Modelling of Innovation Statistics.

Erik Andersson

- Introduction
- Theory + computational methods
- Temporal aspects
- Validation of background error modelling
- Observation error correlation
- Summary and conclusions

Acknowledgement:
F. Bouttier, C. Cardinali, L. Isaksen, S. Saarinen

The new information

The innovations provide the new information to the assimilation.

The innovations (d) = The observed departures from the background

$$\mathbf{d} = \mathbf{y} - H\mathbf{x}_{\mathrm{b}}$$

If the distribution of the data in time is accounted for, then

$$\mathbf{d} = \mathbf{y} - HM\mathbf{x}_{\mathrm{b}}$$

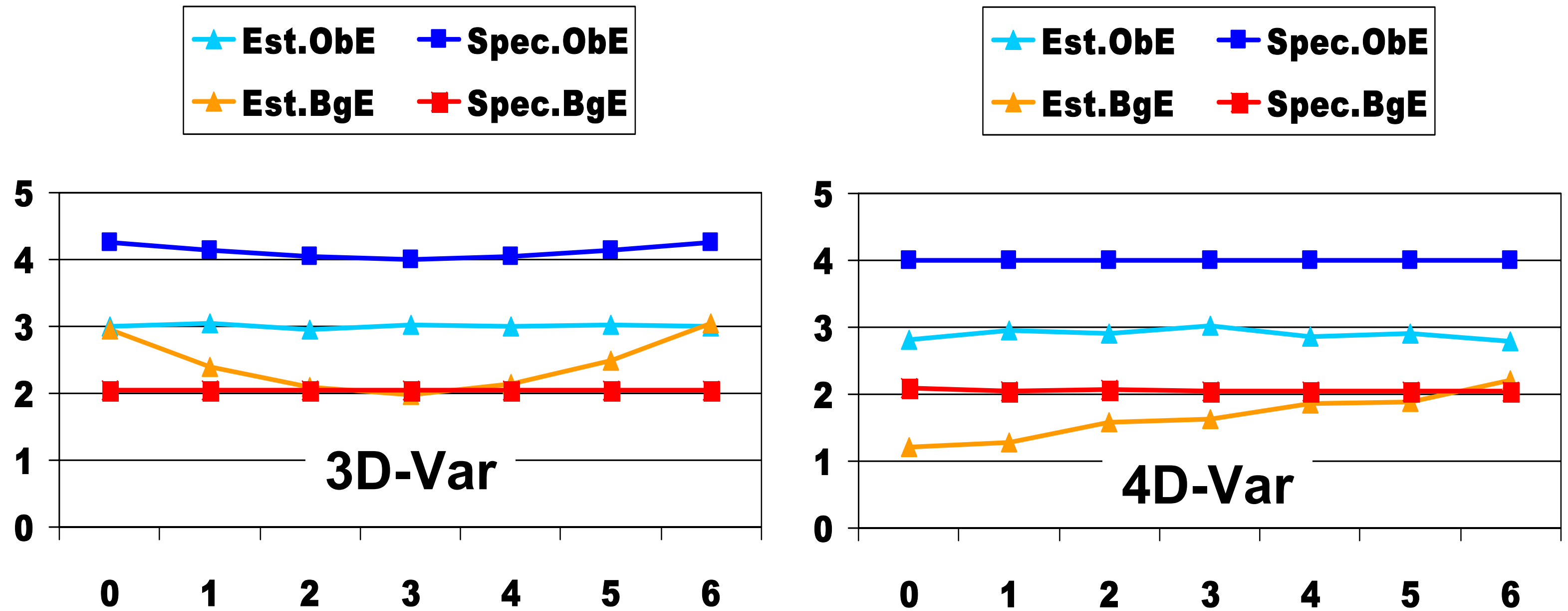
The calculations of the innovations are carried out as accurately as practically possible:

- **We use the full non-linear forecast model M, at highest affordable resolution (T511)**
- **A large effort has been put on developing H to closely mimic the real observation (e.g. RTTOV)**

Temporal evolution of innovations

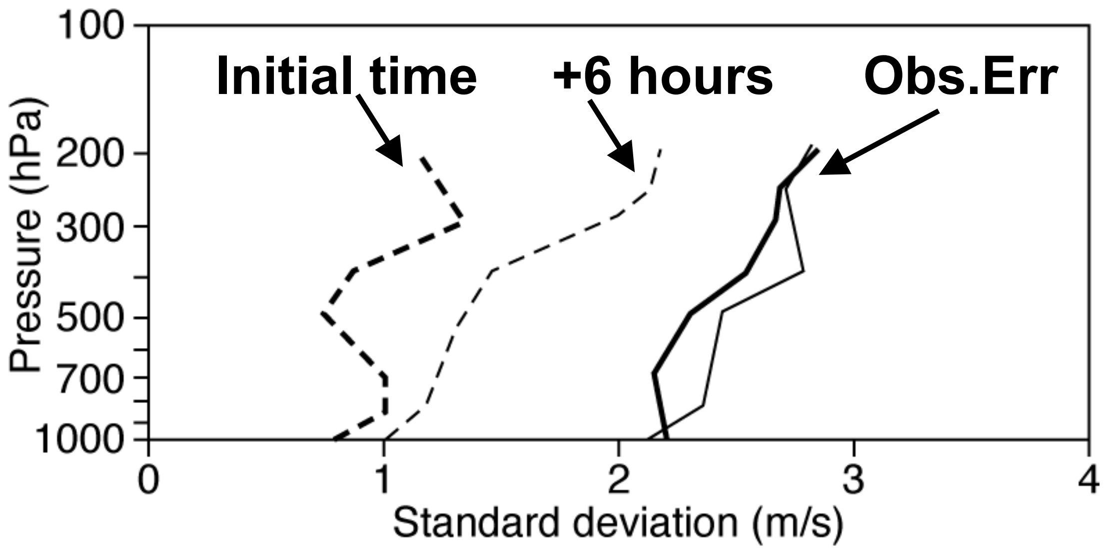
Heikki Järvinen (*Tellus* 2001) studied the innovations within 3D-Var and 4D-Var with a 6-hour assimilation window. Used Hollingsworth-Lönnberg (*Tellus* 1986) de-correlation method to isolate Obs and Bg errors.

Aircraft data, N.Amer, 200 hPa

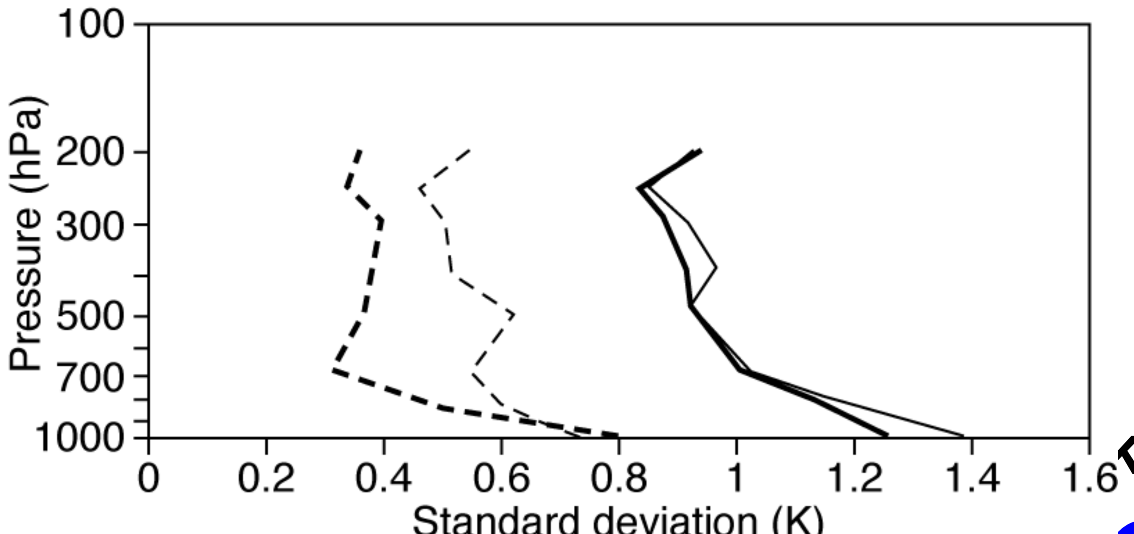


Temporal evolution of innovations
Over 6 hours for Aircraft data, North America

Wind component



Temperature

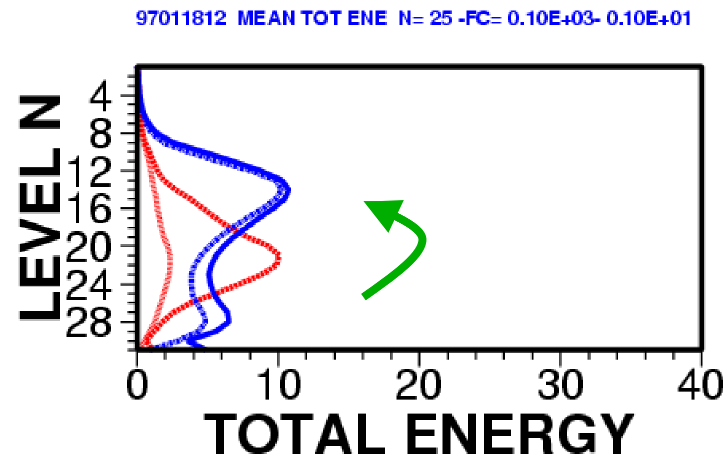


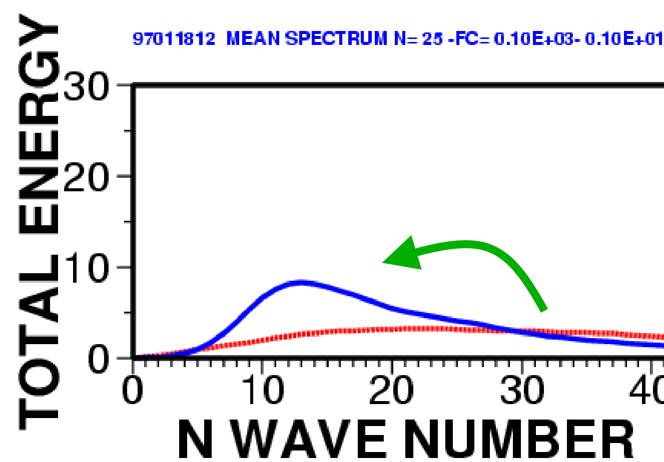
Average SV energy distribution for 18-20 Jan 1997

SV1:25 average vertical distribution at initial time of the kinetic (dotted, x100) and total (solid, x100) energy, and the corresponding final time distributions. The bottom figure shows the SV1:25 average total energy spectrum at initial (x100) and at final time.

Note the SV typical **upward and upscale energy transfer/growth**, and the transformation **from initial potential to mainly final kinetic energy**.

(R. Buizza)





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The errors

Unfortunately, **the observations** $\mathbf{y}$,

the background $\mathbf{x}_b$,

the model M **and**

the observation operators H

are all affected by errors.

Let hât denote 'the truth', and ε **the error, then**

$\mathbf{y} = \hat{\mathbf{y}} + \boldsymbol{\varepsilon}_o$	$\left\langle \boldsymbol{\varepsilon}_o, \boldsymbol{\varepsilon}_o^T \right\rangle = \hat{\mathbf{O}}$	Observation error covariance
$\mathbf{x}_b = \hat{\mathbf{x}}_b + \boldsymbol{\varepsilon}_b$	$\left\langle \boldsymbol{\varepsilon}_b, \boldsymbol{\varepsilon}_b^T \right\rangle = \hat{\mathbf{B}}$	Background error
$M\hat{\mathbf{x}}_{(t=0)} = \hat{\mathbf{x}}_{(t=T)} + \boldsymbol{\varepsilon}_q$	$\left\langle \boldsymbol{\varepsilon}_q, \boldsymbol{\varepsilon}_q^T \right\rangle = \hat{\mathbf{Q}}$	Model error
$H\hat{\mathbf{x}}_{(t)} = \hat{H}\hat{\mathbf{x}}_{(t)} + \boldsymbol{\varepsilon}_f$	$\left\langle \boldsymbol{\varepsilon}_f, \boldsymbol{\varepsilon}_f^T \right\rangle = \hat{\mathbf{F}}$	Representativity error

The innovation covariance

The innovation covariance can be written

$$\langle \mathbf{d}, \mathbf{d}^{\mathrm{T}} \rangle = \hat{\mathbf{H}}\hat{\mathbf{P}}^{f}\hat{\mathbf{H}}^{\mathrm{T}} + \hat{\mathbf{O}} + \hat{\mathbf{F}} - (\hat{\mathbf{H}}\hat{\mathbf{X}}^{\mathrm{T}} + \hat{\mathbf{X}}\hat{\mathbf{H}}^{\mathrm{T}})$$

with

(Joiner and Dee, *QJ* 2000)

$$\hat{\mathbf{P}}^{f} = \hat{\mathbf{M}}\hat{\mathbf{B}}\hat{\mathbf{M}}^{\mathrm{T}} + \hat{\mathbf{Q}}$$

Confusion surrounding '*Model error*':

- Q = Model error, due to imperfections in M
- MBM^T = Predictability error, due to evolution of errors in the initial conditions
- $P^f = MBM^T + Q$ = Forecast error
- B = Bg-error = Initial condition error

4D-Var approximations

In our 4D-Var the 'true' co-variances are approximated:

$\hat{\mathbf{O}} + \hat{\mathbf{F}} \approx \mathbf{R}$ — R diagonal

$\hat{\mathbf{X}} \approx 0$ — No cross co-variances

$\hat{\mathbf{Q}} \approx 0$ — Perfect model assumption

$\hat{\mathbf{H}}\hat{\mathbf{P}}^f\hat{\mathbf{H}}^T \approx \mathbf{HMBM}^T\mathbf{H}^T$ — Tangent linear obs. operators

Tangent linear forecast model

- TL dynamics?
- TL physics?
- Truncation?

B given through J_b-modelling

4D-Var: $HMBM^TH^T+R$

3D-Fgat: HBH^T+R

OI: B_o+R

Which data are useful?

At this point we can conclude that:

- **An analysis scheme which models innovation co-variances well is a good analysis scheme. We put our effort on**
  - Characterizing background error – but still simplified
  - Using the forecast model to evolve errors – but Q=0
  - Developing accurate observation operators – but R=O+F%I

Conversely:

- **Observations whose innovations are easily modelled, are useful observations**
  - Un-correlated with the background
  - Un-correlated with other observations
  - Accurately characterized by H

Validation

From samples of innovations we can compute

$$\langle \mathbf{d}, \mathbf{d}^{\mathrm{T}} \rangle$$

If we knew how to diagnose

$$\mathbf{HMBM}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}} + \mathbf{R}$$

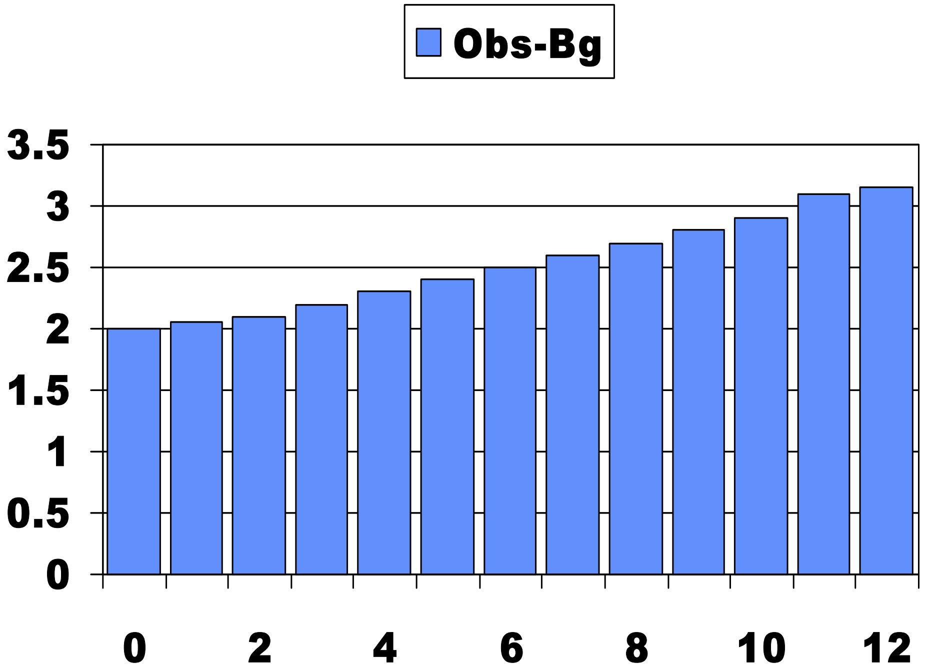
in the full 4D-Var system,

then the two could be compared, and some shortcomings due to the modelling assumptions might become apparent.

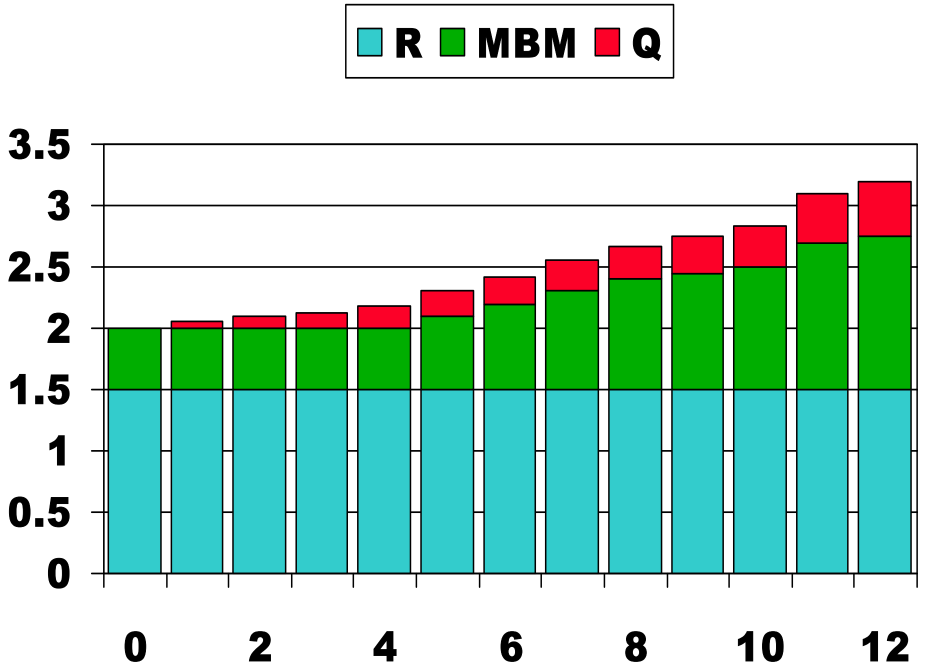
Discrepancies could be due to H, M, B, R or Q !!!

In a 'well-tuned' system: $\langle \mathbf{d}, \mathbf{d}^{\mathrm{T}} \rangle \approx \mathbf{HMBM}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}} + \mathbf{R}(+\mathbf{Q})$

What we expect...
(Fictitious example, for illustration only)



Innovations



Modelling of $HMBM^TH^T + R + Q$

Diagnosing HMBM^{T}H^T in 4D-Var

Due to its definition, the 4D-Var control-variable χ **is a standard multivariately normal quantity, I.e.:**

$$\chi \sim \mathcal{N}(0, \mathbf{I})$$

and:

$$\mathbf{L}\chi = \delta\mathbf{x} \sim \mathcal{N}(0, \mathbf{B})$$

$$\mathbf{HM}\delta\mathbf{x} \sim \mathcal{N}(0, \mathbf{HMBM}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}})$$

Randomization: **Generate a random sample of** N **vectors,** $\chi^{(N)}$, **with zero mean and unit variance, then**

$$\chi^{(N)}(\chi^{(N)})^{\mathrm{T}} \equiv \mathbf{I}^{(N)}$$

$$\mathbf{L}\chi^{(N)}(\mathbf{L}\chi^{(N)})^{\mathrm{T}} \equiv \mathbf{B}^{(N)}$$

Similarly, an estimate of HMBM^{T}H^T, can be obtained from

$$\mathbf{HML}\chi^{(N)}(\mathbf{HML}\chi^{(N)})^{\mathrm{T}} \equiv \mathbf{HMBM}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}(N)}$$ **(M. Ehrendorfer)**

Diagnosing $\mathbf{HMBM^T H^T}$ in 4D-Var

We compute

$$\mathbf{HMBM}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}} \approx \frac{1}{N}\sum_{i=1}^{N} \mathbf{HML}\chi_i (\mathbf{HML}\chi_i)^{\mathrm{T}}$$

for a sample of $N=100$ vectors, accumulating variance-contributions for the diagonal elements only.

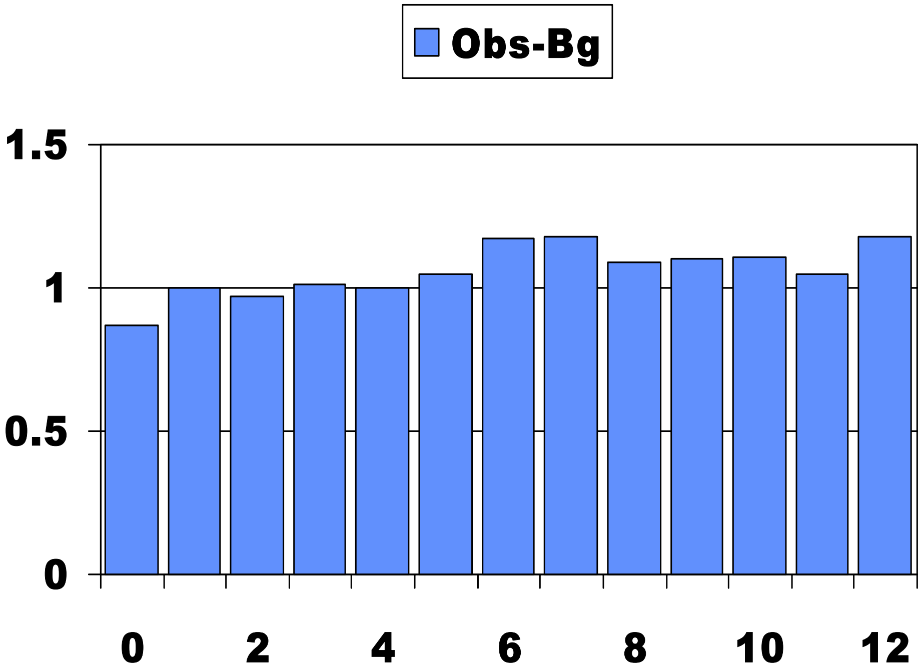
The number of estimated diagonal elements = the number of used observations (. 3,500,000).

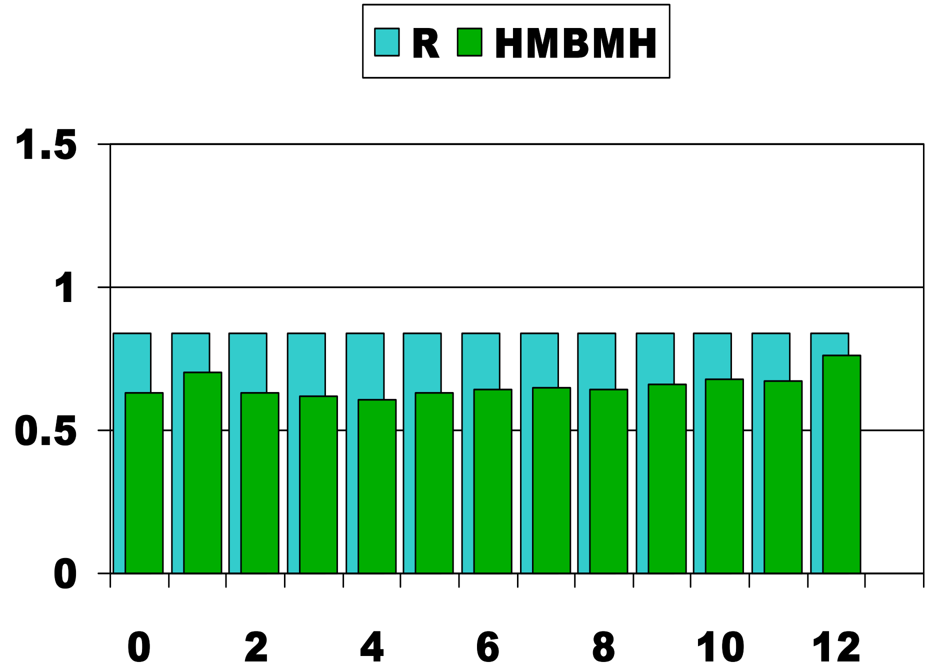
The uncertainty in the resulting randomization estimate is about 3% .

In the following, we will see results of such calculations with $N=100$, for:

- **A few H of used data in 4D-Var**
- **Current L (that is, J_b), and some earlier versions**

DRIBU, North Atlantic
Surface pressure data (hPa)

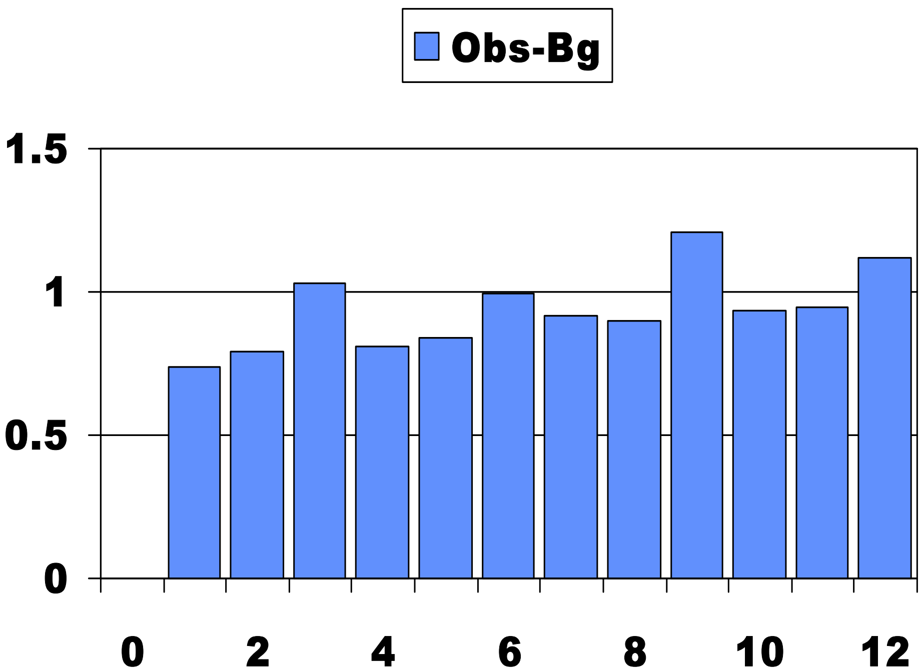


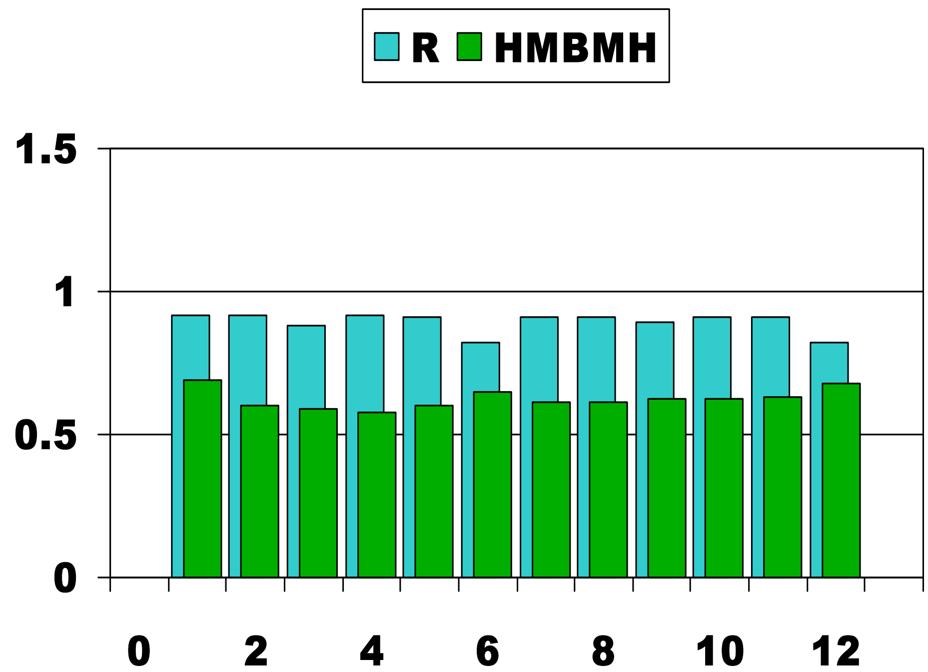


20030205-12 to 20030211-12,

About 600 data per bin

SYNOP, North Atlantic
Surface pressure data (hPa)

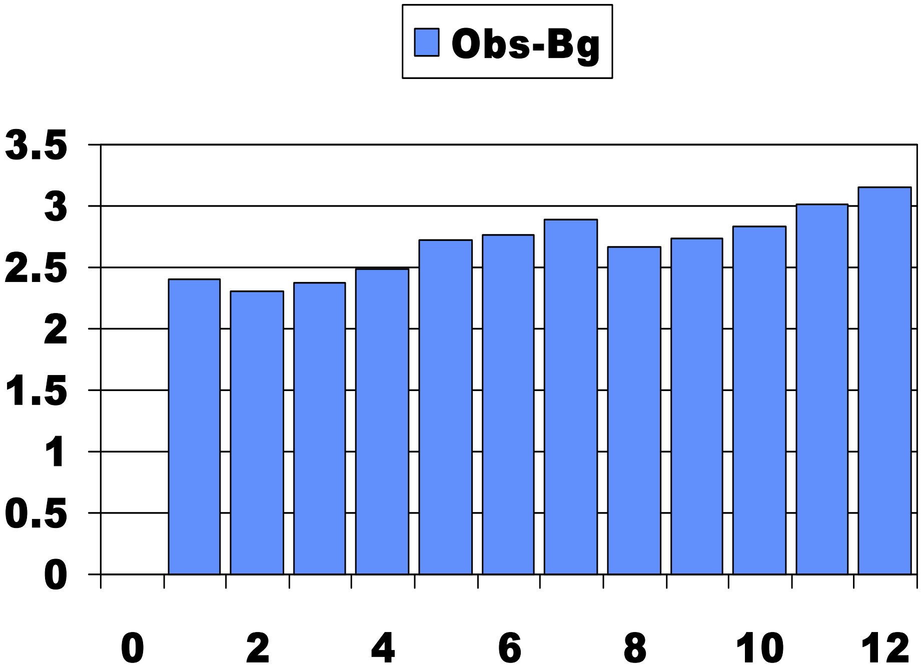


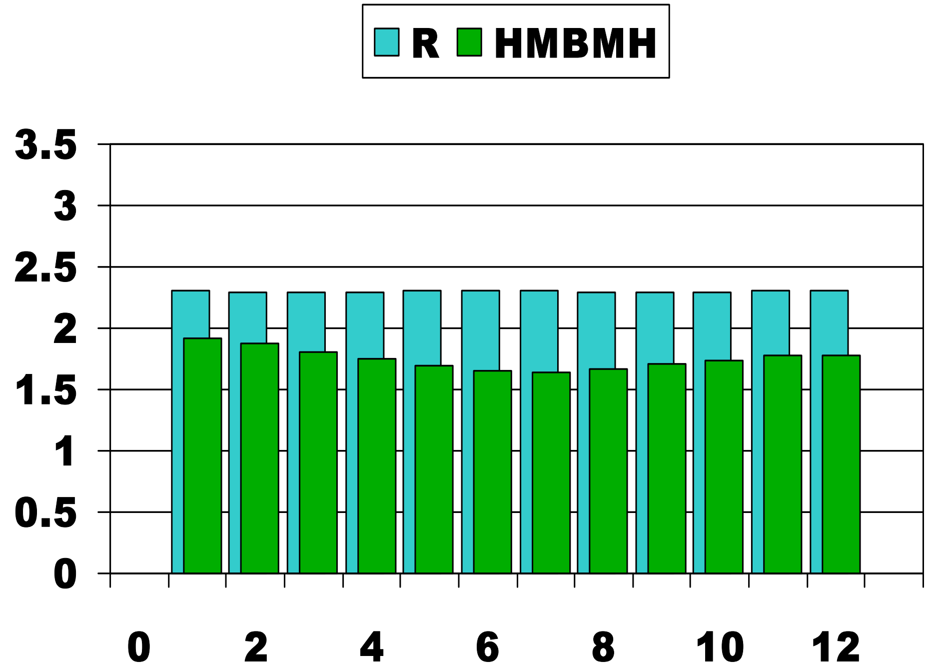


20030205-12 to 20030211-12,

About 4,000 data per bin 6-hourly, 2,000 3-hourly, 1,000 1-hourly

American wind profilers,
U-component (m/s), 300-200 hPa





20030205-12 to 20030211-12,

About 12,000 data per bin

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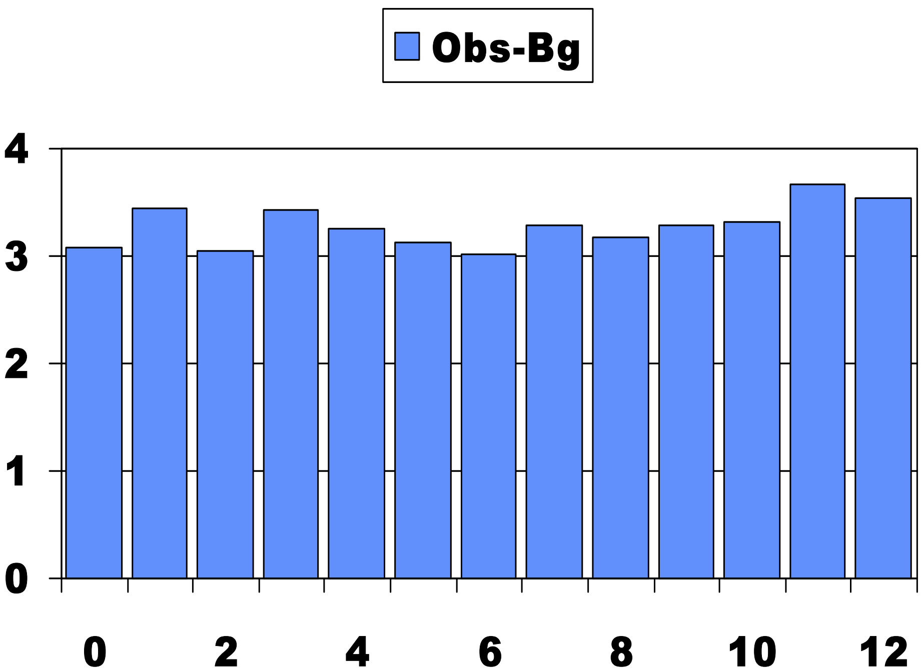
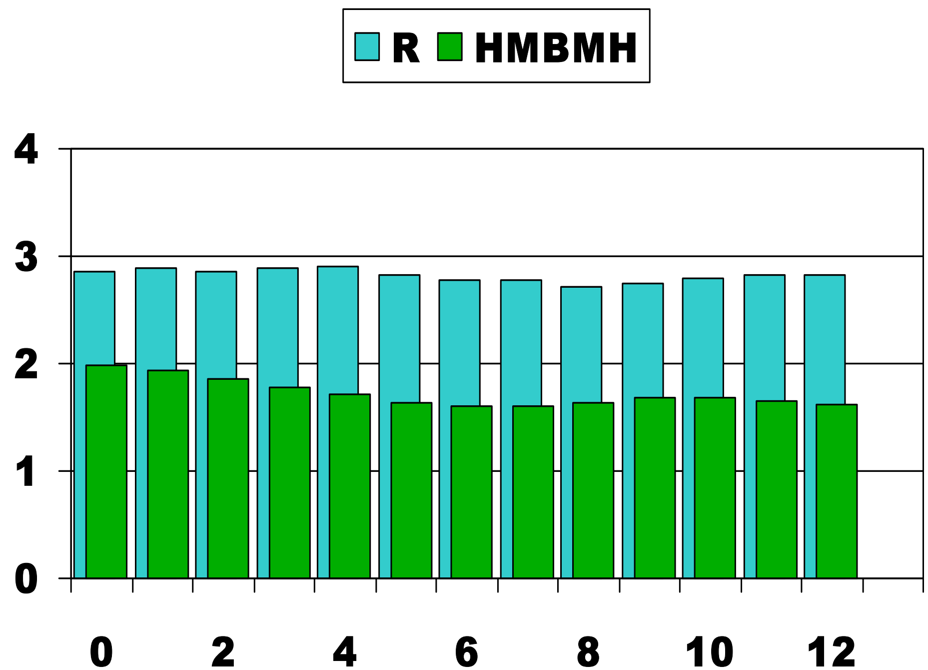
Aircraft data, North Atlantic
U-component (m/s), 300-200 hPa





20030205-12 to 20030211-12,

About 2,200 - 4,500 data per bin

Used data (Sept 2003)

Conventional

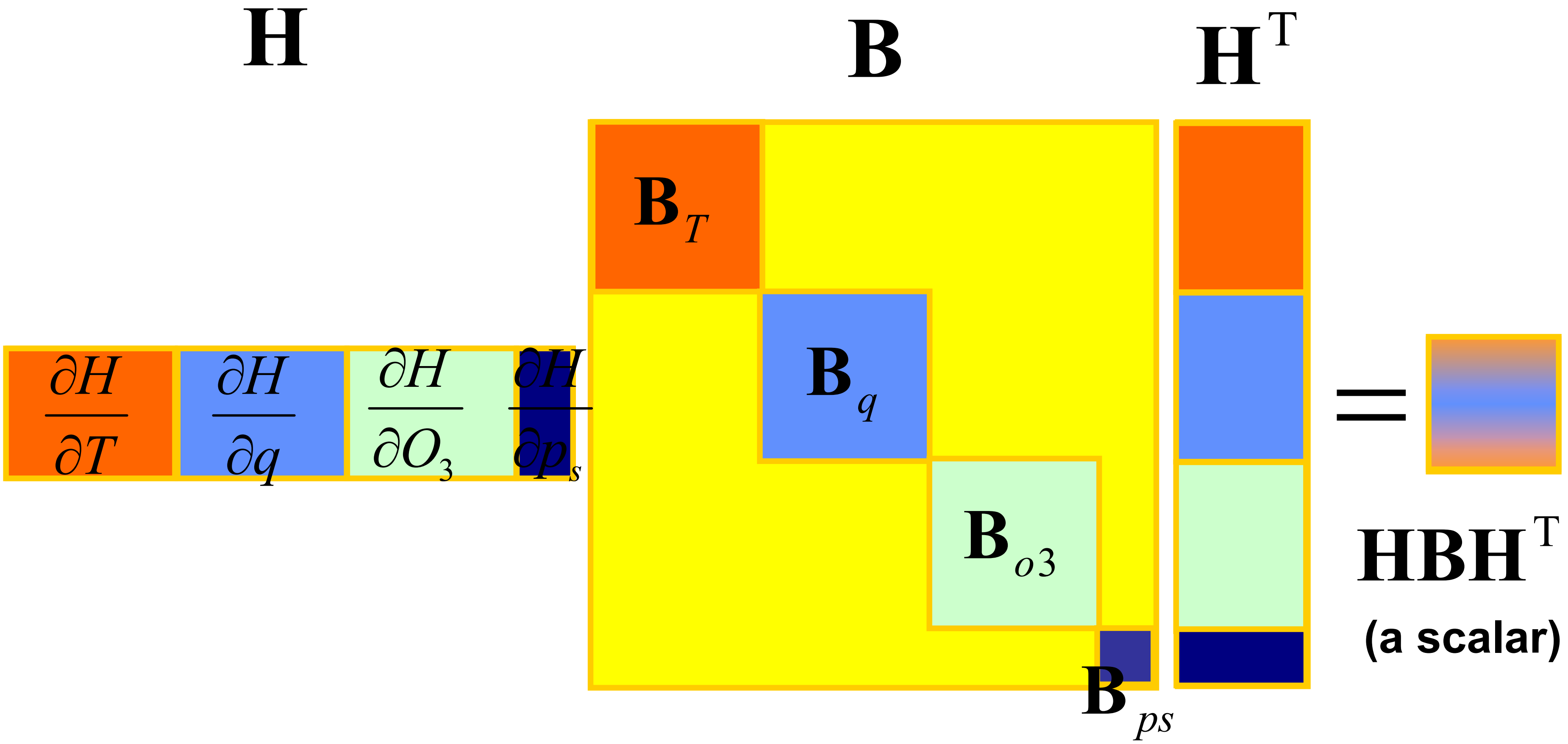
- SYNOP
  Surf.Press, Wind-10m, RH-2m
- AIREP
  Wind, Temperature
- SATOB AMVs
  Meteosat, GOES, MODIS
- DRIBU
  Surf.Press, Wind-10m
- TEMP
  Wind, Temp, Humidity profiles
- DROPSONDE
  Wind and Temp profiles
- PILOT, Am+Eu+Jp Profilers
  Wind profiles
- PAOB
  Surface pressure proxy

Satellite

- NOAA-15/16/17
  HIRS, AMSU-A&B radiances
- AQUA
  AIRS and AMSU-A radiances
- DMSP-13/14/15
  SSMI radiances
- Meteosat-5/7, GOES-9/10/12
  Water Vapour radiances
- QuikScat
  Ambiguous winds
- SBUV, (GOME), MIPAS
  Layer ozone

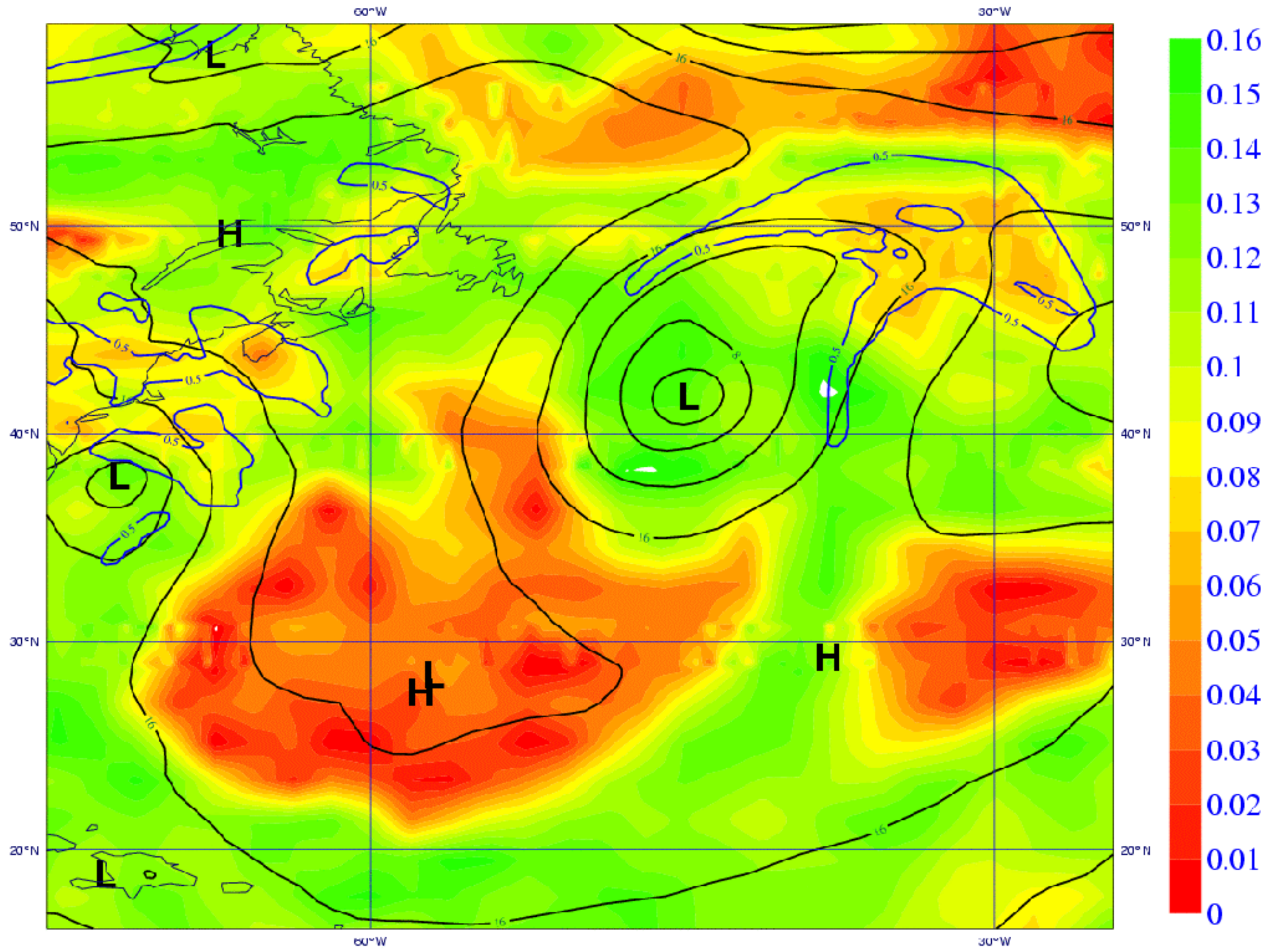
In preparation: MSG, SSMI/S, Cloud and precipitation data…

One vertical column



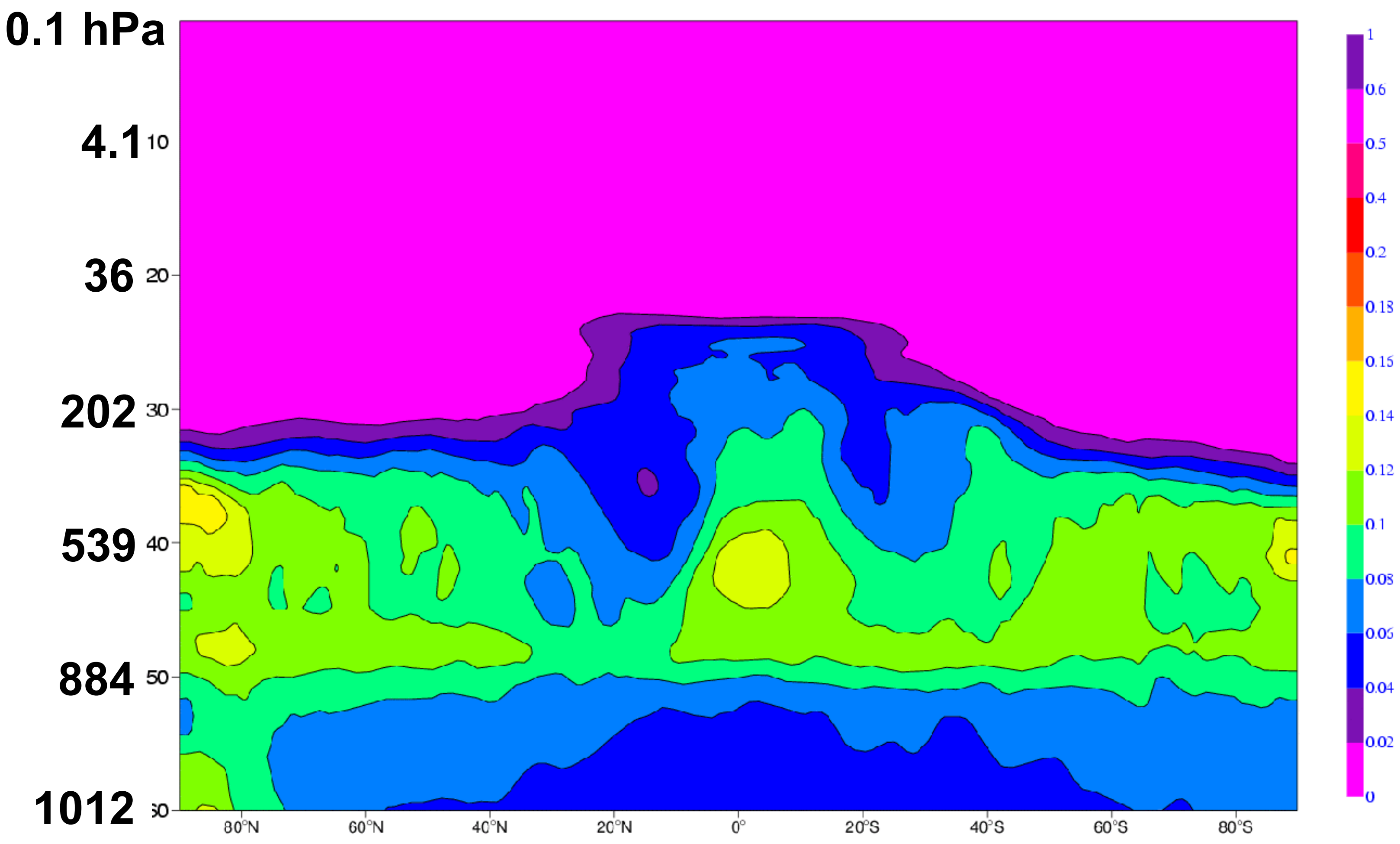
Relative Humidity HBH^T

H is H(T,q,p)



New humidity analysis formulation, E.Holm

Zonal-mean Relative Humidity HBH^T
H is H(T,q,ps)



New humidity analysis formulation, E.Holm

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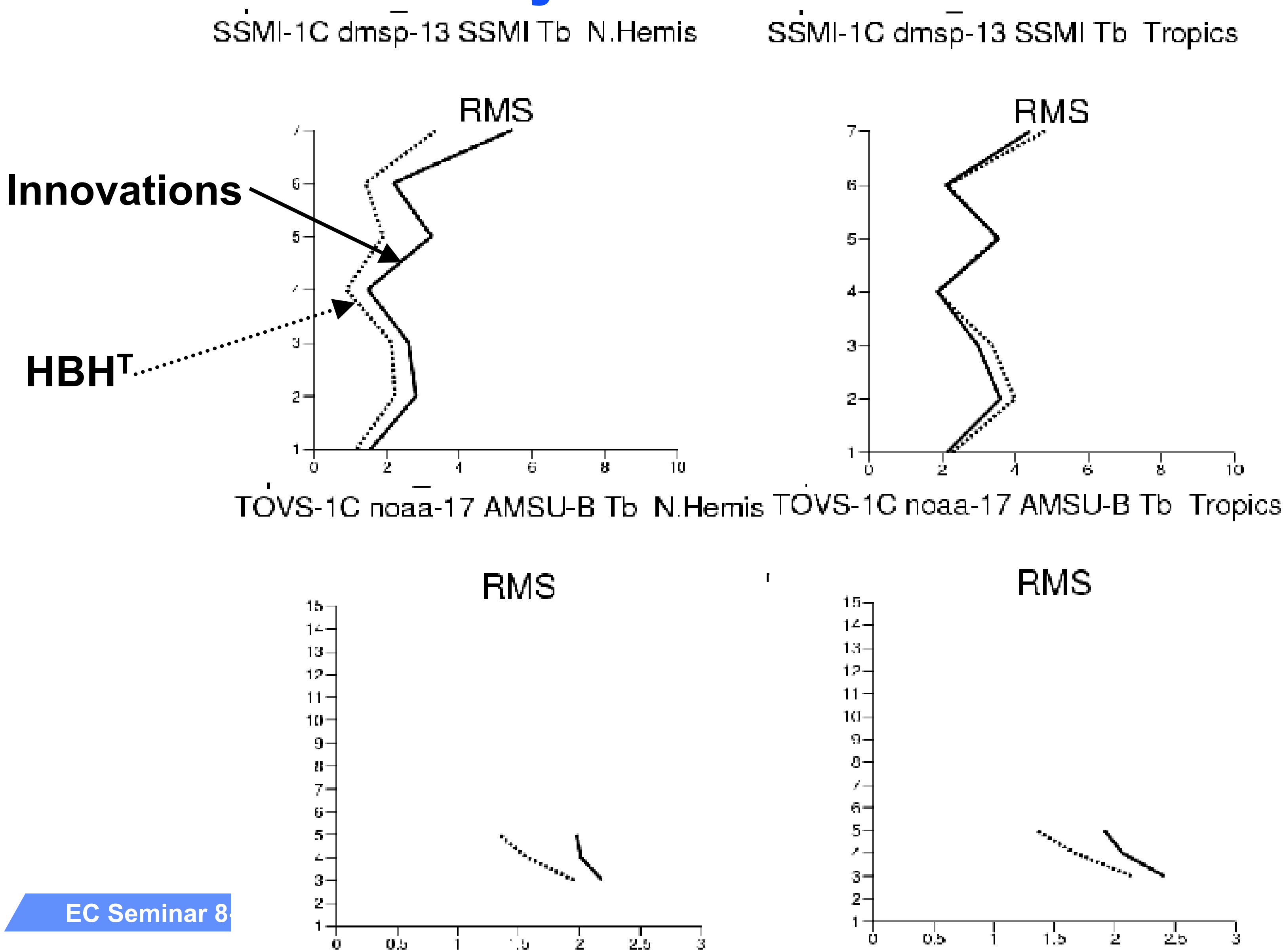
Comparison with innovation statistics

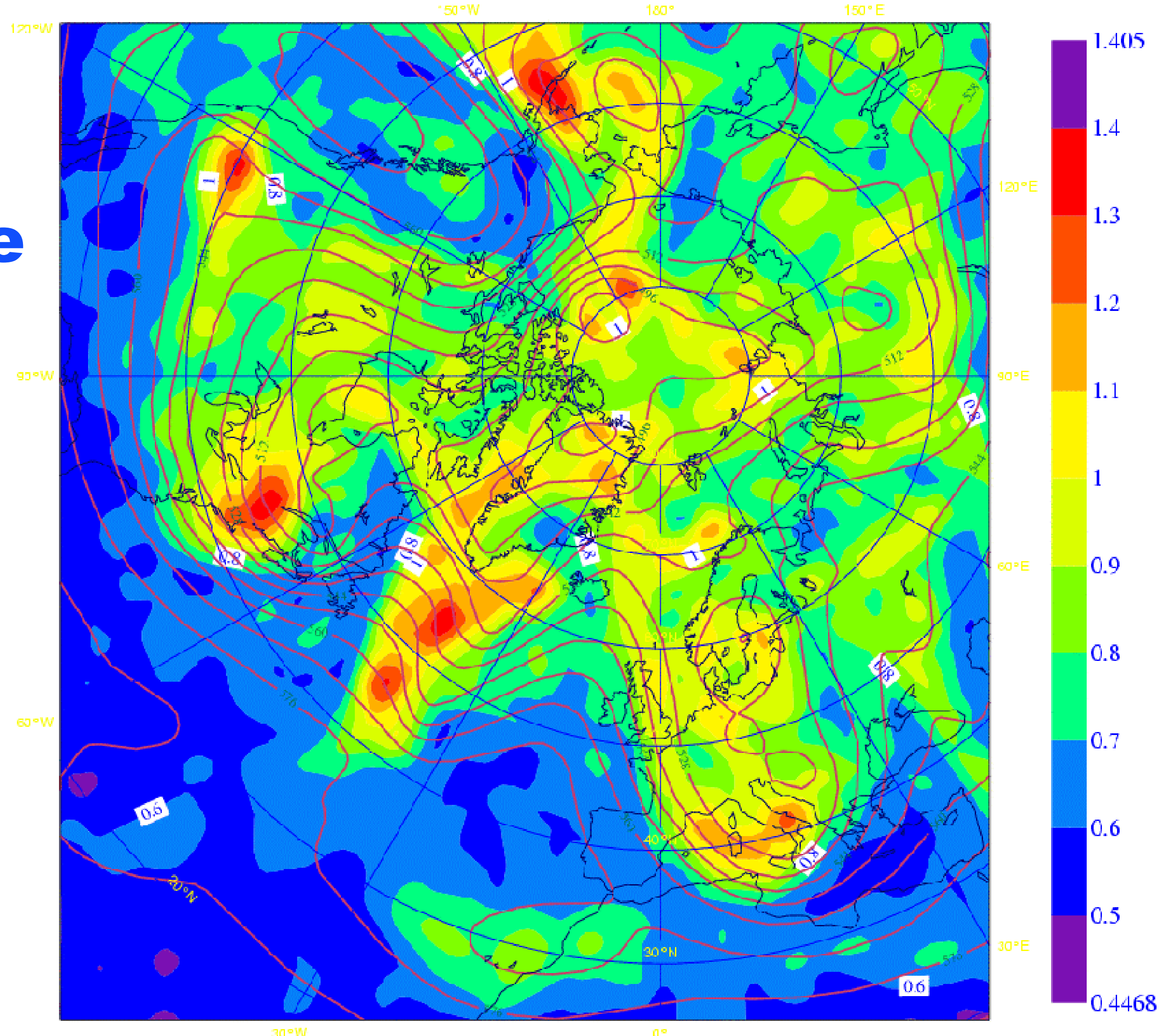
Humidity-sensitive radiances



Temperature HBH^T (K)

T lev39 HBH^T (shaded),

Z 500 hPa (contoured)



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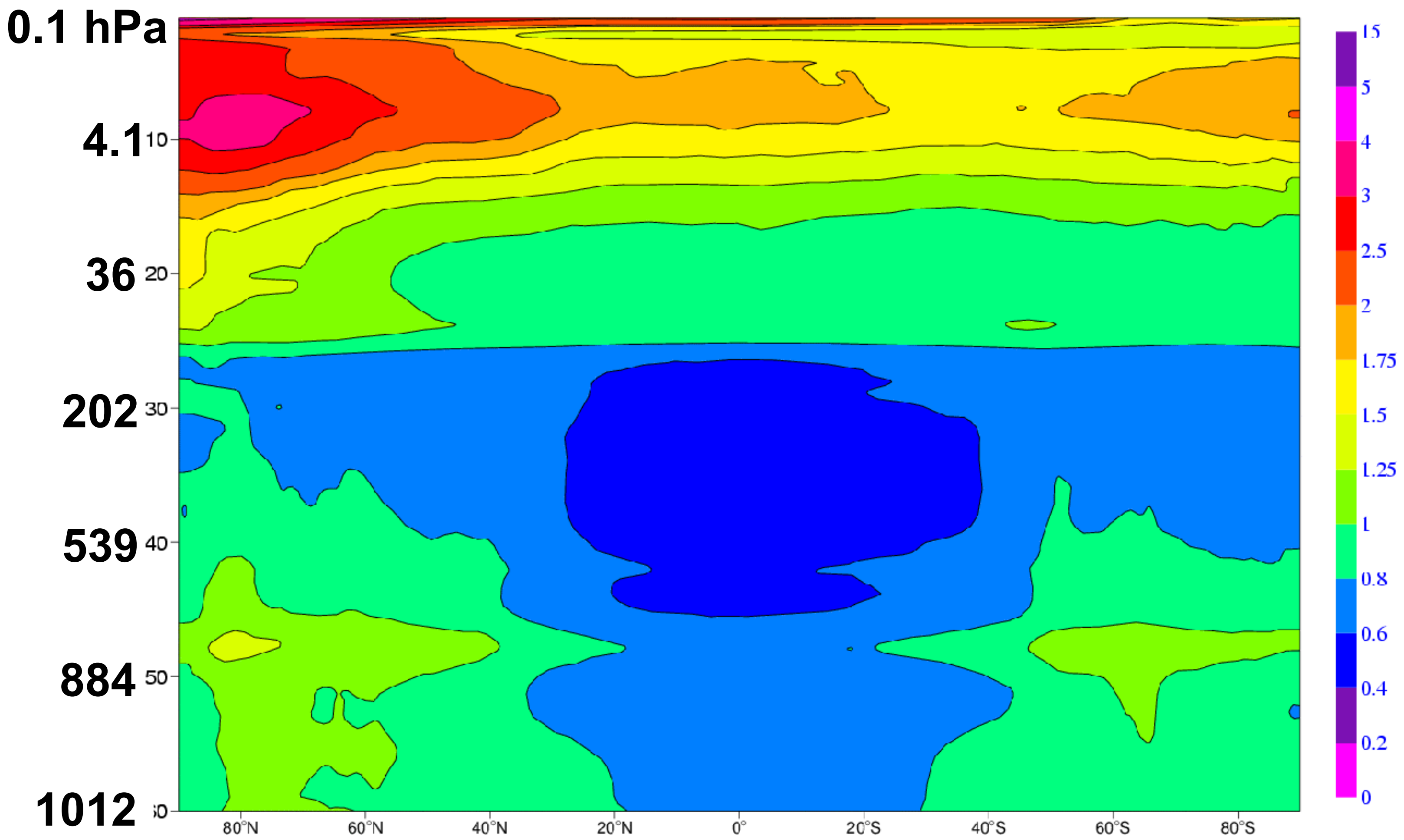
Temperature HBH^T (K)
zonally averaged cross-section



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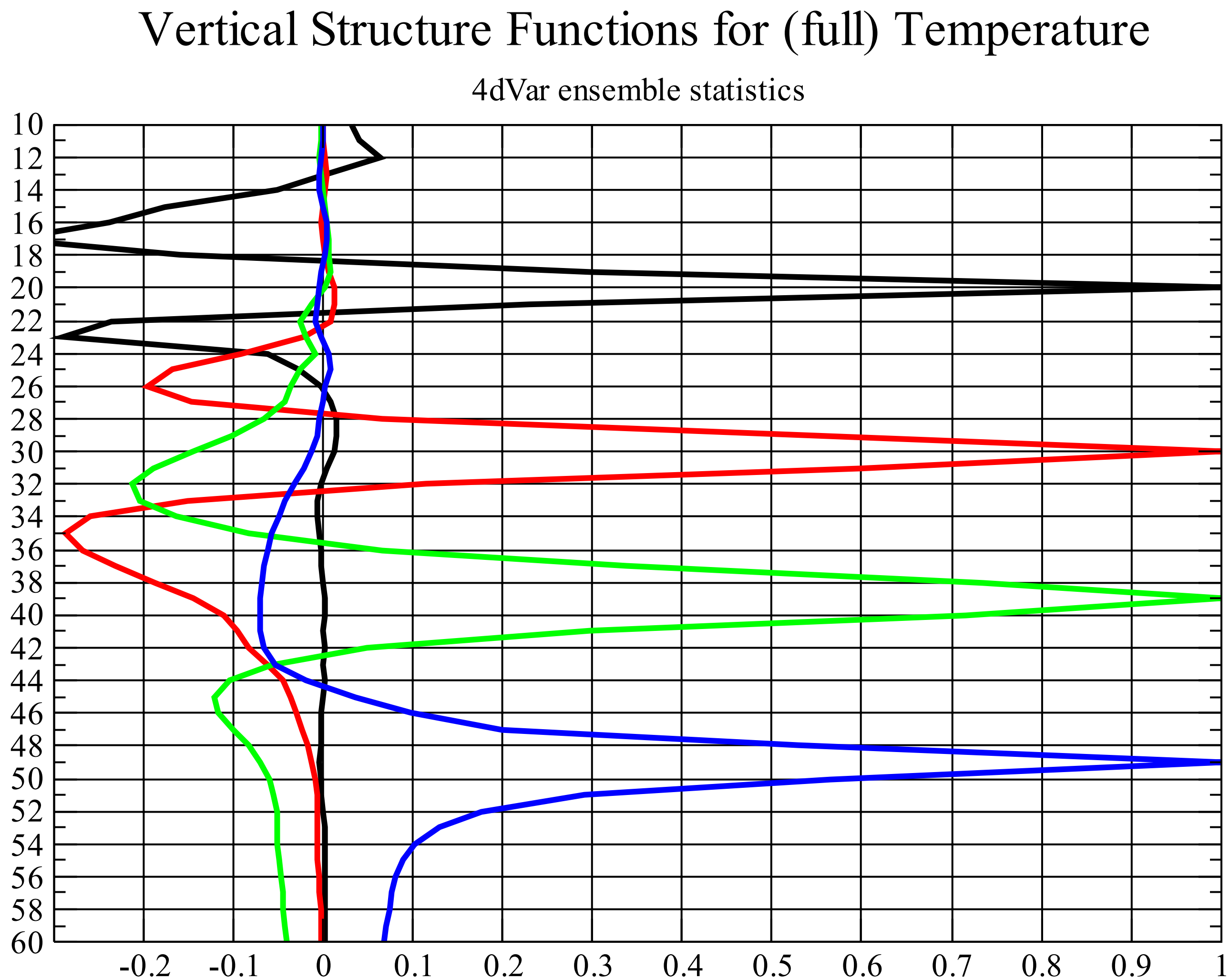
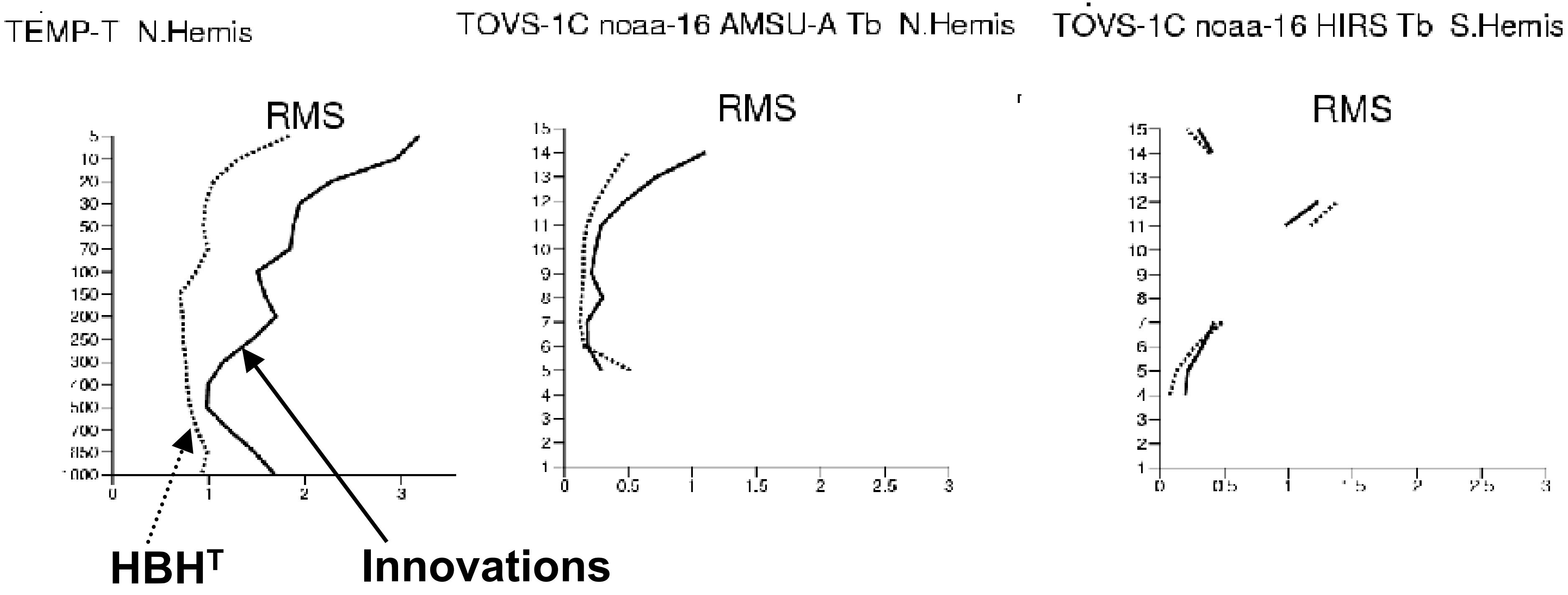
BG-error correlations (temperature)
from an Ensemble of 4D-Var assimilations



Comparison with innovation statistics
Temperature: radiosonde and radiances



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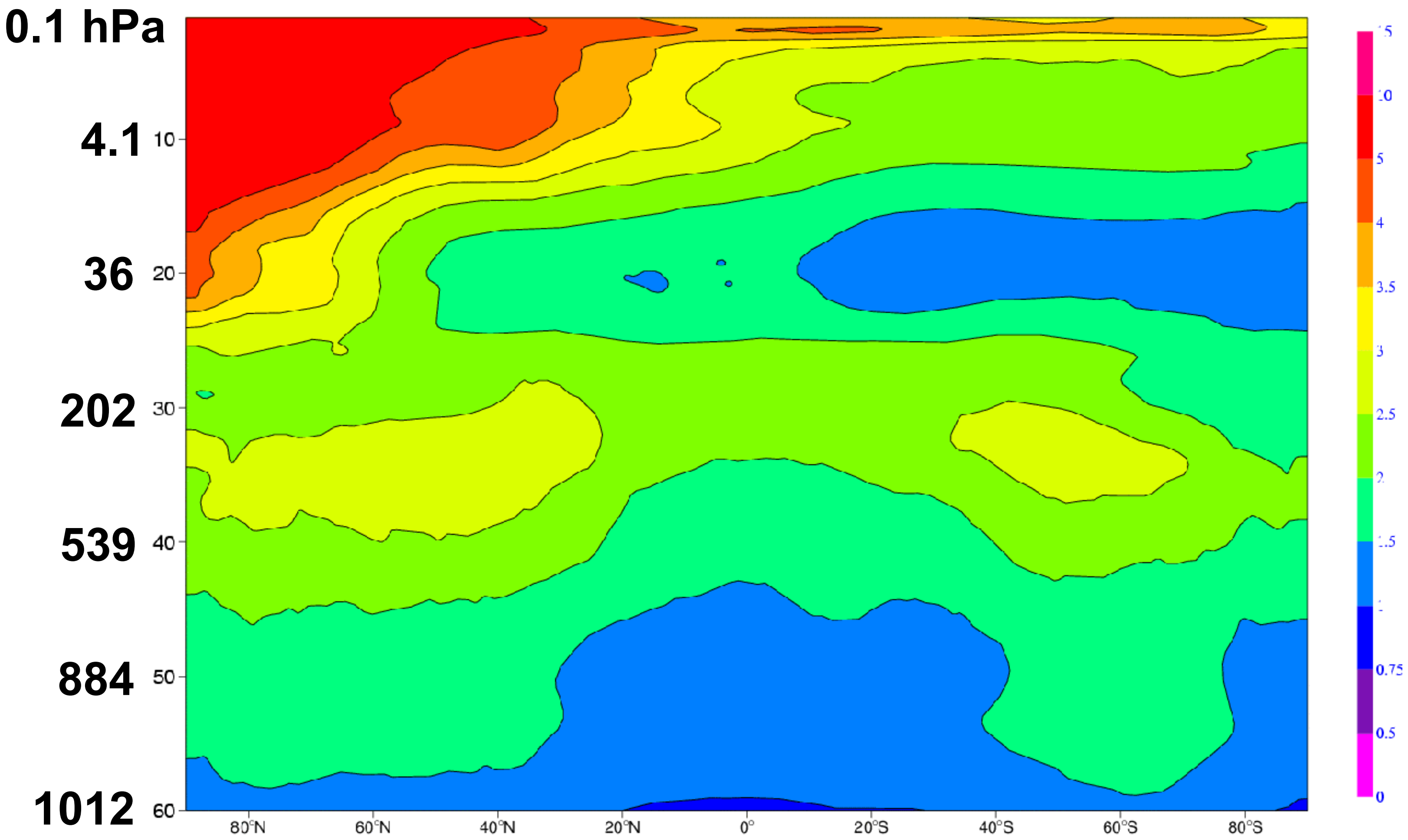
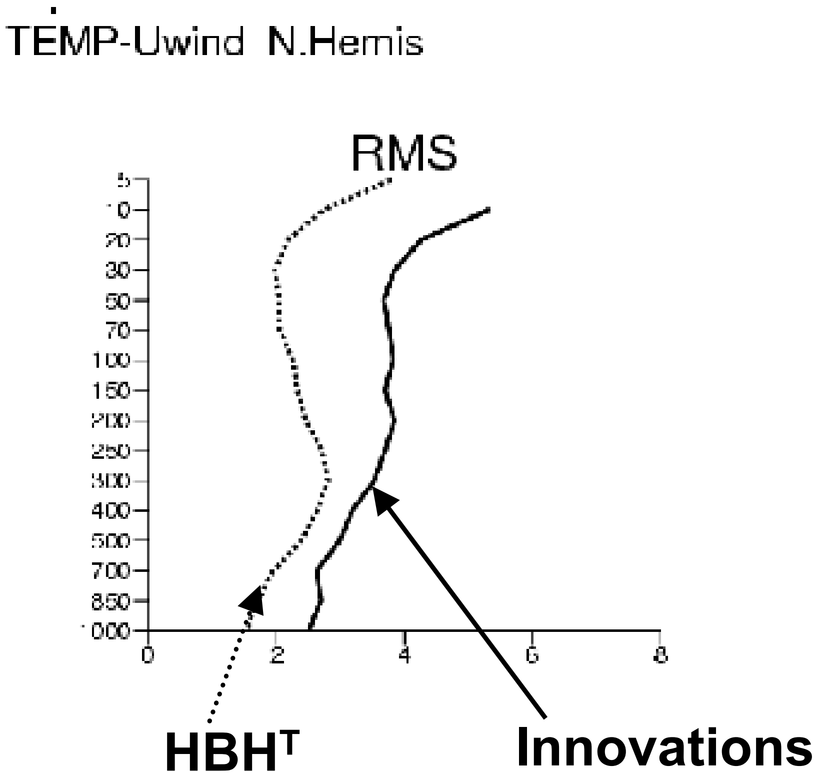
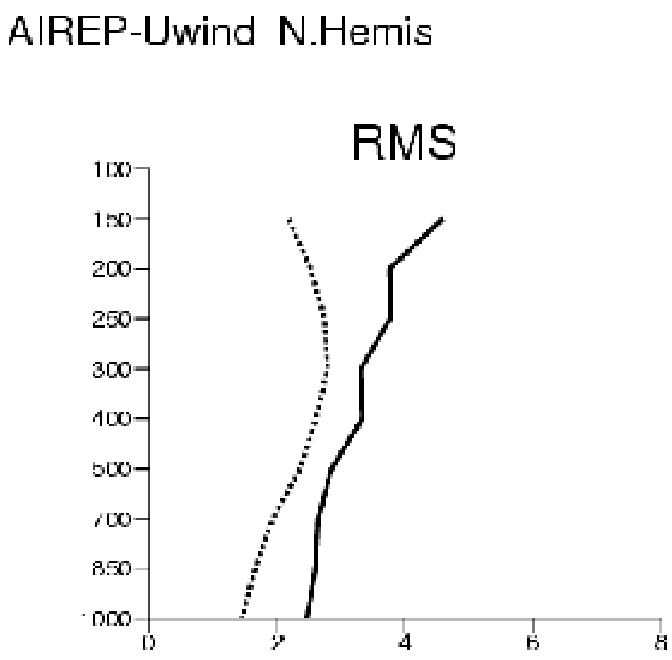
Zonal-mean U-component HBH^T (m/s)



Comparison with innovation statistics
Wind: radiosonde and aircraft





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Jb modelling developments

- Base Jb statistics on an ensemble of 4D-Var assimilations, rather than the lagged forecast (NMC-) method which uses 48-24 hour forecast differences.
  - Forecast errors become more large-scale (both vertical and horizontal) with time. Wind errors grow large particularly in upper troposphere.
  - Ensemble spread provides a more direct estimate of errors in short-range forecasts (M. Fisher).
- Allow vertical correlations to vary with horizontal wave-number, and horizontal correlations to vary with vertical level (non-separability).
  - This is a prevailing feature of short-range forecast error (Phillips 1986; Courtier et al., Andersson et al., Rabier et al. 1998)

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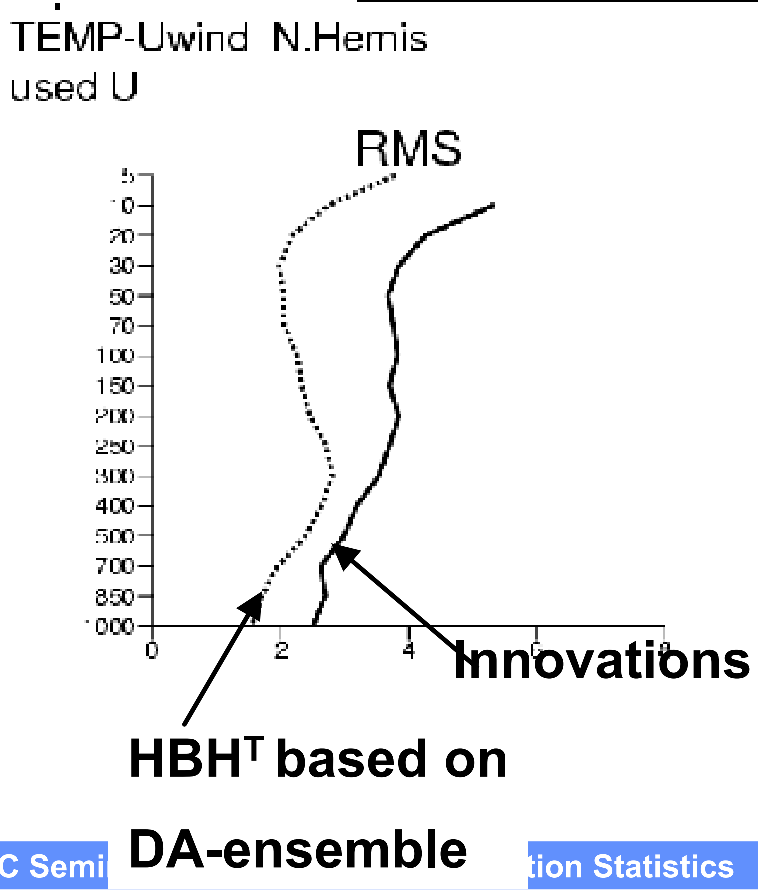
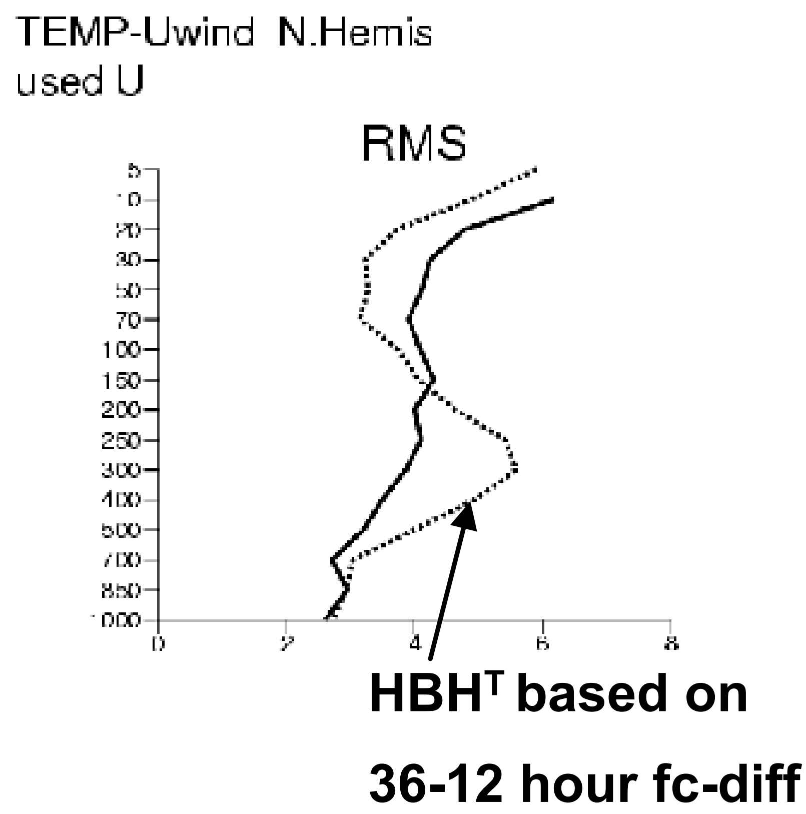
Jb modelling developments

DA-ensemble vs ‘NMC-method’

	Ps (hPa) N.Hem
Innovations	0.92
HBH^T DA-Ensemble	1.03
HBH^T NMC-method	3.83





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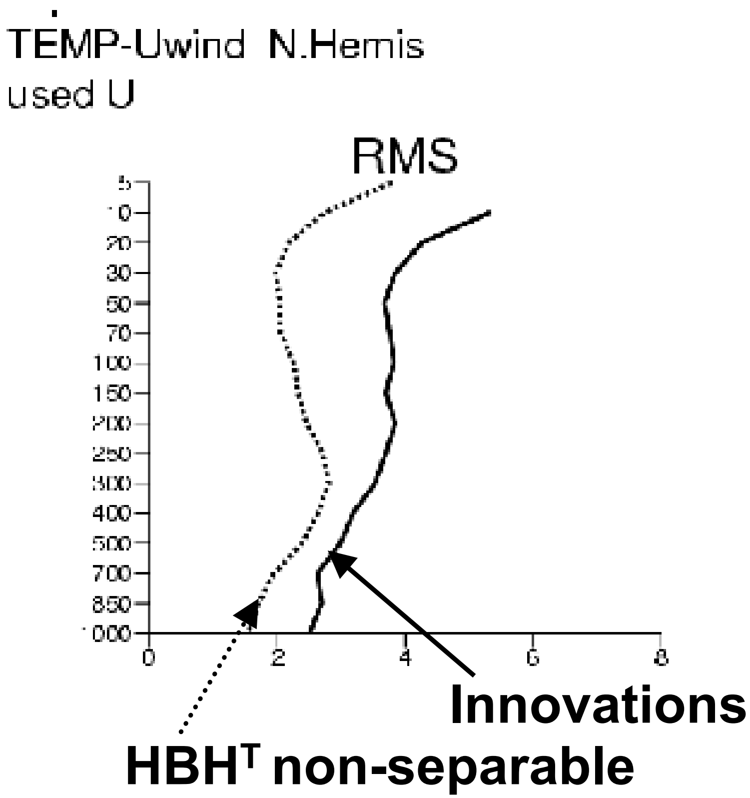
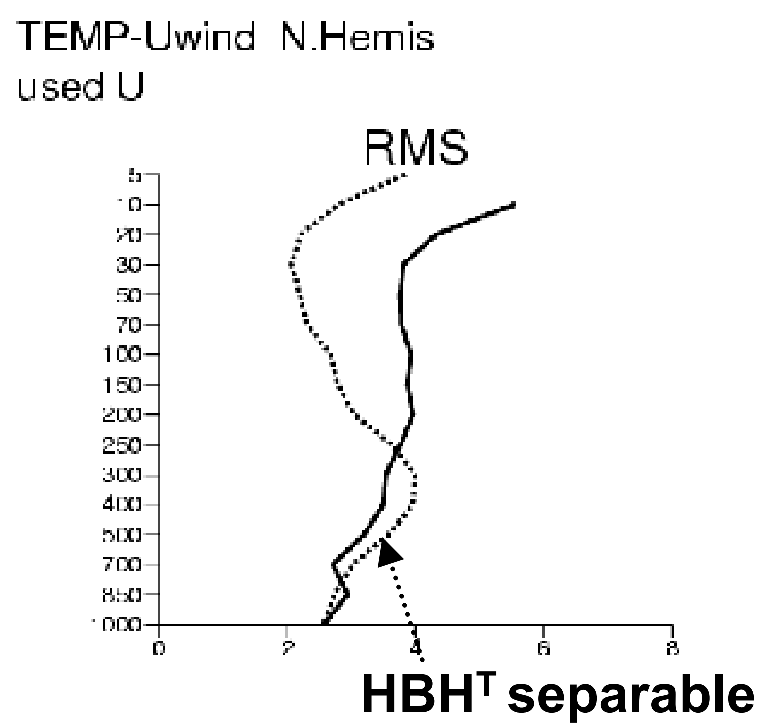
Jb modelling developments
Non-separable vs separable

	Ps (hPa) N.Hem
Innovations	0.92
HBH^T Non-Sep	1.03
HBH^T Sep	1.76





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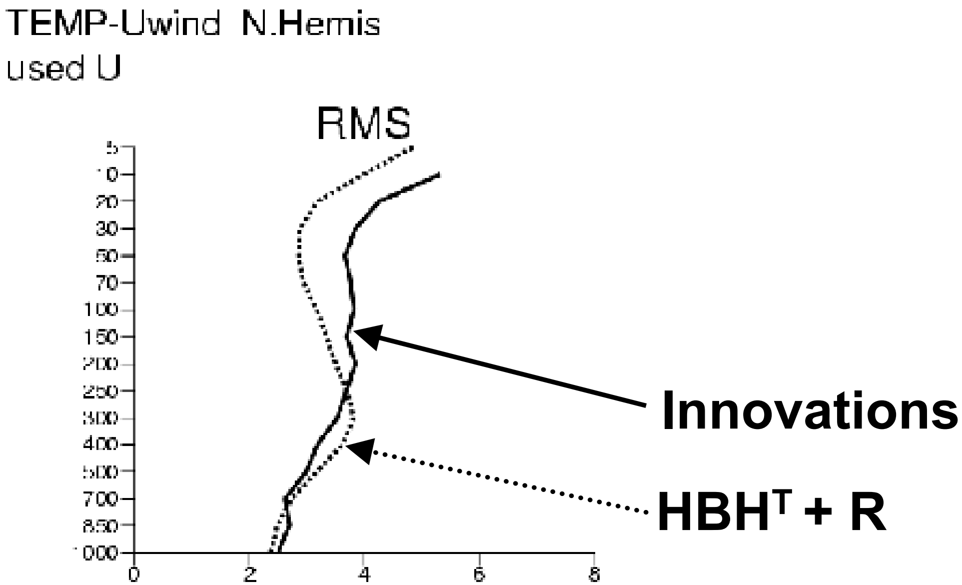
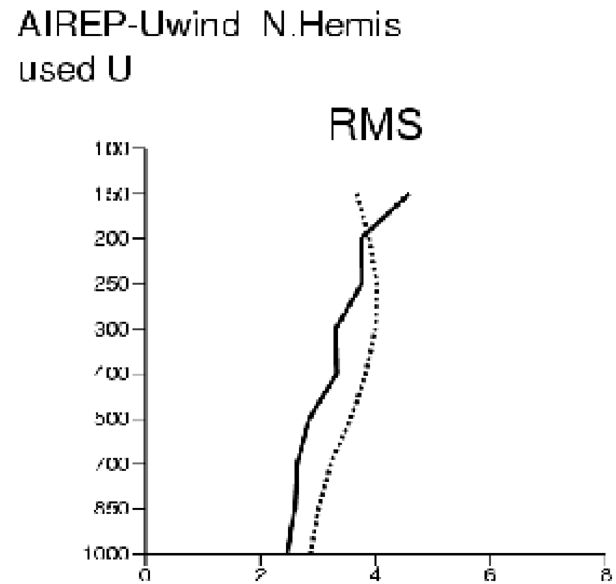
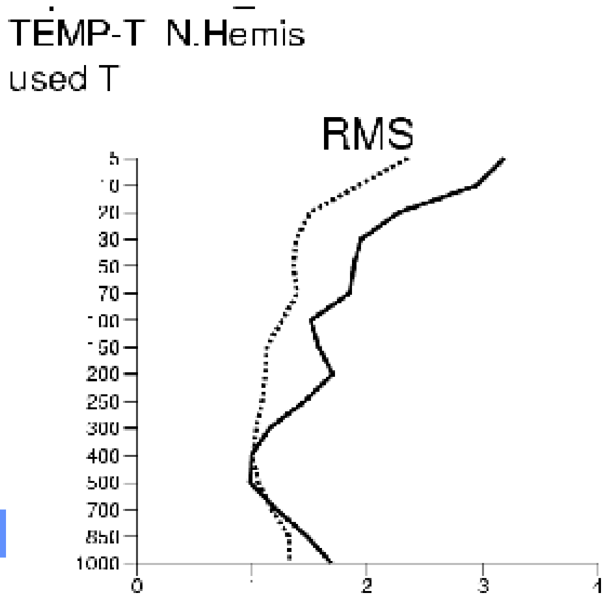
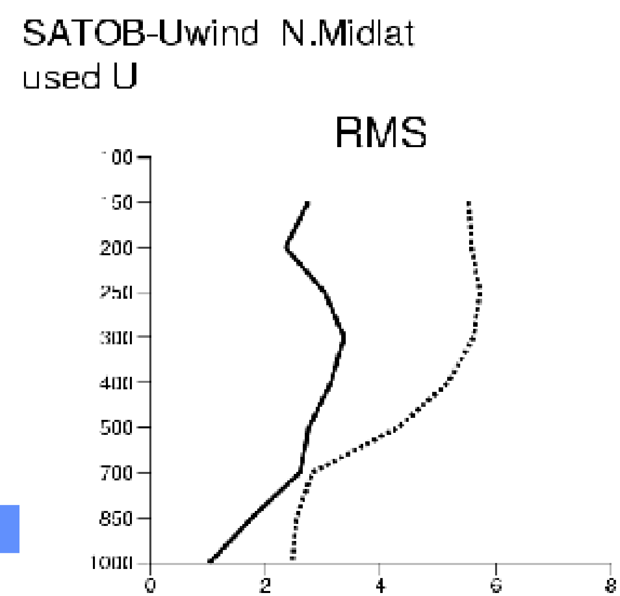
HBH+R
for a range of different data type



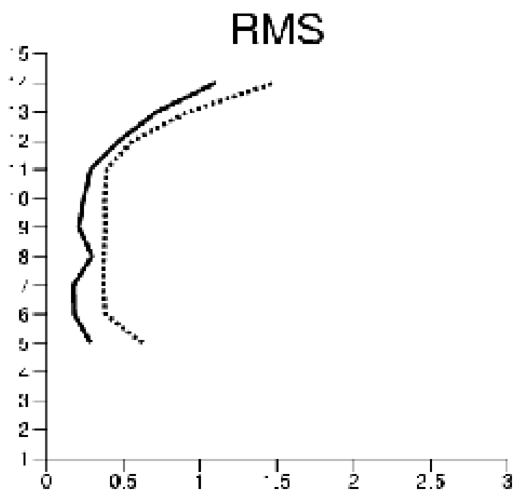




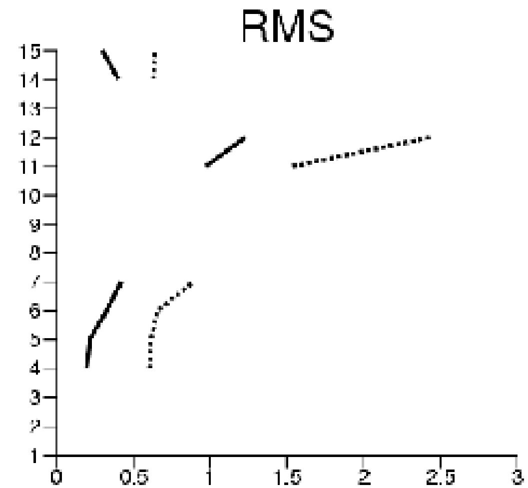


HBH+R
for a range of different data type

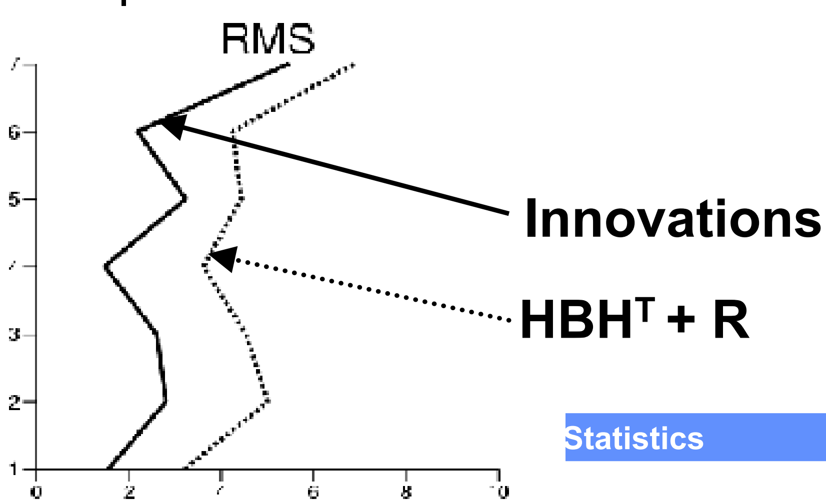
TOVS-1C noaa-16 AMSU-A Tb N.Hemis
used Tb noaa-16 amsu-a



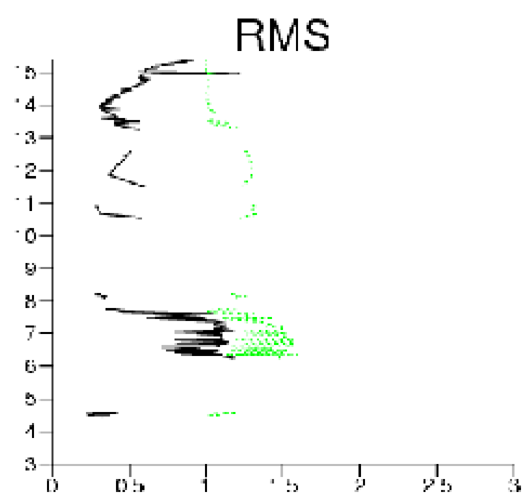
TOVS-1C noaa-16 HIRS Tb S.Hemis
used Tb noaa-16 hirs



SSMI-1C dmsp-13 SSMI Tb N.Hemis
used Tb dmsp-13 rad



AIRS NASA-1 Tb S.Hemis
used Tb nasa-1 AIRS



Observation error correlation

Has most recently been studied by Bormann et al. (*MWR* 2003) and Liu and Rabier (*QJ* 2002).

SATOB (or AMV=Atm. Motion Vectors) and radiosonde co-locations were studied, and error correlation functions were fitted:

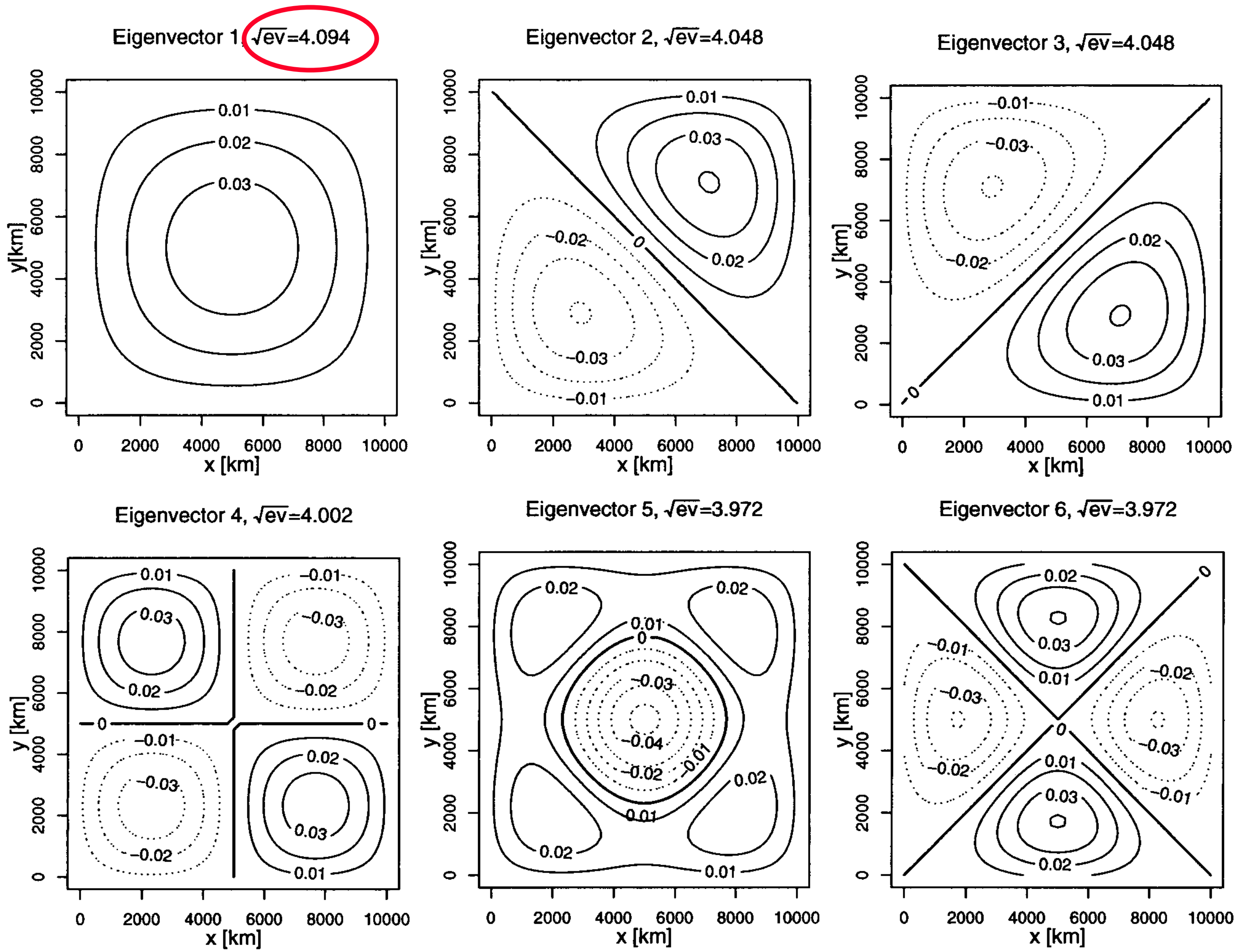
$$R(r) = R_0\left(1 + \frac{r}{L}\right)e^{-r/L}$$

With R_0 the intercept and L the length-scale.

Found L=190 km.

Eigenvectors of R were computed for an idealized observational data set with regular 200 km spacing.

Eigenvectors of R
for idealized observational dataset



L=190 km.

200 km spacing.

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Summary and Conclusions (1)

- A method to diagnose the modelled evolution of background error within 4D-Var has been developed.
- The modelling of innovations within 4D-Var has been studied, and compared to actual innovation statistics.
- Discrepancies can be due to deficiencies in the specification of **B, R, H, M or Q.**

- The evolution of MBM^T is not as expected.
- There seems to be insufficient projection of B onto growing modes – I,e. there is insufficient flow-dependence in B.
- Comparison with EnKF could be performed by replacing the vectors $\delta \mathrm{x} = \mathrm{L}\chi$ with vectors obtained from ensemble differences. Also for DA-ensemble.

Summary and Conclusions (2)

- It has not been possible to identify the model-error Q contribution to the innovations, at this stage.
- Observation errors are specified far too large for many satellite data types.
- Taking account of observation error correlations within 4D-Var, would now seem important.

- DA-ensemble has provided noticeable improvement over the lagged-forecast (NMC) method.
- The separability assumption (in B-modelling) is not appropriate for joint analysis of stratosphere + troposphere.

Summary and Conclusions (3)

- There is insufficient regional variation in B. Tropopause height and Boundary Layer height variations are poorly represented. Wavelet-Jb.
- Current T, RH (and Z) BgErrors show marked flow dependence. This should be validated against innovations.
- The rapid baroclinic error growth within the 12-hour assimilation window is currently underrepresented, probably due to the (relatively) static nature of B.