METHODS AND PROBLEMS OF FINITE DIFFERENCE REPRESENTATION OF MOUNTAINS IN NUMERICAL WEATHER ANALYSIS AND PREDICTION

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Summary

This note attempts to give a brief outline of the existing methods and problems of the representation of mountains in finite difference models for numerical weather prediction, as well as of the associated problems in data analysis and initialization. The emphasis is on providing guidance to the bibliography on these subjects.

1. Finite difference horizontal and vertical representation of mountains in numerical models

There exists a variety of possibilities for choosing the vertical coordinate (e.g. Kasahara 1974, 1977a), and each is associated with its own problems of representing topography.

A number of numerical models uses coordinates that intersect the earth's surface. Some of these coordinates are time-dependent, e.g. isentropic coordinates (Eliassen and Raustein 1968, 1970), or pressure coordinates. Fixed geometric height has also been used as the vertical coordinate, notably in the NCAR general circulation model (Kasahara and Washington 1969, 1971; Oliger et al. 1970).

With such coordinates usually differencing schemes are used which require vertical extrapolation to obtain fictitious subterranean variables (Bleck 1974; Shapiro 1975; Trevisan 1976). An alternative is to physically block the flow. This presents inconveniences if the coordinate surfaces are moving (Katayama et al. 1974), and/or may be associated with difficulties in trying to make an efficient use of fast parallel processor or pipeline computers (Washington and Williamson 1977). It would appear, however, that even apart from this point of "vectorization" the experience with the treatment of mountains in the NCAR z coordinate model was not entirely satisfactory. It is possible that better results would have been obtained by use of a space staggered grid, since it would require no space-uncentered differencing and no fictitious subterranean variables. The blocking method is the standard method of incorporating terrain effects in ocean models.

In atmospheric prediction models by far the most frequent choice is the so-called sigma system (Phillips, 1957), in which the earth's surface is always a coordinate surface. The situation is essentially the same if a "transformed coordinate" system is chosen with some other coordinates following the ground surface.

The pressure gradient force in the sigma system takes the form

$$- \nabla_\sigma \Phi - \sigma\alpha\nabla p_s \cdot \qquad (1)$$

The sigma coordinate is here being defined as

$$\sigma \equiv (p-p_T)/(p_s-p_T),$$

with p_T denoting pressure at the top of the model atmosphere, p_S pressure at the surface, and other symbols having their usual meaning.

Over sloping terrain the two terms in (1) may in absolute value individually be more than an order of magnitude greater than their sum. A relatively small error in one of these terms, therefore, can result in a large error in the sum. Thus, great care is needed if the computation of the pressure gradient force is to remain realistic over steep mountains. For a dramatic illustration of the problem, note that it seems impossible to achieve an error-free representation of an atmosphere in hydrostatic equilibrium, with no horizontal pressure gradient force. Namely, the two terms of the pressure gradient force will, in general, not cancel. That means that the vertical change in geopotential is partly misrepresented as a horizontal change in the geopotential of a constant pressure surface. Various procedures have been suggested for minimization of this error.

The GFDL group (Smagorinsky et al., 1967) has found it useful to calculate the pressure gradient force on pressure surfaces, after a vertical interpolation from sigma to pressure surfaces. However, this still did not appear to be quite satisfactory (Kurihara, 1968). Corby et al. (1972) proposed an elegant scheme in which an exact cancellation of the difference analogues of the two pressure gradient force terms is achieved when the temperature is a linear function of ℓnp.

The method used by Arakawa consists of maintaining the property of the atmosphere that the vertical integral, with respect to mass, of the pressure gradient force is a potential vector when the ground surface is horizontal. Namely, we have

$$- \frac{1}{g} \int_{p_T}^{p_s} \nabla_p \Phi \; dp = - \frac{1}{g} \left[\nabla \int_{p_T}^{p_s} \Phi \; dp - \Phi_s \nabla p_s \right]. \qquad (2)$$

Thus, only when there is a non-horizontal boundary surface the integral of the left side of (2) along an arbitrary closed curve can be different from zero, resulting in an acceleration of the circulation of the vertically integrated atmosphere by the pressure gradient force. Arakawa has shown (1972; Arakawa and Lamb, 1977) that this property can be maintained in the finite difference analogue of the equations.

The Arakawa method, however, is associated with use of different analogues of the hydrostatic equation for the two terms of the pressure gradient force. Values of geopotential, needed for the analogue of the first term of the right hand side of (1), are calculated using all temperatures of a vertical column of the grid; the second term, giving the difference in slopes of the constant sigma and the constant pressure surface, is calculated locally. It is possible that hydrostatic inconsistency of this kind can lead to serious errors in the vicinity of steep mountains (Rousseau and Pham, 1971; Janjić, 1977).

In a similar way, the method of Corby et al. is also hydrostatically inconsistent. A space centered scheme is used to calculate geopotentials of sigma surfaces, and a forward scheme (in vertical) is used to calculate the correction term (the second term of the pressure gradient force).

The difficulty is, of course, a result of the relatively large hydrostatic variation that is present in sigma surfaces over a sloping terrain (e.g. Sundqvist, 1975a). Therefore, Phillips (1973) and Gary (1973) suggest that a suitably chosen hydrostatic component be removed from each of the two terms of (1). This can be done by defining

$$\Phi(x,y,p,t) \equiv \bar{\Phi}(p) + \Phi'(x,y,p,t),$$

where $\bar{\Phi}(p)$ is a conveniently chosen simple function of p. Substituting this into

$$-\nabla_\sigma \Phi + \sigma \frac{\partial \Phi}{\partial \sigma} \nabla \ln (p_s - p_T), \qquad (3)$$

which is equivalent to (1), one finds that the pressure gradient force can be written as

$$-\nabla_\sigma \Phi' + \sigma \frac{\partial \Phi'}{\partial \sigma} \nabla \ln (p_s - p_T). \qquad (4)$$

The magnitudes of the two terms now being considerably reduced, a higher accuracy of the finite different calculation may be expected.

An additional device that can be used to reduce a specific error or achieve another desirable objective is to have a "layer" (instead of a "level") model (Arakawa, 1972), and then optimize the heights of the velocity components within the layers. Brown (1974) in this way obtained a dramatic improvement in calculating temperatures from the observed geopotentials with the Arakawa pressure gradient force formulation. This procedure has also been used by Phillips (1974), combined with his "reference atmosphere" technique, for a comprehensive study of the Arakawa's method, its possible modifications and integral properties. The resulting schemes of Phillips, the one of Corby et al. and a number of their variants, as well as associated integral constraints, have been further investigated by Nakamura (1978).

A novel approach has been introduced by Janjić (1977). He points out that instead of (1) or (3) the pressure gradient force in the sigma system can be written in a more general form as

$$-\nabla_\sigma \phi + \frac{\partial\phi}{\partial\zeta}\nabla_\sigma \zeta \qquad (5)$$

where ζ is an arbitrary monotonic function of pressure. For hydrostatic consistency, the same function is chosen for vertical differencing in the hydrostatic equation. ζ can now be defined so as to minimize the non-linear part of ϕ (ζ), and this will reduce the error. In the HIBU model (Janjić, 1977; Mesinger, 1977), Janjić has chosen the function $\zeta = \ln^2 p$.

The schemes of Arakawa, Corby et al. and Janjić, and some variations of the scheme of Janjić, have recently been compared, in a number of ways, by Lipovscak (1979). For the profile $\phi(p)$ used in the note of Phillips (1974), he obtains smallest errors with the Janjić scheme, and largest errors with the Arakawa scheme.

However, even the hydrostatically consistent schemes, such as those of Janjić and that used in the ECMWF model (Burridge and Haseler, 1977), as can be seen from an example given by Janjić (1977), have a consistency criterion. It can be written as

$$(\delta_s \phi)_\sigma \, \Delta s \leqslant \delta_\sigma \phi \, \Delta\sigma \qquad (6)$$

where Δs stands for Δx or Δy. Thus, increasing the steepness of model mountains, and increasing the vertical resolution, may lead to a violation of the hydrostatic consistency of the scheme.

A combination of the sigma system and a blocking procedure to deal with steep wall-type mountains has been proposed by Egger (1972a, 1972b, 1974), and used for studies of lee cyclogenesis.

One disadvantage of the sigma system are large errors away from mountains, at the tropopause level and in the stratosphere. They can be eliminated by changing to quasi-horizontal (e.g. pressure) coordinates above a given level, for example as done by Arakawa and Lamb (1977).

For several of these schemes (Kurihara 1968; Arakawa 1972; Corby et al 1972; Miyakoda 1973; Gilchrist 1975; Corby et al 1977; Arakawa and Lamb 1977; Janjić 1977) an associated procedure to ensure consistency in transformation between the kinetic and potential energy has also been developed.

It has been suggested by Arakawa that conservation of potential enstrophy within advection terms may be very important for a realistic simulation of the dynamical effect of mountains. Schemes that conserve potential enstrophy have been constructed by Sadourny (Burridge and Haseler 1977), and Arakawa and Lamb, both for the fully-staggered "C" grid. With the C grid, however, use of the pressure gradient averaging technique of Janjić (1977), eliminating a topographically induced sigma system inconsistency in elevations of the pressure gradient and the Coriolis force, appears not to be feasible.

An improvement in simulation of the barrier effects of mountains can be achieved by a judicious construction of the grid point terrain height values (Mesinger 1976, 1977; Bleck 1977). However, the increase in horizontal resolution appears to be the only ultimate solution to this problem. On parameterization of the effects of still smaller-scale mountain elements very little progress has so far been made (Sawyer 1959; Cressman 1960; Egger 1970, 1971; Miyakoda 1975).

Recent experiments of Rowntree (1978) give a sensitivity test of the barrier effect. With enhanced mountains, doubled in height with less enhancement for larger massifs, the mean 500 mb map was generally more like the observed one for the forecast period.

A number of other computational problems are produced by mountains or related to the possibilities for representation of mountains in prediction models. For example, in defining boundary conditions for a nested grid model, care must be taken not to violate the hydrostatic equilibrium in interpolations from the coarse to the fine mesh (Miyakoda and Rosati 1977). Inside the integration region, a careful treatment of the smallest resolvable scale motion components, e.g. of the propagation of gravity waves between neighbouring grid points if a semi-staggered ("B" or "E") grid is used (Mesinger 1975; Janjić 1974; Mesinger and Arakawa 1976; Janjic 1979), enstrophy conservation in advection terms (Arakawa 1966; Grammeltvedt 1969; Arakawa and Lamb 1977; Janjić 1977; Mesinger 1979), and energy consistency in the thermodynamic equation, will to a large extent eliminate the need for an artifical lateral diffusion and for the smoothing of model topography, and thus will improve the ability of the model to represent smaller scale mountains.

2. Data analysis, initialization and assimilation problems

A non-trivial problem in sigma coordinate models is how to convert the initial geopotential (or temperature) from pressure to sigma system. Two main alternatives appear available. The usual one is to interpolate vertically the geopotential (or temperature) to obtain initial conditions on sigma surfaces. However, in anticipation of the inability of the two-term sigma system representation of the pressure gradient force to reproduce an atmosphere in a hydrostatic balance, it was suggested by Sundqvist (1976) to attempt to reduce the error by performing a vertical interpolation of the pressure gradient force, rather than of geopotential, and then solve for temperature on sigma surfaces. The method has recently been tested by Mihailović (1979). The results show high sensivity of forecasts to the choice of one of these two vertical interpolation methods, existence of technical problems in the non-uniqueness of the solution for temperature, and possible advantage of the Sundqvist proposal. However, the errors in the temperature field may then be larger than in the case of the conventional procedure. When, on the other hand, the geopotential (or temperature) is vertically interpolated it is the pressure gradient force that is subject to an error.

A very difficult problem is posed by the need to obtain initial data in a proper balance with the mountain as it is present in the model. A lack of such balance creates spurious disturbances in the initial stages of the forecast (Egger 1972b; Bleck 1977). It is, of course, possible to solve the balance equation with the mountains present (Holmann 1971; Sundqvist 1973b). A procedure to achieve a more refined initialization with quasi-geostrophic divergence included (Phillips, 1960), has recently been suggested by Kasahara (Kasahara, 1977b; Browning et al., 1978). Some work is under way in this field in the Meteorological Research Institute, and at the Japan Meteorological Agency, Tokyo (Masuda, 1978; Kondo, 1978). It is being attempted to obtain a field adjusted to a model mountain by a numerical experiment, with a view to possibly superimposing the mountain perturbation field when initializing the real data; and to solve the balance equation on pressure surfaces which include holes produced by mountains.

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