LOCAL WEATHER ELEMENT GUIDANCE FROM THE ECMWF WEATHER FORECASTING SYSTEM IN THE MEDIUM RANGE: AN INTERPRETATION STUDY

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· 1. INTRODUCTION

Until now the statistical interpretation of numerical model output has been applied mainly to short range forecasts. Some national weather centres have an operational system based on Model Output Statistics (MOS) or the "Perfect Prog" (PP) method to produce forecasts for many individual stations (Cater et al, 1979; Finizio, 1982; Kruizinga, 1982; Rousseau, 1982). The data base for the development of the statistical equations is usually very large as short range numerical forecasts have been performed for quite a long time.

Medium range forecasts have been carried out on a daily basis at ECMWF for a comparatively short period. The special archive at the Centre (covering the European area with data from forecasts up to 10 days) goes back to only 1 March 1981. The contents of this archive is documented in the ECMWF Meteorological Bulletin M 1.9. The software offered by the ECMWF to process this data makes it comparatively easy for the user to carry out interpretation studies on single stations.

The major target of this study is to show how the data handling system provided by ECMWF has been used for statistical interpretation. The results can only be very preliminary as the forcast database is not large enough. However, we will try to gain some indications of what benefits we can expect from the application of statistical interpretation to medium range forecasts.

2. DATA AND METHODOLOGY

One of the most important parameters in local weather forecasting is the temperature at $2\ m$. The problem tackled here is the production of a $5\ day$

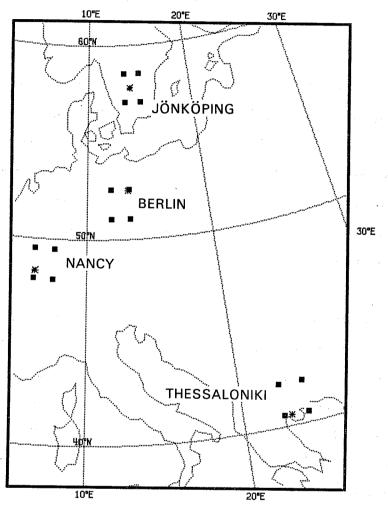


Fig. 1 Stations used in the interretation studies. Location of the station (π) and the four surrounding gridpoints (\blacksquare) of the "European archive grid" (1.5° x 1.5°).

forecast of the 2 m temperature. The statistical interpretation will be carried out for the winter and summer season, and for a small number of stations in Europe. The locations chosen for the winter studies (Fig.1) are Jönköping, Berlin, Nancy and Thessaloniki; for the summer studies the stations are Berlin and Nancy only.

The derivation of the 2 m temperature in the post-processing of the forecast data is explained by Louis (1982). When we verify this direct model output parameter against the observed 2 m temperature, we must keep in mind that the observed vertical temperature gradient at low levels is not used by the analysis. Below 850 mb the model contains 4 sigma levels apart from the surface σ =1.0. In the data assimilation cycle the model produces a first guess low level temperature profile which is very much dependent on the parameterization of physical processes and on the formulation of the tendency equation for the surface temperature. The analysis then adds only an even distributed increment to the mean temperature in the layer 1000 to 850 mb.

Serious problems for the application of statistical techniques may arise from changes in the numerical forecast model. This will affect forecasts in the medium range far more than in the short range (Glahn, 1982) making it more difficult to derive stable equations.

There were some modifications to the ECMWF forecasting system between the winters 1980/81 and 1981/82 which could cause the kind of problem mentioned above. The most important change was probably the introduction of a more realistic topography in April 81. At the same time the surface fluxes for stable conditions were changed. Model modifications like these may have changed the forecast error pattern. It is therefore interesting to see if the model errors in the two winters 1980/81 and 1981/82 were different for the observation station near Jönköping. The scatter diagram (Fig.2a and 2b) of the 2 m temperature (observed against direct model output) shows that the

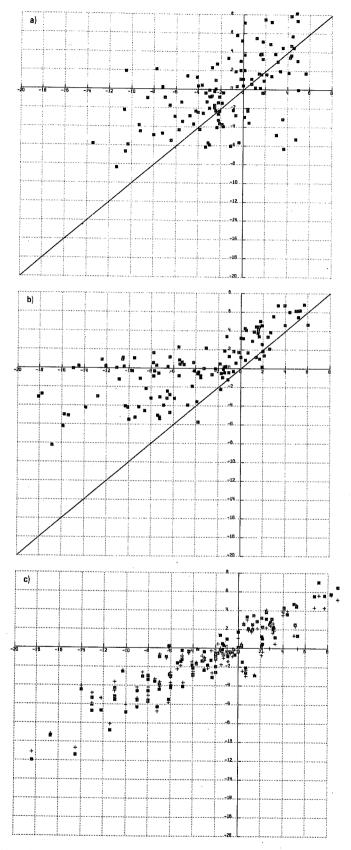


Fig. 2 Scatter diagram for the 2 m temperature (C) at Jönköping

- a) observed (abscissa) against direct model output (ordinate) at D+5 for winter 1980/81
- b) the same as a) but for winter 1981/82
- c) observed (abscissa) against initialised analysis (2) and observed (abscissa) against initialised model surface temperature (+)

model errors in both winters are similar. The model is not able to forecast very low temperatures, an error which can also be found in the initialized analysis (Fig.2c). As we have pointed out, this does not necessarily mean that the analysis itself is the cause since the model influence (via the first guess in the data assimilation cycle) on the low level temperature is very strong.

The positive bias of the initialised 2 m temperature at Jönköping where the surface is covered in snow, draws our attention to the surface temperature which has almost the same bias (Fig.2c). Over land a thin soil layer of specified heat capacity is defined in the model which exchanges heat and moisture with the atmosphere and with the deep soil. From recent investigations (private communications with J.F.Louis) it seems that the thin upper soil layer assumed for snow covered land is too deep. Thus the surface temperature in the model will fall only slowly when strong radiative cooling or cold air advection is present. This is in contrast to observations which show that snow cover prevents almost any exchange between the soil and the surface above the snow.

If we now look into the domain of positive temperatures of the scatter diagrams (Fig.2a and b), we see that the distribution in the last winter has sharpened; this should be beneficial for statistical interpretation.

To get a reasonably long period for the derivation of the regression equations, the definition of the winter season has been extended to include the months from November to March. The dependent database contains six months, from 1 December 1980 to 31 March 1981 plus 1 November to the 31 December 1981. Three months (January, February and March 1982) are then left as independent data to compare the direct model output with the regression equations results.

The summer cases are taken from the periods are 15 May to 15 September 1981 for the dependent data, and 15 May to 15 August 1982 for the independent set.

The list of predictors offered to the regression program, Table 1, contains firstly, the direct model output parameters, and secondly, derived predictors like the windspeed and the thickness. To derive the forecast equations both the forward stepwise regression described by Draper and Smith (1966) and the optimum regression (Grönaas, 1981) were used. But there were only minor differences in the result in the sense that the equation selected by the forward regression was among those 5 best relations found by the optimal regression.

3. RESULTS

3.1 MOS technique, effect of area and time averages

In the following we will be interested in the kind of predictors that are selected when we apply the multiple regression to the longer (dependent) data set. The explained variance of the first predictor and the increase of the explained variance when additional predictors enter the regression equation will be interpreted as a measure of the quality of our derived equation on dependent data. The real independent test will then be made by applying the derived predictor equation to the dataset which is not part of the development sample. In this test we will compare the error statistics of the direct model output with those calculated from the predictor equation. In both cases errors are defined as deviations from observations. First we will concentrate on one station and show the kind of influence that different time and space means can have on our results.

Table 1 List of predictors

Z,T,u,v,w at 1000, 850, 750, 500 mb

Large scale rain, convective rain, snowfall, cloud cover

b) Derived from direct model output

ΔZ 500/1000 mb

 $\begin{vmatrix} \dot{v} \\ v \end{vmatrix}$ 1000, 850, 700, 500 mb and 10 m

c) Sine and cosine of the day of the year

Forecast time: 96, 108, 120, 132, 144 hours

Table 2 Predictor selection for Jönköping from a dataset of 6 winter months with no area mean and no time mean. Predictand 2 m temperature, verification time 120 h.

	Predictor	Explained variance
1.	T _{2m} (120h)	0.29
2.	U ₅₀₀ (108h)	0.34
3.	T ₇₀₀ (120h)	0.36
4.	v ₅₀₀ (120h)	0.38

Jönköping, the first station selected for the interpretation study, is situated in the south of Sweden (Fig. 1) well away from the coast. Despite the problems in calculating the surface temperature for snow covered soil, which has a pronounced effect on the low level temperatures, the first predictor selected is the direct model output 2 m temperature (Table 2). The second predictor increases the explained variance from 29 to 34 per cent, whereas the increase by the third and fourth predictor is much smaller. Hammons et al (1976) examined a dataset containing 410 events for a winter season (compared to 180 available here) and found that the first three predictors usually accounted for most of the explained variance of the temperature. In our experiments where different area and time means were applied, a very stable predictor selection was found. The 2 m model temperature always entered the equation first and the zonal wind component second. The most frequent third predictor was the temperature at 1000 mb. After that, very different predictors were selected in the fourth place; this supported our findings that when these predictor equations were applied to independent data, the largest improvement on the model ouput was produced with 3 predictors. The explanation for having an optimum equation with only 3 predictors in the equation is probably associated with the rather short dependent data set.

In our first set of experiments, the effects of using various area means of the forecast data was tested. The derived statistical equation was applied to the independent data sample and the result was compared with the direct model output (Table 3) for the same period. The area averaging did not change the standard deviation of the model temperature very much, thus indicating that this parameter is already rather smooth. Overall the use of smoothed data to derive the prediction equations improved the error statistics. The only exception was the bias which was least when the equation was derived from the next north western gridpoint.

Table 3 Influence of area mean on statistical interpretation. Verification of one predictor $(T_{\rm 2m}$ direct model output) or a predictor equation (Y) against 2 m temperature observed. Station is Jönköping and verification time is 120 h. Independent data for period 1/1/1982 to 31/3/1982. Units of $T_{\rm 2m}:{\rm C}.$

Area mean over N points	0		4		16	
Time mean over N steps	0		0		0.	
Mean observed	-4.1		-4.7		-4.7	
Standard deviation of predictand	6.5		6.5	*	6.5	
Standard deviation of predictor	3.5	3.4	3.4	3.5	3.0	3.1
Mean error (bias)	5.0	0.4	5.6	1.2	5.4	6.0
Mean absolute error	5.5	4.	6.0	3.9	5.7	4.0
rms error	7.4	5.6	7.8	5.3	7.6	5.3
Explained variance	.31	. 31	,31	.38	.35	.38
	$^{\rm T}_{\rm 2m}$	Y	T_{2m}	×	$\mathrm{T}_{2\mathrm{m}}$	Y

Table 4 The same as Table 3 except that it shows the influences of time mean on statistical interpretation. Units of $\rm T_{2m}:C.$

7	7	. 6	 					0.59 0.64	$^{\mathrm{T}}_{\mathrm{2m}}$ Y
		ļ						0.64	X
4	Ŋ	-2.5	5.2						T.Zm
				3.4	0.4	2.7	3.7	0.56	¥
4	m	-3.0	5.4	3.2	4.0	4.2	5.5	0.53	T_{2m}
					1.2		5.3		×
4	0	-4.7	6.5	3.5	5.6	0.9	7.8	0.32	T_{2m}
Area mean over N points	Time mean over N steps	Mean observed	Standard deviation of predictand	Standard deviation of predictor	Mean error (bias)	Mean absolute error	rms error	Explained variance	

A substantial improvement in the relationship between observation and forecast was gained by applying a time mean of at least 3 forecast steps (12 hours apart) to both the predictor and predictand. This procedure reduced the standard deviation (Table 4) of the predictand whereas that of the direct model output remained almost the same. From the viewpoint of available model information, a part of the non explainable variance in the observation had been removed. A 3 time level mean compared to no mean improved the reduction of variance significantly. The results for the error statistics became better as well. When we extended the period for the mean to 5 and then 7 steps of 12 hours, we found that the 5 step mean gave the best relationship between observation and direct model ouput. The verification of the derived regression equations on independent data suggests the same conclusion. The best result was again obtained by using an equation developed on a 5 timelevel mean predictor-predictand set. The large positive bias could be corrected completely and the mean absolute error and the rms error was reduced by about 30%.

Looking now at some of the details (Fig.3) of the best result obtained so far (4 point mean/5 timestep mean), we see that the regression equation corrected the direct model output in the right direction for the second and third cold spells which were not captured by the model forecast. In the transition period to spring temperatures (day 55 to day 75) the statistically interpreted values were too low, but towards the end of the period the correction of the forecast was successful again. Fig.4a, 4b show the scatter diagrams for the direct model output and the statistical interpretation results for Jönköping using independent data. The most obvious improvement is in the low temperature forecasts. However there are still a large number of forecasts which are much too warm after the correction. The reduction of the bias for the total sample had the effect of producing forecast temperatures which are too low when the observed temperature was in the range of -4 to +5 degrees.

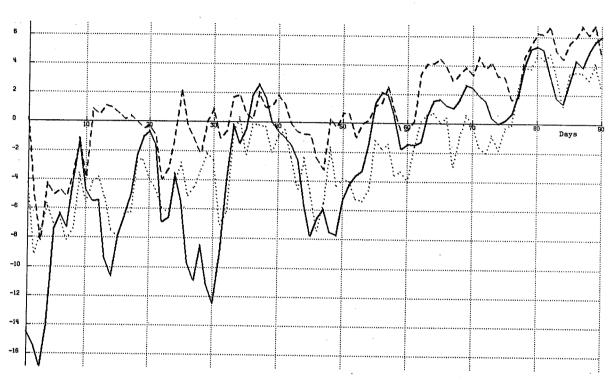


Fig. 3 2 m temperature (C) Jönköping at D+5. Observed (solid line) direct model output (dashed line) and three predictor equation applied on this independent data (dotted line). Period: 1 January 1982 to 31 March 1982.

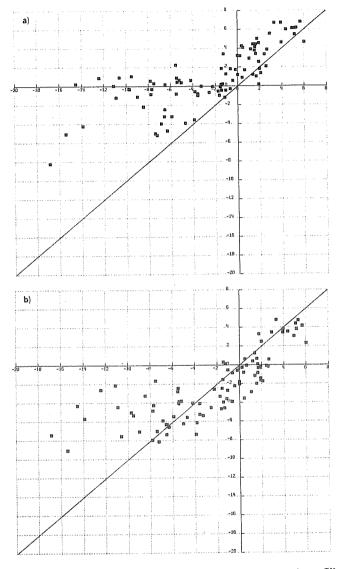


Fig. 4 Scatter diagram for the 2 m temperature (C) in Jönköping. Period: 1 January 1982 to 31 March 1982.

- a) observed (abscissa) against direct model output (ordinate)
- b) observed (abscissa) against a three predictor equation (ordinate) applied on this independent data.

3.2 Winter cases

In the following studies of three further stations in winter (Berlin, Nancy, Thessaloniki) only the most successful time mean (i.e. over 5 steps) was used to derive the forecast equation.

Again in all three cases the first predictor selected was the direct model output 2 m temperature either at the verifying time or 12 to 24 hours earlier (Table 5). For Thessaloniki and Berlin the second and the third predictors are a wind field parameter and the 1000 mb temperature respectively. The order of selection was the same as for Jönköping. Quite frequently the forecast equation contained predictors valid 12 to 24 hours before the verifying time. However, this cannot be interpreted as an indication that synoptic waves are systematically too fast. As a sufficient drop in the skill in the forecast will also lead to a foreward time offset in the selection of predictors; in other words, persistance from earlier forecast steps may be better than the forecast itself.

Compared to the direct model output temperature at 2 m, the statistical equation reduced the bias in all cases (Table 6). The improvement of the mean absolute error and the rms error was rather small and, in terms of explained variance, the statistical interpretation gave rather poor results for Berlin and Thessaloniki.

3.3 Summer cases

For the summer season only two stations, Berlin and Nancy, were chosen.

Compared to Berlin, which has a subcontinental climate, there is a maritime component in the climate for Nancy.

During the summer season fewer occasions with strong inversions are observed.

Therefore the temperature at upper levels should be a good predictor. But for

Berlin the 2 m temperature was still the first predictor (Table 7) and the

Predictor selection for Berlin, Nancy and Thessaloniki for 5 winter months. Predictand 2 m temperature, verification time 120 h. 4 point area mean, 5 timestep mean. Forecast time for which the predictors are valid is given in brackets. Table 5

DNIKI	Explained variance	. 53	. 55	.57
THESSALONIKI	Predictor selected	T _{2m} (120)	U ₈₅₀ (120)	T_{1000} (108)
ICY	Explained variance	.61	. 64	99°
NANCY	Predictor selected	T _{2m} (108)	z_{1000} (108)	T ₁₀₀₀ (108)
NI	Explained variance	0.71	0.73	0.74
BERLIN	Predictor	1. T _{2m} (108)	2. $ \vec{\nabla} _{10m}$ (96)	3. T ₁₀₀₆ 108)
		-	2	3.

Verification of one predictor (T_{2m} direct model output) or a predictor equation (Y) against 2 m temperature observed. Stations are Berlin, Nancy and Thessaloniki and verification time is 120 h independent data for period 1/1/1982 to 31/3/1982. Units of T_{2m} : C. Table 6

1										
THESSALONIKI					2.0		1.7			A
HESSA	5	ιΩ	6.2	က	3.0	3.0-	1.9	2.4	π.	${\rm T}_{\rm 2m}$
H										
									eņ.	
					2.8	1.8	2.9	4.0	. 43	¥
NANCY	4	5	2.5	4.8	3.0	2.5	3.2	4.6	.36	$^{\mathrm{T}}_{\mathrm{2m}}$
					3.7	1.6	2.4	3.1	.77	×
BERLIN	4	ເວ	1.4	5.3	3.5	2.2	2.7	3.5	62.	$^{\mathrm{T}}_{\mathrm{2m}}$
BER					က	63	87	က		H"
				Standard deviation of predictan	Standard deviation of predictor					
	nts	S.C.		f pre	f pre					
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			pe,	viati	viati	(bias	te er		arian	
	ean c	ean c	bserv	rd de	rd de	rror	psolu	ror	ned v	
	Area mean over	Time mean over	Mean observed	tanda	tanda	Mean error (bias)	Mean absolute error	rms error	Explained variance	
	7		~	ω	ν <u>α</u>	X	E	H	国	

850 mb temperature entered the equation only as the second predictor.

The summer dataset for Nancy is an exception to all cases considered so far.

The 2 m temperature of the model was not selected at all; instead the geopotential height entered the equation first. This is also in contrast to the predictor selection for the same station in the winter season. The correlation between the observed temperature and all predictors shows that the geopotential height always has a better correlation than the temperature from any pressure level offered to the regression. The explanation may be that the temperature field at Nancy in the model may have been affected by mountain gridpoints as the peaks of the Alps in the model are only one gridlength away. In reality the major influence of the Alps will be more confined to the mountainous region itself. To confirm the explanation for the unusual predictor selected in this case, more stations near large mountains should be investigated.

When the predictor equation for Nancy was applied to independent data the low skill 2 m temperature forecast for this station improved substantially (Table 8). This is not true for Berlin where the statistical interpretation was not able to add anything to the high skill of 2 m the temperature forecast.

3.4 Perfect prog and modified MOS technique

A typical winter forecast error, such as too high temperatures near the surface, may be traced back to the formulation of the models surface exchange processes in the data assimilation cycle. A typical example of this is Jönköpping (Fig.2c) where the forecast model has introduced a large positive bias for low temperatures. As the correlation between analysis and observation is usually very high, a regression equation derived from the analysis and observation data sample will correct a large amount of this bias. There are now two ways to use this equation on sample forecasts. In the usual

Table 7 Predictor selection for Berlin and Nancy with a dataset of 4 summer months. Predictand 2 m temperature, verification time 120 hours. 4 point area mean, 5 timestep mean. Forecast time for which predictors are valid is given in brackets.

	BE	RLIN	NAN	CY
	predictor selected	explained variance	predictor selected	explained variance
1.	T _{2m} (120)	0.51	Z ₅₀₀ (108)	0.46
2.	T ₈₅₀ (108)	0.54	v 850 ⁽⁹⁶⁾	0.47
3.	v ₅₀₀ (96)	0.55	v 850 (108)	0.49

Table 8 Verification of one predictor (T_{2m} direct model output) or a predictor equation (Y) against 2 m temperature observed. Stations are Berlin and Nancy and verification time is 120 h. Independent data for period 15/5/82 to 15/8/82.
4 point mean, 5 timestep mean. Units for T_{2m}:C.

	BER	LIN	NAN	CY
Mean observed	19.3		21.0	
Standard deviation of predictand	3.7		3.2	
Standard deviation of predictor	3.1	3.7	2.7	2.0
Mean error (bias)	0.0	0.0	-2.5	-1.6
Mean absolute error	1.6	1.7	3.0	2.4
rms error	2.0	2.1	3.8	3.0
Explained variance	•72	. 68	•29	-40
	T 2m	Y	T 2m	Y

"Perfect prog" method, this equation would be directly applied to later forecast steps (Glahn, 1982). Generally this procedure will lead to an improvement of the direct model output only if the data sample for deriving the equation is very long. This requirement is not yet fulfilled by the present ECMWF database.

In a second approach, the combination of predictors in the equation derived from the analysis-observation sample can be offered as an additional predictor to the MOS predictor selection algorithm. The MOS-type statistical interpretation procedure is thereby only slightly modified by offering a predictor derived from the existing ones. Though the data sample to develop forecast equations is certainly not yet long enough for real "Perfect prog" studies, the two procedures proposed above have been tested. Instead of being developed on the analysis, the "Perfect prog" equation was developed on a set of predictors valid for a 24 hour forecast representing time mean values of 3 timesteps (12,24,36 hours). The first predictor selected from this dataset was the 2 m temperature giving an explained variance of 81%. In second and third places followed the 1000/500 mb thickness and the windspeed at 850 mb respectively. When this equation derived from 24 h forecast data is applied to independent forecast data valid for 120 hours (labelled PP in Table 9) we find an overall improvement in the error statistics, apart from the explained variance which is lower compared to the direct model output 2 m temperature.

The second approach, i.e. offering to the MOS procedure an addition predictor consisting of the combination of variables occurring in the 24 hr "Perfect prog" equation, led to an equation which gave nearly the same results on independent data as the direct Perfect prog method. Both techniques, however, do not perform as well as the "pure" MOS approach for this special station. As mentioned above the result would probably have been better for a larger dependent data sample.

Verification of one predictor (T_{2m} model output) or a predictor equation (Y) against 2 m temperature observed. (a) MOS-technique, (b) Perfect prog, (c) combined Perfect prog-MOS. Station is Jönköping and verification time 120 h. Independent data for period 1/1/82 to 31/3/82. Units T_{2m} :C. Table 9

Mean observed	-2.5	3.4		
Standard deviation of predictand	5.2			
Standard deviation of predictor	3.1	3.4	3.4	3.4
Mean error (bias)	3.5	0.2	1.1	89.
Mean absolute error	3.6	2.3	2.2	3.0
rms error	4.9	3.2	4.0	4.0
Explained variance	.56	.64	.54	.41
	$^{\mathrm{T}}_{2\mathrm{m}}$	×	×	X
		MOS	дд	MOS-PP

3.5 Dependency of interpretation results on the forecast range

Up to now we have concentrated on the statistical interpretation of a five day forecast. In this section we will examine the performance of the statistical interpretation over a wider forecast range. So far we have split the available data for winter months into dependent and independent data. Here we will merge all months to one dataset to see what amount of explained variance on independent data we can get when we use the longest possible time series. In this procedure we will allow 3 predictors to enter the regression equation. The improvement of the explained variance by the 3 predictor equation over the direct model output will then be interpreted as a maximum which could be achieved on independent data.

Two stations, Jönköping and Berlin, have been chosen; for these the upper level dynamical forecast has almost the same skill but the low level temperature forecast is very much better for Berlin. As shown before, the 2 m temperature forecast at Jönköping is influenced by a positive bias in the models surface temperature due to problems of snow cover in rather cold regions like central Sweden.

Fig. 5 depicts the explained variance of the direct model output and that of the predictor equation as a function of the forecast time, both for Jönköping and Berlin. The amount by which the explained variance of the 3 predictor equation is better than the direct model output has a maximum at D+5 for Jönköping. The difference is smaller at D+3 when the skill of the forceast is comparatively high and becomes smaller again towards D+7 when the skill of the forecast is very low. Beyond that the skill of both types of forecasts for Jönkoping levels off to rather low values. For Berlin the 3 predictor equation adds only a small percentage to the existing high level of explained variance. But the added amount increases slightly with advancing forecast time.

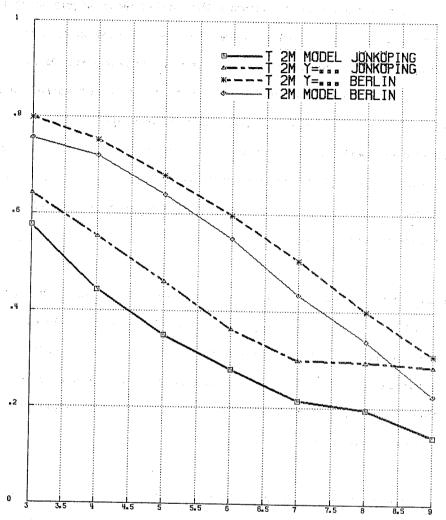
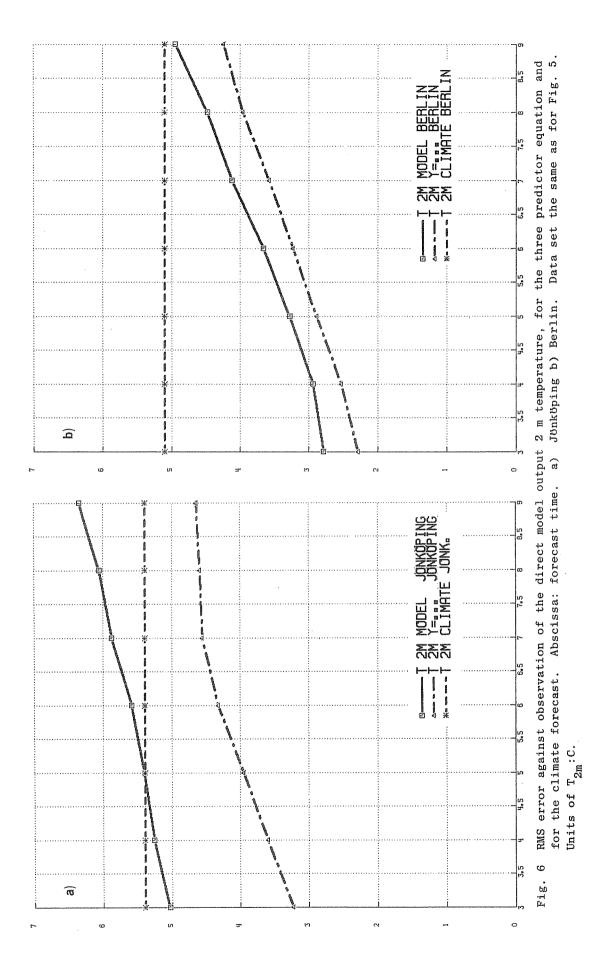


Fig. 5 Explained variance of the observed 2 m temperature by the direct model output 2 m temperature (first and third graph from below) and explained variance by the first three predictors in the regression equation (y = ...) second and fourth graph from below. Abscissa: forecast time. The datasets for the two stations Berlin and Norrköping contain 8 months from winter 80/81 and 81/82.



In terms of rms error (Fig.6), the gain by statistical interpretation can be substantial. For Jönköping, the direct model output error becomes as large as the error of the climatological forecast at D+5, whereas the rms error of the 3 predictor equation stays well below that level. Again for a high skill 2 m temperature forecast like Berlin, the improvement by statistical interpretation on the rms error is comparatively small.

4. SUMMARY AND CONCLUSIONS

The time series available from medium range weather forecast are still too short to develop equations for statistical interpretation with a large number of predictors. For a selection of 4 European stations it was found that more than 3 predictors in the regression equation gave an overfitting of the data leading to a poor verification when applied to independent samples. The reason for that need not be the short development sample only; it seems also to be connected to the high quality of the direct model output 2 m temperature. This parameter turned out to be the best predictor in almost all cases examined here and indicates the usefulness of the model derived 2 m temperature. Changes in the model formulation could have a large effect on interpretation results and should be carried out very carefully. But so far model modifications seem to have no serious effect on the statistical interpretation.

In the verification of the regression equations on independent data the most successful improvement could be gained by correcting the model bias. In most cases the mean absolute error and the rms could be reduced as well. However in terms of reduction of variance, the interpretation result was sometimes inferior to the direct model output.

The usefulness of a MOS type interpretation for medium range weather forecast can not be decided now - it has to wait for a larger sample of forecasts. A Perfect prog method based on analysis could be developed now, but then a

very useful parameter like the discrete model output 2 m temperature would not be available in the development sample.

We can expect a large variation in the performance of the low level temperature forecast even for stations where we have nearly the same quality of upper level forecasts. The skill of the 2 m temperature forecast will very much depend how the model is able to reproduce the local climate near the surface. From the results presented here, it seems that the local statistical interpretation can improve the 2 m temperature forecast by a large amount where local climatology is not well simulated by the forecast model. The amount we can gain in explained variance by using a predictor equation instead of the direct model output 2 m temperature appears to increase with advancing forecast time well into the medium range. For a high skill forecast the gain will be less than for a low skill forecast.

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