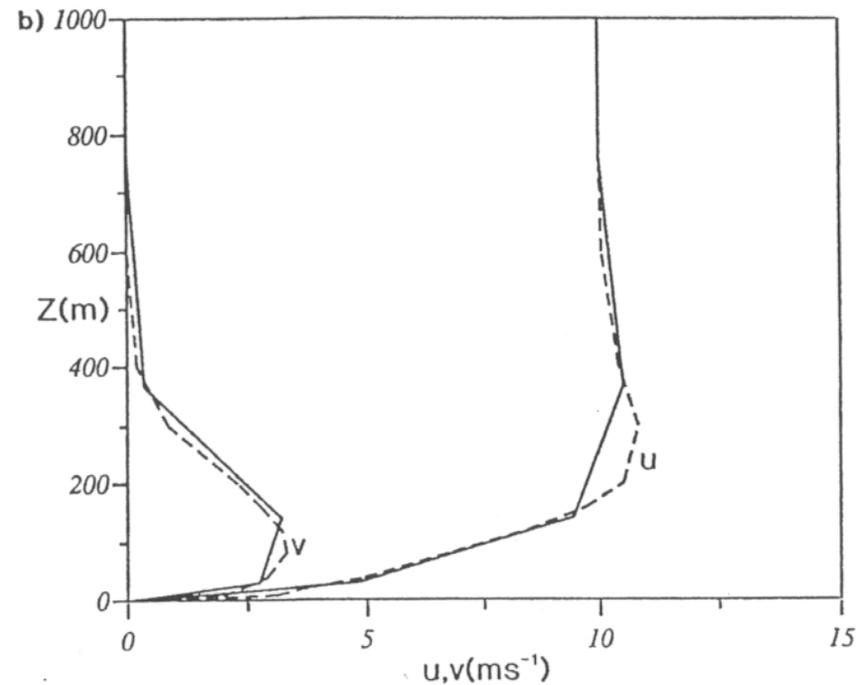
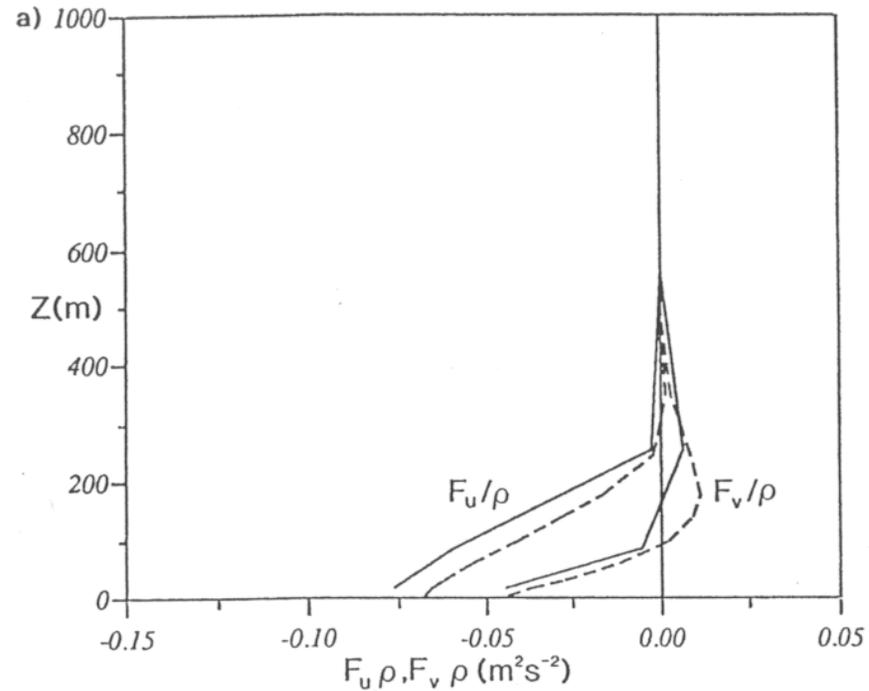
Processes in the IFS: Vertical Diffusion

Issue:

◆ Vertical resolution

Comments:

◆ In spite of the low number of levels in the stable boundary layer the solution is surprisingly insensitive to resolution



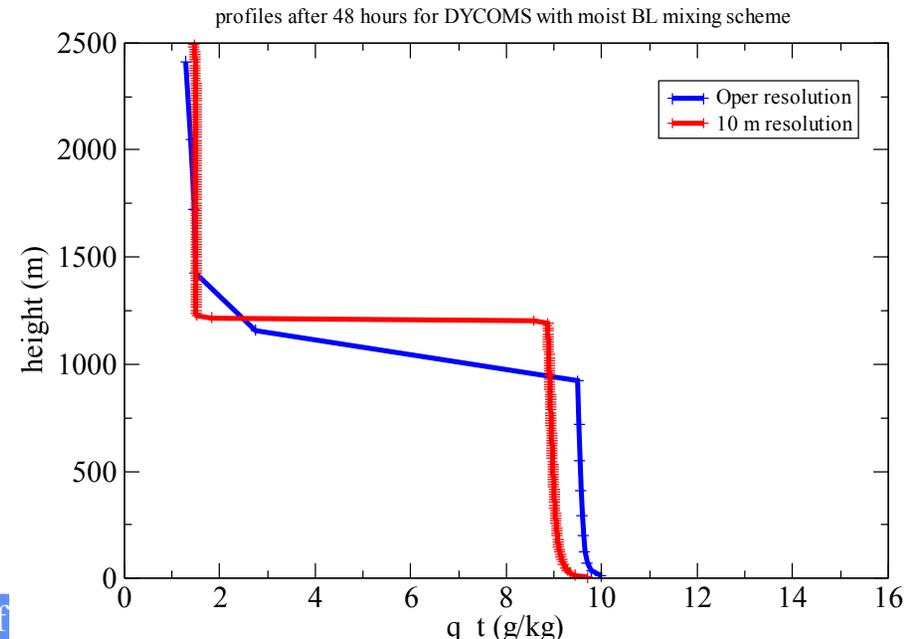
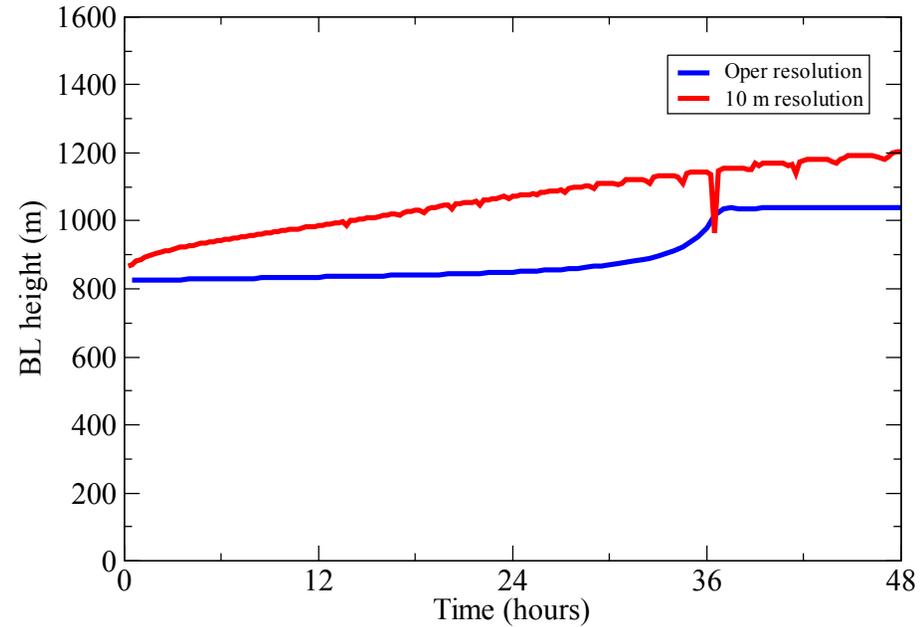
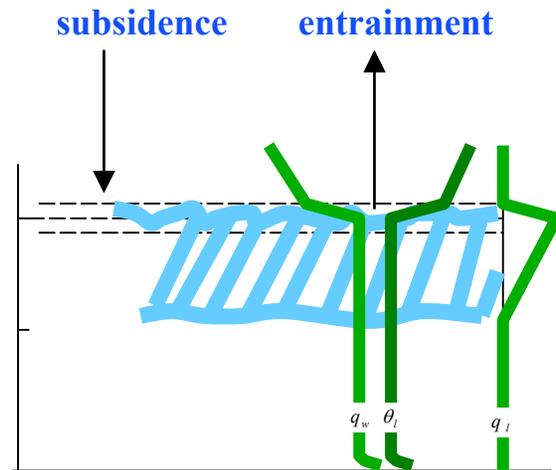
Processes in the IFS: Vertical Diffusion

Issues:

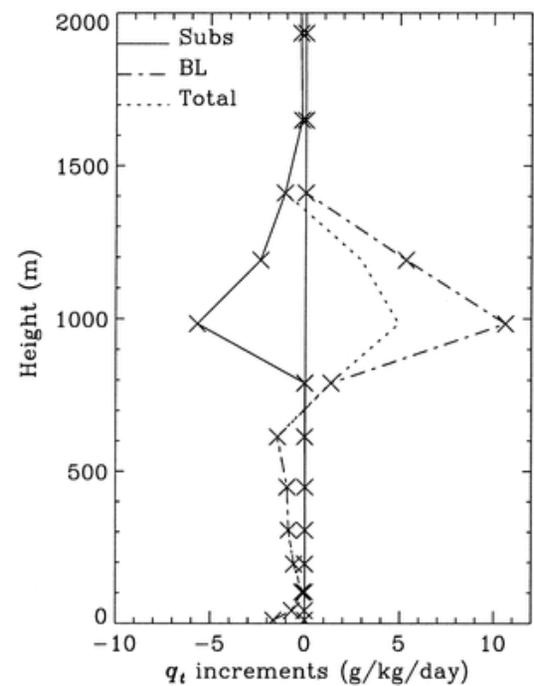
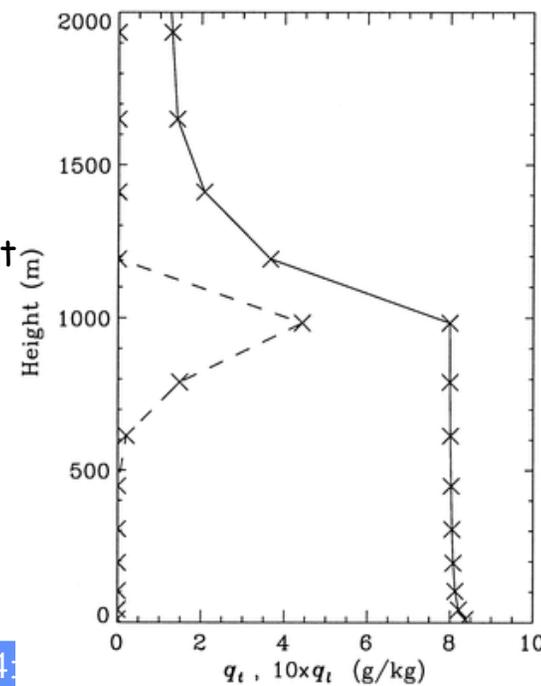
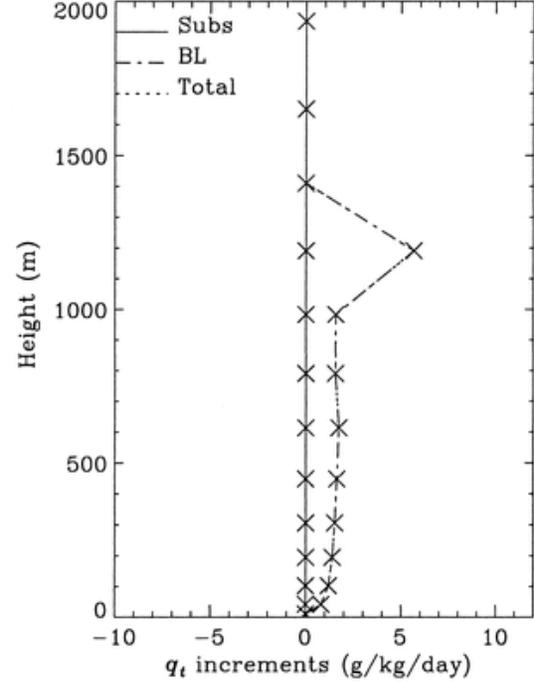
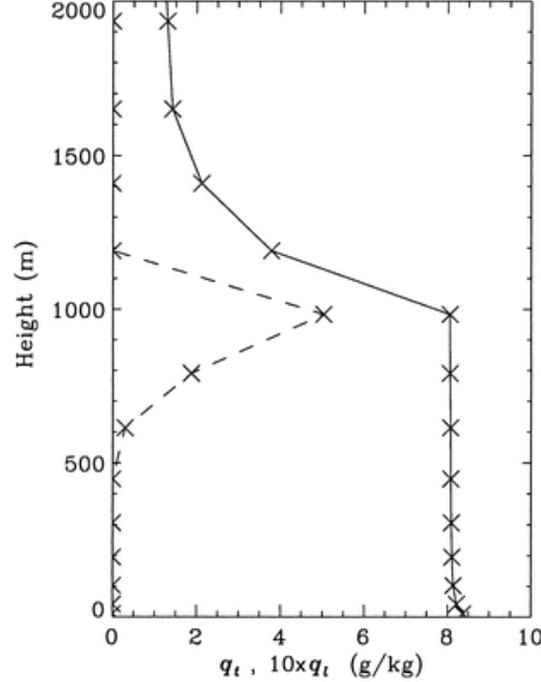
- ◆ Vertical resolution
- ◆ Handling of inversions

Comments:

- ◆ Mixing through inversions is often in balance with subsidence



Inversion numerics



Figures by A. Lock: The numerical representation of entrainment in parametrizations of boundary layer turbulent mixing, MWR, 2001, 129, 1148-1163.

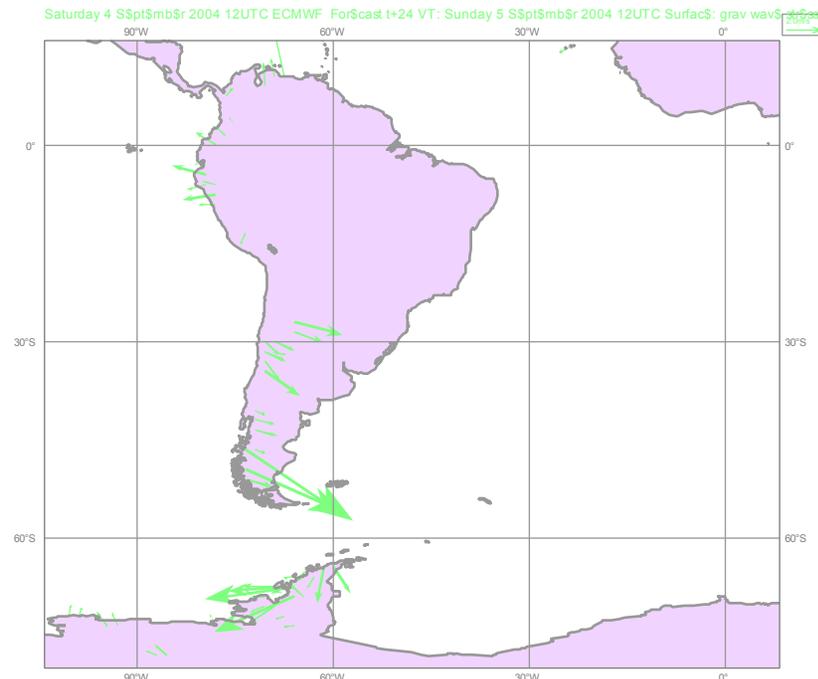
See also:

Grenier and Bretherton (2001): MWR, 129, 357-377.

Lenderink and Holtslag (2000): MWR, 128, 244-258.

Processes in the IFS: Subgrid orography

- ◆ Low level blocking + gravity wave drag
- ◆ Low level tendencies can be very large on isolated points
- ◆ Good balance would benefit from simultaneous solution of vertical diffusion and subgrid orography with the same tri-diagonal solver



Processes in the IFS: Convection

$$\frac{\partial \Psi}{\partial t} = g \frac{d}{dp} [M_u (\Psi_u - \Psi)] + S$$

$$\begin{aligned} (\Delta \Psi_j)_{\text{cnv}} = & \frac{\Delta t g}{\Delta p_j} \left[(M_u \Psi_u)_{j+1/2} - (M_u \Psi_u)_{j-1/2} \right. \\ & \left. - (M_u)_{j+1/2} \Psi_j + (M_u)_{j-1/2} \Psi_{j-1} \right] + S_j \end{aligned}$$

- ◆ Upwind differencing in vertical
- ◆ Mass flux limiter to prevent instability
- ◆ Shallow convection is closed by assuming balance of moist static energy between dynamics, vertical diffusion and convection in subcloud layer i.e. the convection scheme needs surface fluxes from vertical diffusion as input
- ◆ For deep convection cloud base mass flux is based on CAPE reduction over a specified relaxation time (1 hour for low resolution to 15 minutes at high resolution, which is close to the time step)
- ◆ Subcloud layer fluxes are specified as a linear profile with zero at the surface



Processes in the IFS: Convection

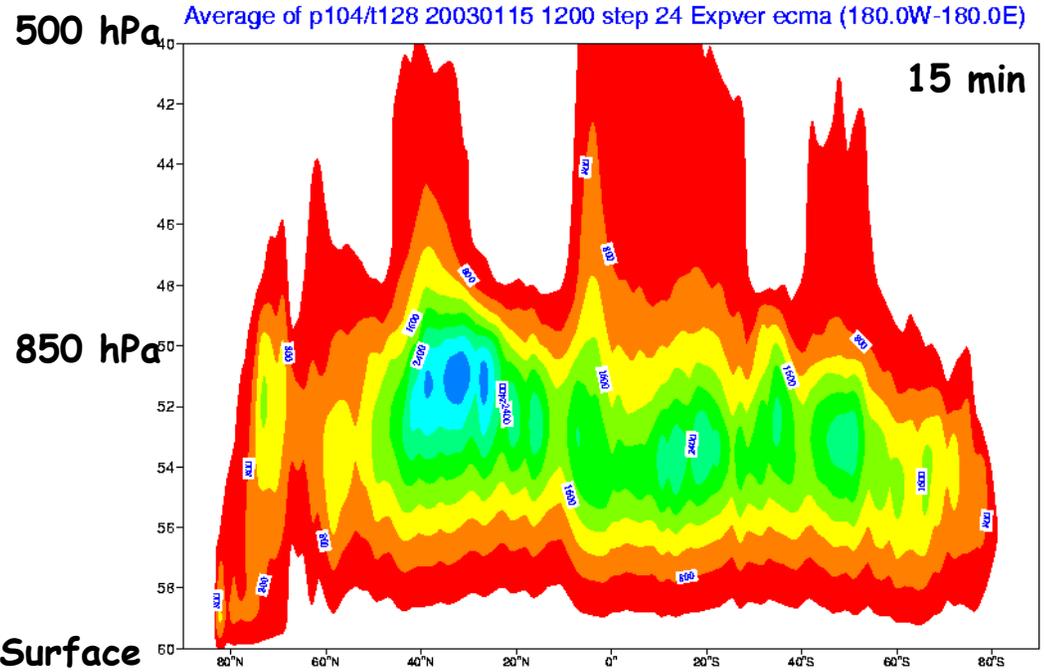
Issues:

- ◆ Mass flux limiter introduces time step dependency with high vertical resolution
- ◆ Implicit formulation is desirable, but specification of linear flux profile below cloud base turns out to be essential to balance fluxes from vertical diffusion scheme
- ◆ Input profile is crucial for convection triggering and for CAPE diagnosis
- ◆ Should convection be seen as a slow process that can be handled with explicit numerics or as a fast process that needs implicit numerics?
- ◆ Which are the critical processes that balance convection? (dynamics, radiation, vertical diffusion, clouds)

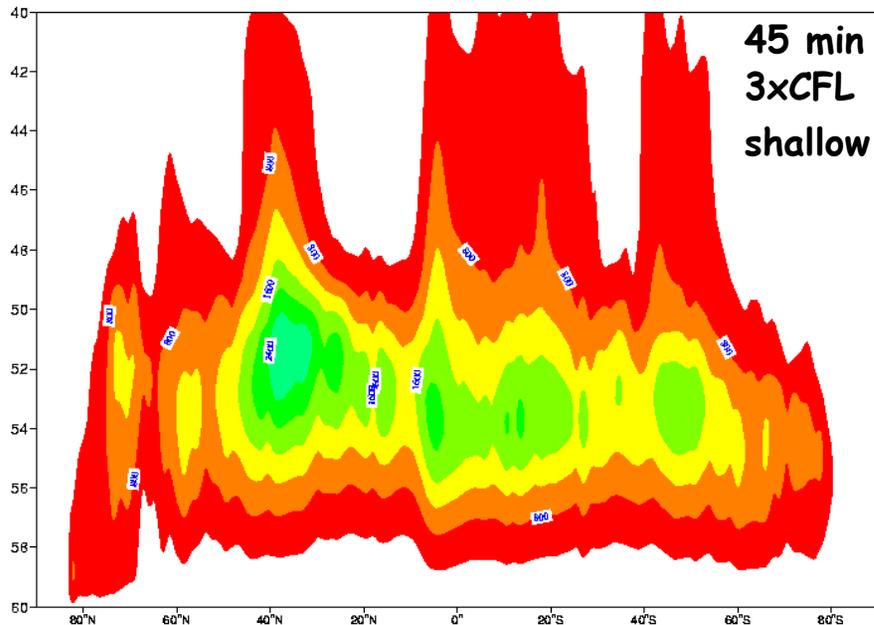


Mass flux limiter in convection

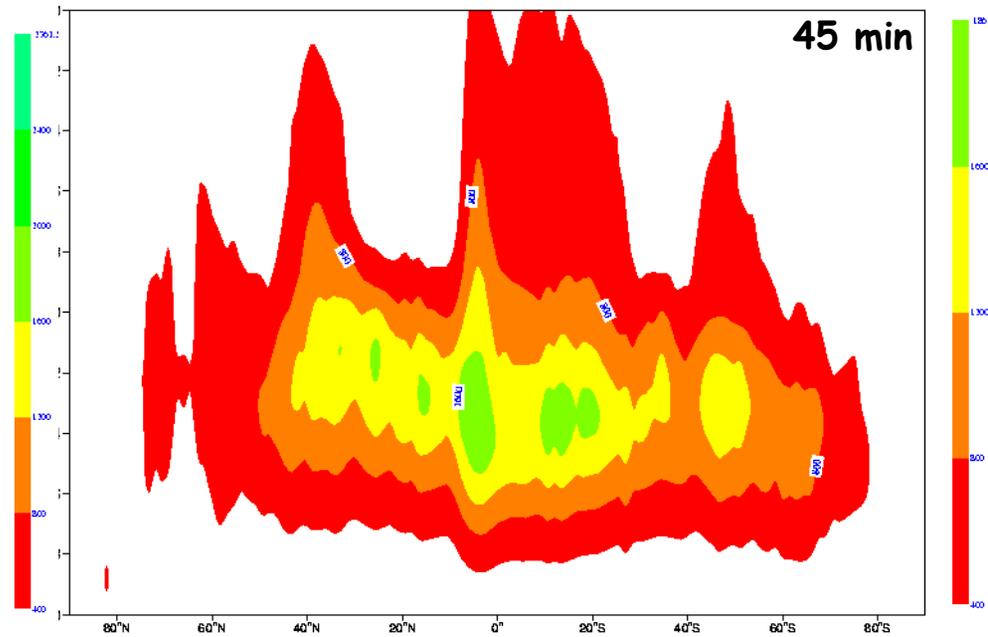
T255 24-hour zonally averaged updraught mass flux (cnt. int.: 400 kg/m²/day).



Average of p104/t128 20030115 1200 step 24 Expver ecqx (180.0W-180.0E)



Average of p104/t128 20030115 1200 step 24 Expver ecm6 (180.0W-180.0E)



Processes in the IFS: Clouds

$$\frac{\partial \ell}{\partial t} = S_{\text{cnv}} + S_{\text{vdf}} + S_{\text{strat}} - E_{\text{cld}} - G_{\text{prec}} - \frac{1}{\rho} \frac{\partial}{\partial z} \rho (\overline{w' \Gamma'})_{\text{entr}}$$

$$\frac{\partial \mathbf{a}}{\partial t} = \delta \mathbf{a}_{\text{cnv}} + \delta \mathbf{a}_{\text{vdf}} + \delta \mathbf{a}_{\text{strat}} - \delta \mathbf{a}_{\text{evap}}$$

Equations are written (level by level) as:

$$\frac{\partial \ell}{\partial t} = \mathbf{C} - \mathbf{D} \ell$$

$$\frac{\partial \mathbf{a}}{\partial t} = \mathbf{A} - \mathbf{B} \mathbf{a}$$

with **A,B,C,D** from processes, e.g. vertical motion, convective detrainment, precipitation, turbulence, cloud erosion. An exponential solution over single time step is used to integrate in time.



Processes in the IFS: Clouds

Example of convective detrainment and ice fallout

$$\frac{\partial l}{\partial t} = \mathbf{C} - \mathbf{D} l$$

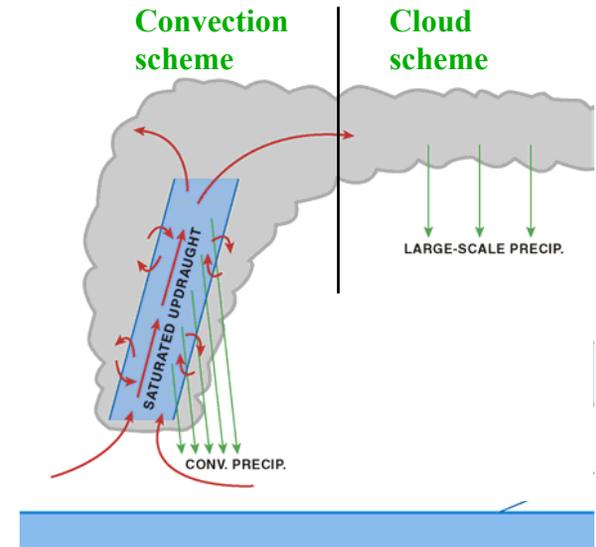
$$\frac{\partial l}{\partial t} = \frac{1}{\rho} \mathbf{D}_{\text{up}} (l_u - l) - \rho \mathbf{g} \mathbf{w}_{\text{ice}} \frac{\partial l}{\partial p}, \text{ with } \mathbf{w}_{\text{ice}} = c_1 l^{c_2}$$

$$\text{for level } j: \quad \mathbf{C} = \frac{1}{\rho_j} \mathbf{D}_{\text{up},j} (l_{u,j} - l_j) - \rho_{j-1} \mathbf{g} \mathbf{w}_{\text{ice},j-1} \frac{l_{j-1}}{\Delta p_{j-1}}$$

$$: \quad \mathbf{D} = -\rho_j \mathbf{g} \mathbf{w}_{\text{ice},j} \frac{1}{\Delta p_j}$$

Comments:

- ◆ Detrainment source term has to be part of the implicit time integration of ice fallout for proper balance (sequential splitting)
- ◆ Ice fallout needs to be computed at full time level



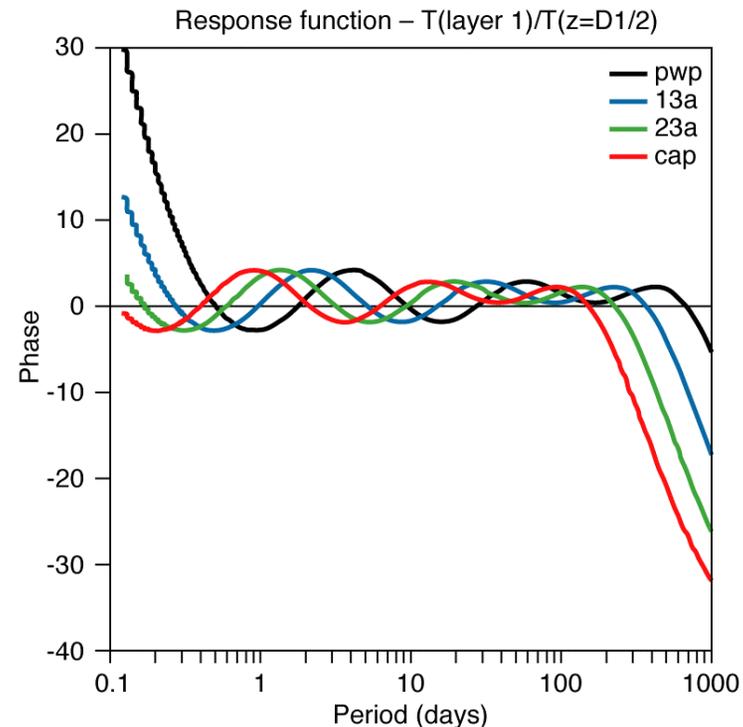
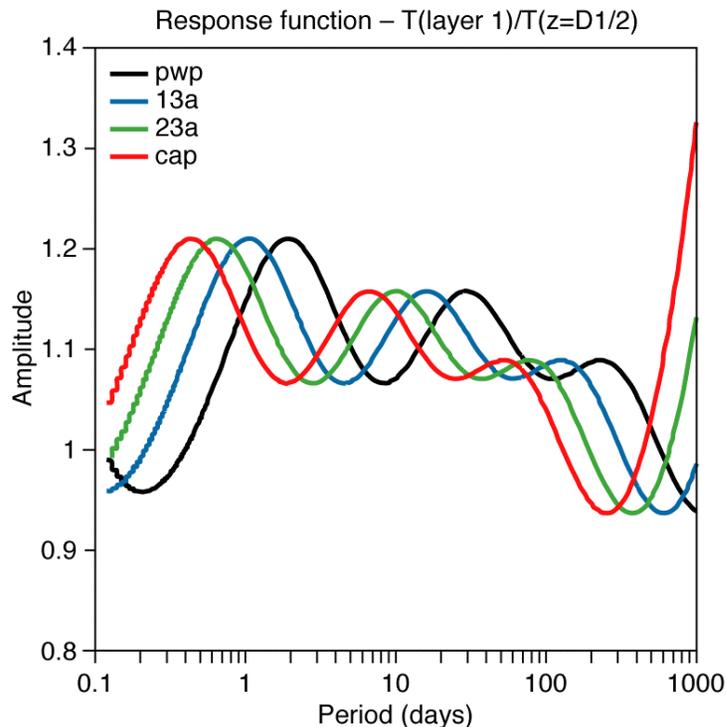
Processes in the IFS: Land surface (TESSEL)

Non-linear diffusion equations for temperature and soil water:

$$C_s \frac{\partial T}{\partial t} = K_T \frac{\partial^2 T}{\partial z^2}$$

Soil numerics:

- ◆ Implicit solution as vertical diffusion with implicitness factor equal to 1.
- ◆ Crude vertical discretization to cover time scale from hours to one year.
- ◆ Layer depths: 0.07, 0.21, 0.72 and 1.89 m



Processes in the IFS: Coupling of TESSEL to the atmosphere (Best coupler)¹

Coupling includes skin layer with instantaneously responding skin temperature for each tile.

$$\mathbf{H} = \rho C_H^n |\vec{U}| (S_1^{n+1} - S_{sk}^{n+1}), \quad S = C_p T + gz$$

$$\mathbf{E} = \rho C_Q^n |\vec{U}| \beta (q_1^{n+1} - \alpha q_{sat}(T_{sk}^{n+1}))$$

Eliminate T_{sk} by linearizing and using the surface energy balance equation (i.e. derive Penman /Monteith equation):

$$\mathbf{R}_{sw} + \mathbf{R}_{lw} + \mathbf{H} + \mathbf{LE} = \Lambda (T_{sk}^{n+1} - T_s)$$

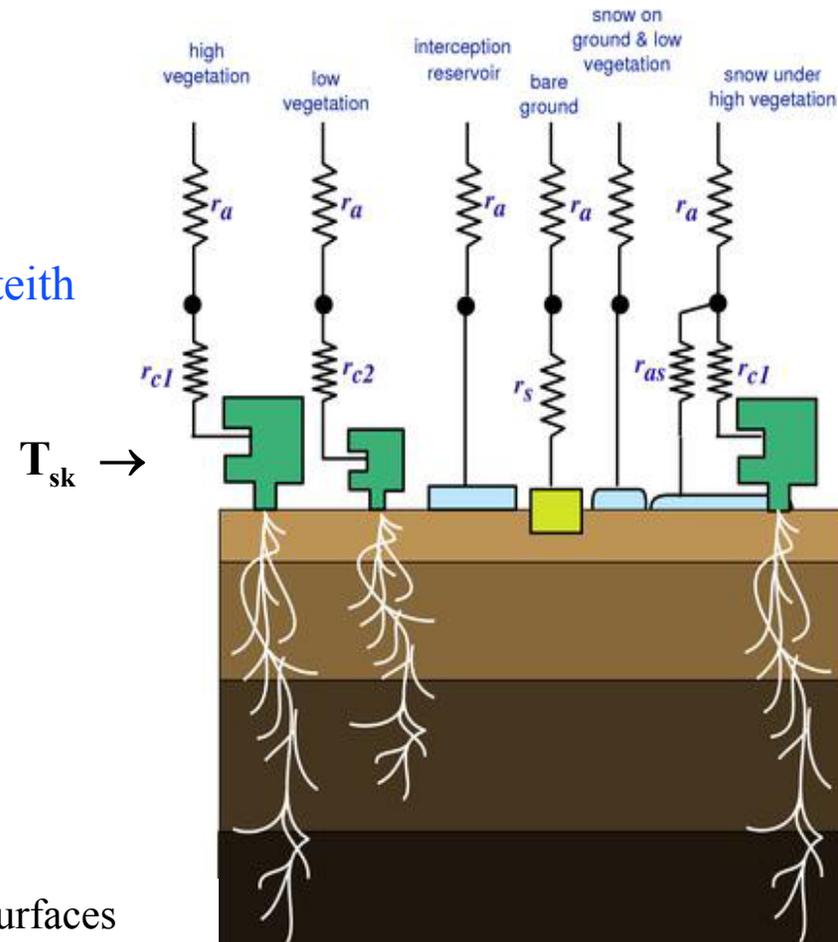
The result is two linear relations between lowest model level variables and fluxes with tile dependent coefficients D

$$\mathbf{H} = \mathbf{D}_{H1} S_1^{n+1} + \mathbf{D}_{H2} q_1^{n+1} + \mathbf{D}_{H3}$$

$$\mathbf{E} = \mathbf{D}_{E1} S_1^{n+1} + \mathbf{D}_{E2} q_1^{n+1} + \mathbf{D}_{E3}$$

Land surface tiles:

- ◆ High vegetation
- ◆ Low vegetation
- ◆ Wet surface
- ◆ Bare ground
- ◆ Exposed snow
- ◆ Snow under vegetation



¹Best et al. (2004): A proposed structure for coupling tiled surfaces with the planetary boundary layer, JHM

Processes in the IFS: Coupling of TESSEL to the atmosphere (Best coupler)¹

Averaging of fluxes over tiles, by averaging coefficients:

$$\bar{\mathbf{H}} = \mathbf{S}_1^{n+1} \sum_i \mathbf{v}^i \mathbf{D}_{H1}^i + \mathbf{q}_1^{n+1} \sum_i \mathbf{v}^i \mathbf{D}_{H2}^i + \sum_i \mathbf{v}^i \mathbf{D}_{H3}^i$$

$$\bar{\mathbf{E}} = \mathbf{S}_1^{n+1} \sum_i \mathbf{v}^i \mathbf{D}_{E1}^i + \mathbf{q}_1^{n+1} \sum_i \mathbf{v}^i \mathbf{D}_{E2}^i + \sum_i \mathbf{v}^i \mathbf{D}_{E3}^i$$

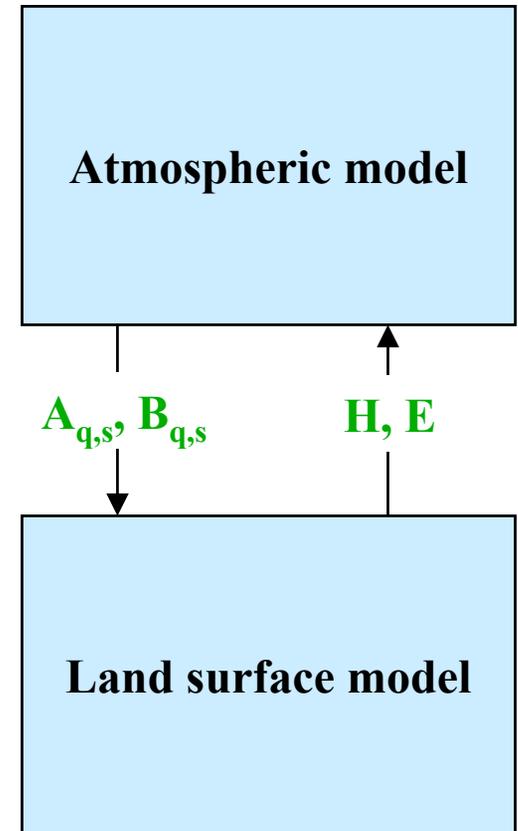
Combine with result of downward elimination of tri-diagonal matrix for vertical diffusion:

$$\mathbf{S}_1^{n+1} = \mathbf{A}_s \bar{\mathbf{H}} + \mathbf{B}_s$$

$$\mathbf{q}_1^{n+1} = \mathbf{A}_q \bar{\mathbf{E}} + \mathbf{B}_q$$

Comments:

- ◆ The atmospheric surface layer is part of the LSM
- ◆ All the tile dependent parameters are part of the LSM



¹Best et al. (2004): A proposed structure for coupling tiled surfaces with the planetary boundary layer, JHM



Coupling of TESSEL to the atmosphere (Best coupler)¹

Fractions

1: 0.0

2: 0.0

3: 0.53

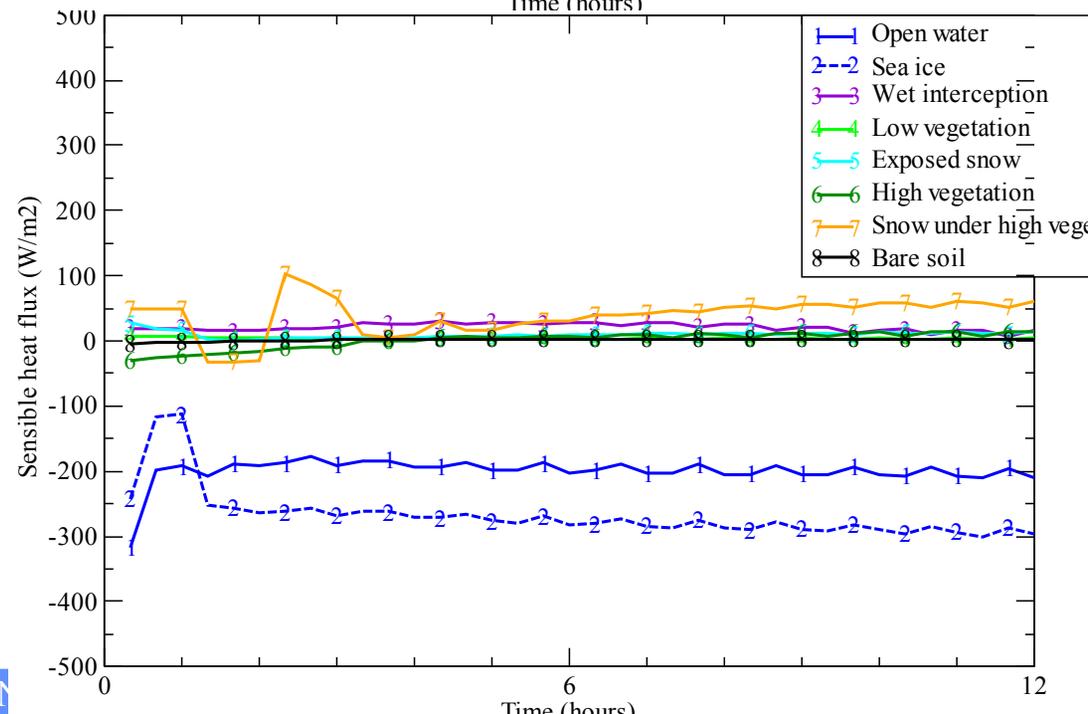
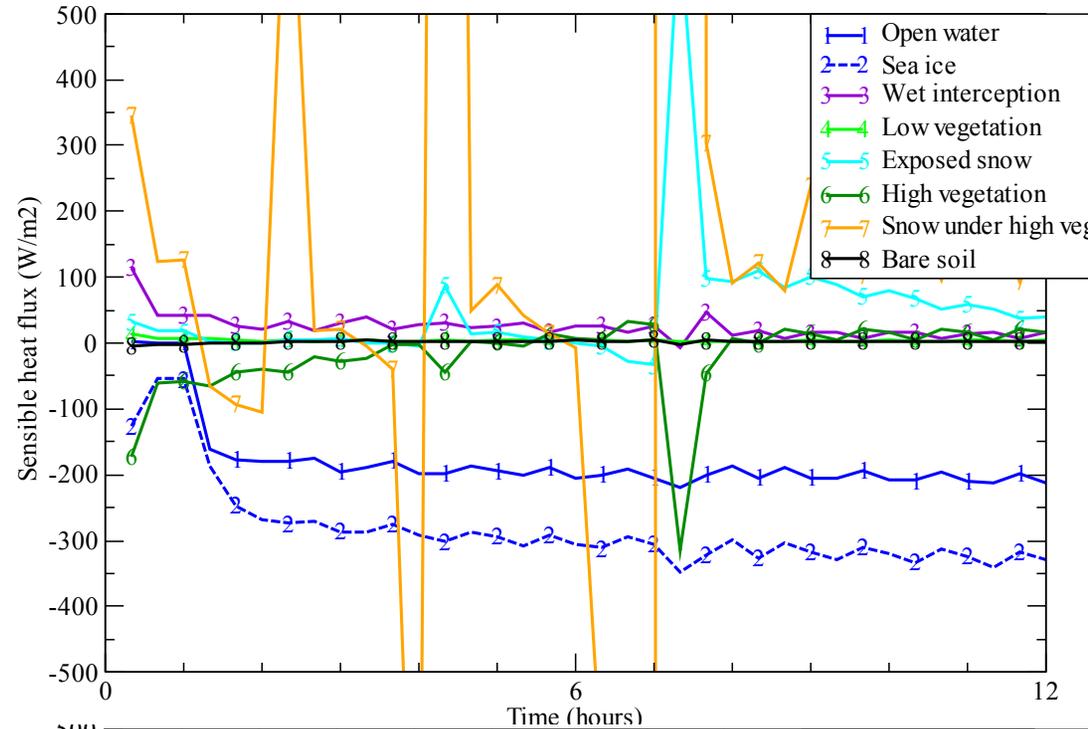
4: 0.04

5: 0.00

6: 0.37

7: 0.00

8: 0.06



Conclusions

- ◆ The physics of a process and its coupling to other processes is important from the numerical point of view
- ◆ Splitting is major issue
- ◆ Sequential split with slow process first and a single fast implicit process is preferred option
- ◆ Unification of fast processes is desirable (e.g. BL, subgrid orography and shallow convection)
- ◆ Balance is important
- ◆ 2nd order physics in ECMWF model should be reformulated considering convection and clouds as implicit processes
- ◆ Different processes have different problems e.g.:
 - convection needs implicit numerics at high vertical resolution,
 - microphysics is fast and therefore difficult,
 - vertical diffusion is noisy.

